Simple and Causal Twisted-Pair Channel Models for G.fast Systems

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Abstract—The use of a hybrid copper and fiber architecture is attractive in both fixed access and mobile backhauling scenarios. This trend led the industry and academia to start developing the fourth generation broadband system, which aims at achieving bit-rates of 1 Gb/s over short copper loops. In this context, accurate models of short twisted-pair cables operating at relatively high frequencies are key elements. This work describes new parametric cable models that incorporate four important characteristics: support of frequencies up to 200 MHz, few parameters, causal impulse responses and require relatively easy fitting procedures. The results show that the models achieve good accuracy for single segments, which are the expected topology for G.fast deployments.

I. INTRODUCTION

ADSL constitutes today the main access technology for fixed broadband, reusing the copper plant from the central office (CO) supporting loop lengths of up to 7 km to the end customer. Gradually, this is now replaced by the VDSL technology, utilizing bandwidths up to 30 MHz over the twisted-pair copper lines. For loops shorter than approximately 1.5 km this gives superior bit-rates compared to ADSL. Due to the increasing bit rate demand, the idea is then to deploy fiber for the part of the telephony loop where many customers share the same cable. The best known connectivity in access networks is achieved by fiber-to-the-home (FttH) taking fiber all the way along individual paths to each customer. Secondly, the cost of fiber deployment, which increases as the fiber termination is moved closer to the customer. Therefore, the final drop of 20-200 m is the most expensive part of the access section, normally meaning digging along individual paths to each customer. Secondly, the cost of installing new fiber plant inside the customer’s premises is higher than utilizing existing wires. If instead the final drop of the existing telephone grid is used to transmit the signals, these inconveniences and expenses can be avoided.

The advantages of these hybrid fiber-copper architectures has led the industry and academia to start developing the fourth generation broadband systems [1], [2]. Since the beginning of 2011, the International Telecomunicações Union (ITU) is working on the G.fast standard [3], which aims at achieving up to 1 Gb/s over very short copper loops. To reach these data rates a bandwidth of up to 200 MHz should be used for signaling. Even though the signal attenuation over twisted-pairs increases with both frequency and cable length, this will be feasible over short loops, i.e. up to 250 meters. In this context, accurate models of short twisted-pair cables for these bandwidths are needed for the development of the G.fast systems.

In the literature there are many parametric cable models described, e.g. [4]–[8], operating over the VDSL spectra up to 30 MHz. It has been seen, e.g. [9]–[12], that these models do not take into account effects that occurs at higher frequencies, and that the direct extrapolation do not lead to a reliable model. The reason is that the traditional models disregard aspects that are now important by assuming, for example, that the shunt conductance is zero. Another important aspect, in which some of the existing models fail, is the causality. When performing e.g. simulations in time-domain, clearly such non-causal models should be avoided.

This work presents three new cable models that incorporate four important characteristics; they are valid for frequencies up to 200 MHz, have few parameters, are causal and have a natural way of fitting to measurements. The models are derived through a generalization of the BT0_H model [8] and adopted to Chen’s model [6]. The models, called the KM models, are based on high-frequency approximations of the propagation constant and the simplest of them is a causal version of Chen’s model, which is convenient when modeling single segments.

The paper is organized as follows. The next section gives a brief review of existing cable models, especially the herein generalized models BT0_H and Chen’s model. The proposed cable models are presented in Section III, and in Section IV the parameters fitting for the KM models is described. Experimental results indicating the accuracy of the proposed models for modeling single segments topologies in both frequency and time-domains are discussed in Section V, and the work is concluded in Section VI.

II. CONCEPTS AND REVIEW OF EXISTING CABLE MODELS

According to classical transmission line theory, e.g. [13], the electrical characteristics of twisted-pair copper cables are determined by the frequency dependent primary coefficients, the series resistance \( R(f) \), series inductance \( L(f) \), shunt conductance \( G(f) \) and shunt capacitance \( C(f) \). These coefficients compose the series impedance \( Z(f) = R(f) + j2\pi fL(f) \) and shunt admittance \( Y(f) = G(f) + j2\pi fC(f) \). From the primary coefficients, one can derive the secondary coefficients,
the characteristic impedance and the propagation constant, as
\[ Z_0(f) = \sqrt{\frac{Z(f)}{Y(f)}} \quad \text{and} \quad \gamma(f) = \sqrt{Z(f)Y(f)}, \]
respectively. Writing the propagation constant as a complex function we get
\[ \gamma(f) = \alpha(f) + j\beta(f) \]
where \( \alpha(f) \) is the attenuation constant and \( \beta(f) \) the phase constant.

In general, \( R(f) \ll 2\pi f L(f) \) above few hundred kHz. Considering cables with low-loss conductors like copper or aluminium, this relation is typically valid from about 100 kHz up to the G.fast frequency range. For instance, it can be verified from the cables described in [9], [11] an increase of one order of magnitude between \( R(f) \) and \( 2\pi f L(f) \) from 100 kHz up to 2 MHz. The relation \( G(f) \ll 2\pi f C(f) \) is also true for most cables (especially for the ones with low-loss dielectric, e.g. polyethylene insulated). In cases where \( R(f) \ll 2\pi f L(f) \) and \( G(f) \ll 2\pi f C(f) \), the characteristic impedance can be simplified to \( Z_0(f) = \sqrt{L(f)/C(f)} \).

Assuming a perfectly-terminated homogeneous line, the transfer function becomes
\[ H(f) = e^{-d\gamma(f)} \]
where \( d \) is the cable length. The causality of the system can be determined from the impulse response, \( h(t) = F^{-1}\{H(f)\} \) and the propagation delay \( T_{pd} = d/s \), where \( s \) is the wave propagation velocity. A system where the impulse response has significant energy during the propagation delay is considered to be non-causal.

To categorize the existing cable models we need to slightly formalize the notation. Furthermore, from now on we will drop the notation of the frequency dependency \( f \) of the functions when it is clear from the contents. A cable model is assumed to be composed by a set \( \Theta = \{\Theta_1, \ldots, \Theta_M\} \) of \( M \) parameters and an associated set \( \Gamma = \{\Gamma_1, \ldots, \Gamma_N\} \) of \( N \) equations. Typically, the equations describe the primary coefficients, such that \( \Gamma = \{R, L, G, C\} \) or the secondary coefficients, \( \Gamma = \{\gamma, Z_0\} \). Alternatively, some models describe only the propagation constant, \( \Gamma = \{\gamma\} \) or the attenuation constant, \( \Gamma = \{\alpha\} \).

For the last drop of the telephony loop, we are typically facing a single cable segment, and then it often suffice to consider one of the simpler models where \( \Gamma = \{\gamma\} \) or \( \Gamma = \{\alpha\} \) which typically contains fewer parameters and, hence, gives a lower computational complexity in the fitting process. The model can then be used e.g. for fitting to measurements, as basis of channel model in simulations or in single ended line testing (SELT) or double ended line testing (DELT) measurements. However, for more complex cable scenarios containing multiple segments of distinct gauges, bridged taps, etc, the classical two-port network modeling is used, for example, to obtain an electrical equivalent described by the \( A, B, C, D \) matrices obtained from the primary or secondary coefficients.

A cable model can be used to study the influence of a given physical parameter (related to the geometry or material) on the electrical behavior (transfer function, etc). In such examples it is useful to have a physical interpretation of all parameters in \( \Theta \), such as conductivity and permeability. These models we will classify as physical, while a model is empirical if at least one parameter does not have a physical interpretation [14]. Typically, extra empirical parameters are incorporated to achieve improved matching between the model results and actual measurements.

When considering the time-domain representation of a model, the causality is another important characteristic. It should be noted that some classical cable models can exhibit non-physically realizable behavior in time-domain because they violate the Hilbert transform relations between the real and imaginary parts of the functions (e.g., \( \{Y, Z\} \), or \( \{\gamma\} \)) in \( \Gamma \).

In Table I at the end of this text a classification of existing cable models is given. It can be noted that the BT0 model [4], that has been adopted by the DSL standardizations, has the largest number of parameters. It is also well documented that this model gives a non-causal impulse response, see e.g. [14], [15]. By calculating the series inductance from the Hilbert transform of the series resistance, the non-causality was corrected in the BT0H [8], with the additional feature of significant reduction on the number of parameters. For clarification we here give the definition of the BT0H model. It should be noted that there are \( M = 4 \) cable dependent parameters, \( \Theta = \{R_0, L_\infty, C_\infty, v\} \).

**Definition 1 (BT0H).** The primary coefficients are given by
\[ R = R_0 Q(L) \quad L = R_0 \frac{\Lambda(L)}{2\pi f} + L_\infty \]
\[ C = C_\infty \quad G = 0, \]
where
\[ Q(\phi) = \sqrt{1 + \phi^2} \]
and \( \Lambda(\phi) \) its Hilbert transform,
\[ \Lambda(\phi) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{Q(\eta)}{\phi - \eta} d\eta. \]

In the sequel we will use a generalized version of the BT0H model to derive a new model for the propagation constant. The calculation is based on a series expansion of (2) and is inspired by Chen’s model [6], which we define here.

**Definition 2 (Chen).** The propagation constant, \( \gamma = \alpha + j\beta \), is modeled by
\[ \alpha = k_1 \sqrt{f} + k_2 f \]
\[ \beta = k_3 f. \]
III. PROPOSED MODELS

As indicated in (5), BT0_H assumes that the shunt capacitance is constant in frequency and that the shunt conductance is zero. This approximation has been seen to be invalid for higher frequencies. In the BT0 model the conductance is \( G = G_0|f|^\alpha \), where \( G_0 \) scales the attenuation over frequency and \( g_c \) provides extra freedom on the curve behavior. However, to limit the number of parameters, we notice from measurements that for polyethylene insulated cables it is reasonable to adopt \( g_c = 1 \). Then we get

\[
G = G_0|f|. \tag{10}
\]

Recalling that the shunt admittance is \( Y = G + j\omega C \), where \( \omega = 2\pi f \), and that the causality restricts the real and imaginary parts through the Hilbert transform, we can derive the shunt capacitance. The Hilbert transform of the absolute value is given by

\[
|f| \xrightarrow{H} f = \frac{2}{\pi}(1 - \ln |2\pi f|). \tag{11}
\]

Benefiting from the version of the Hilbert transform presented in [8], which allows impulses and their derivatives in the time-domain signals, adding a term \( C_\infty \) that is proportional to \( f \) does not imply loss of causality. Hence, with the shunt conductance according to (10), a causal model is achieved if the shunt capacitance is set to

\[
C = G_0 \frac{2}{\pi}(1 - \ln |2\pi f|) + C_\infty. \tag{12}
\]

If instead of the primary coefficients we are interested in the propagation constant, we can rewrite \( \gamma \) in (1) as

\[
\gamma = j\omega\sqrt{LC} \frac{1}{1 + \frac{R}{j\omega L}} \frac{1}{1 + \frac{G}{j\omega C}}. \tag{13}
\]

By the assumption of \( R \ll 2\pi f L \) and \( G \ll 2\pi f C \), we can use the Taylor series expansion \( \sqrt{1 + x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + \ldots \) to get the series expansion of (13). Furthermore, by letting

\[
C_1 = C(f_1) = G_0 \frac{2}{\pi}(1 - \ln |2\pi f_1|) + C_\infty \tag{14}
\]

where \( f_1 \) is the highest used frequency, we can approximate

\[
G = \frac{\text{sgn}(f)G_0}{2\pi C_1} - \frac{\text{sgn}(f)G_0}{2\pi C_1} \approx \frac{\text{sgn}(f)G_0}{2\pi C_1}, \tag{15}
\]

since \( G \ll 2\pi f C \). Next the series expansion in \( j\omega \) of the expression in (13) will be used to derive causal models.

A. First order model

The first order series expansion of the square root expressions in (13) yields

\[
\gamma \approx j\omega\sqrt{LC} \left(1 + \frac{R}{j2\omega L}\right) \left(1 + \frac{G}{j2\omega C}\right) = \frac{R}{2} \sqrt{\frac{C}{L}} + \frac{G}{2} \sqrt{\frac{L}{C}} + j \left(\omega \sqrt{LC} - \frac{RG}{4\omega \sqrt{LC}}\right). \tag{16}
\]

From this we conclude that the real and imaginary part of (2) can be approximated by

\[
\alpha = \frac{R}{2} \sqrt{\frac{C}{L}} + \frac{G}{2} \sqrt{\frac{L}{C}} \tag{17}
\]

and

\[
\beta = \omega \sqrt{LC} - \frac{RG}{4\omega \sqrt{LC}}. \tag{18}
\]

A key aspect of the proposed model is to verify the dependency on \( f \) for \( \alpha \) and \( \beta \), and combine the effect of these coefficients in a sensible way that leads to few but powerful model parameters. To achieve this we start with the real part, (17), of the propagation constant.

First we recall that when \( R \ll 2\pi f L \) and \( G \ll 2\pi f C \) the characteristic impedance, \( Z_0 = \sqrt{L/C} \), can be assumed to be constant over the frequency range. This relation has also been verified from measurements on corresponding cables. From the definition of the resistance in the BT0_H model it can be seen that for high frequencies the resistance is proportional to the square root of the frequency, \( R \propto \sqrt{f} \). Hence, the first term in (17) can be approximated with \( k_1 \sqrt{f} \). Similarly, from (10) we notice that \( G \propto |f| \), and we can conclude that (17) can be approximated with

\[
\alpha = k_1 \sqrt{\left|f\right|} + k_2 |f|. \tag{19}
\]

Interestingly, this is the exact same form as defined in Chen’s cable model in Definition 2. In Chen’s model only positive \( f \) are used since it is used to model the propagation constant \( \gamma \).

Now, to similarly approximate (18) we must also check that the causality condition still holds. First we will do an estimation according to (18), and then we will compare with the results from the Hilbert transform.

The first part of (18) can be rewritten as \( \omega C \sqrt{L/C} \). From (12) together with the above reasoning that \( \sqrt{L/C} \) is constant, we see that \( f \sqrt{LC} \propto f(1 - \ln |2\pi f|) \). The second part can be rewritten as \( \frac{1}{2} R \frac{G_0}{\omega C L} \sqrt{L/C} \), and from the approximation in (15) and that \( R \propto \sqrt{|f|} \) we see that it is proportional to \( \text{sgn}(f)\sqrt{|f|} \). Following the results in [8], we can also add a term proportional to \( f \), without violating the causality. Summarizing, we can approximate (18) as

\[
\beta \propto \{\text{sgn}(f)\sqrt{|f|}, f(1 - \ln |2\pi f|), f\}. \tag{20}
\]

To determine the constants we again consider the Hilbert transform. Apart from the previously used Hilbert pair \( |f| \xrightarrow{H} f \frac{2}{\pi}(1 - \ln |2\pi f|) \), we also use \( \sqrt{|f|} \xrightarrow{H} \text{sgn}(f)\sqrt{|f|} \). Then, we get

\[
\beta = k_1 \text{sgn}(f)\sqrt{|f|} + k_2 \frac{2}{\pi} f(1 - \ln(2\pi f)) + k_3 f = k_1 \text{sgn}(f)\sqrt{|f|} - k_2 \frac{2}{\pi} f \ln |f| + k_3 f \tag{21}
\]

where \( k_3 = k_2 \frac{2}{\pi} \frac{\ln \frac{2}{\pi}}{2\pi} + \tilde{k}_3 \). Hence, we can write the model as in the next definition.
Definition 3 (KM1). The propagation constant, \( \gamma = \alpha + j \beta \), can be modeled by

\[
\alpha = k_1 \sqrt{f} + k_2 f + k_3 \frac{1}{\sqrt{f}} + k_4
\]

\[
\beta = k_1 \sqrt{f} - k_2 \frac{2}{\pi} f \ln f + k_3 f - k_5 \frac{1}{\sqrt{f}}.
\]

In the definition we have dropped the absolute value since \( \gamma \) will be used to model the frequency response, which is only used with positive \( f \). It can be noted that the first part, \( \alpha \) is the same as in Chen’s model, and that the difference in the imaginary part, \( \beta \) is due to the Hilbert transform. Hence, the KM1 model is a causal version of Chen’s model. The abbreviation KM stands for the \( k \)-model, since we have only the three constants \( k_1, k_2 \) and \( k_3 \).

B. Higher order models

In KM1 the first order series expansion of the square root expressions in (13) was used. If instead the second order series expansion is used we can identify

\[
\alpha = \frac{R}{2} \sqrt{C} + \frac{G}{2} \sqrt{L} + \frac{R^2G^2LC}{16k_2L^2C^2}
\]

\[
\beta = \frac{R}{4} \omega \sqrt{LC} - \frac{RG}{4 \omega \sqrt{LC}} + \frac{R^2 \sqrt{LC}}{8 \omega L^2} + \frac{G^2 \sqrt{LC}}{8 \omega C^2} + \frac{R^2G^2 \sqrt{LC}}{64 \omega^3 L^2 C^2}.
\]

By identifying, we can see that an extra constant representing the DC resistance should be added, such that \( \alpha = k_1 \sqrt{f} + k_2 f + k_3 f + k_4 \). This added constant does not affect the causality of the system, and we can still use the same \( \beta \) as in the KM1 model. Hence the KM2 model can be written as

Definition 4 (KM2). The propagation constant, \( \gamma = \alpha + j \beta \), can be modeled by

\[
\alpha = k_1 \sqrt{f} + k_2 f + k_3 f + k_4
\]

\[
\beta = k_1 \sqrt{f} - k_2 \frac{2}{\pi} f \ln f + k_3 f - k_5 \frac{1}{\sqrt{f}}.
\]

By using the third order expansion of (13) we will get an extra term, proportional to \( 1/\sqrt{f} \). With the Hilbert pair \( 1/\sqrt{f} = \frac{f}{2\pi} \text{sgn}(f) / \sqrt{|f|} \) the definition for the KM3 model becomes the following.

Definition 5 (KM3). The propagation constant, \( \gamma = \alpha + j \beta \), can be modeled by

\[
\alpha = k_1 \sqrt{f} + k_2 f + k_3 f + k_5 \frac{1}{\sqrt{f}} + k_6
\]

\[
\beta = k_1 \sqrt{f} - k_2 \frac{2}{\pi} f \ln f + k_3 f - k_5 \frac{1}{\sqrt{f}} + k_7.
\]

In KM3, the expression for \( \alpha \) is often used for modeling the amplitude function for Cat5 and Cat6 cables [16]. In the definition above we also get the phase information for a causal version of the model.

IV. FITTING THE KM MODELS

The parameters of the KM models can be found via least-squares fitting from an estimated \( \gamma \). For example, a transfer function measurement can be obtained at \( N \) frequency points \( f_i, i = 1, \ldots, N \) using a vector network analyzer (VNA) and converted to \( \gamma \) via Eq. (3). The squared error is minimized by imposing its partial derivatives with respect to each parameter to be zero.

A. KM1’s estimator

The squared error for the attenuation constant is given by

\[
E = \sum_{i=1}^{N} (k_1 \sqrt{f_i} + k_2 f_i - \alpha(f_i))^2.
\]

Taking partial derivatives with respect to each parameter of the error sum at different frequency points, and making them equal to zero, results in the following equations

\[
k_1 = \frac{\sum_{i=1}^{N} \sqrt{f_i} \alpha(f_i) - \sum_{i=1}^{N} f_i}{\sum_{i=1}^{N} f_i^2}
\]

\[
k_2 = \frac{\sum_{i=1}^{N} f_i \alpha(f_i) - \sqrt{\sum_{i=1}^{N} f_i^2}}{\sum_{i=1}^{N} f_i^2}
\]

which are solved as a system of two equations and two unknowns.

Similarly, the squared error for the phase constant is given by

\[
E = \sum_{i=1}^{N} (k_1 \sqrt{f_i} - k_2 \frac{2}{\pi} f_i \ln f_i + k_3 f_i - \beta(f_i))^2
\]

and the least-squares estimation for the \( k_3 \) parameter results in

\[
k_3 = \frac{\sum_{i=1}^{N} \beta(f_i) - \sum_{i=1}^{N} f_i \ln f_i}{\sum_{i=1}^{N} f_i^2}
\]

B. KM2’s estimator

The squared error for the attenuation constant is given by

\[
E = \sum_{i=1}^{N} (k_1 \sqrt{f_i} + k_2 f_i + k_4 - \alpha(f_i))^2
\]

that leads to the following equations

\[
k_1 = \frac{\sum_{i=1}^{N} \sqrt{f_i} \alpha(f_i) - \sqrt{\sum_{i=1}^{N} f_i^2}}{\sum_{i=1}^{N} f_i^2}
\]

\[
k_2 = \frac{\sum_{i=1}^{N} f_i \alpha(f_i) - \sqrt{\sum_{i=1}^{N} f_i^2}}{\sum_{i=1}^{N} f_i^2}
\]

\[
k_3 = \frac{\sum_{i=1}^{N} \alpha(f_i) - \sqrt{\sum_{i=1}^{N} f_i^2} - k_2 \sum_{i=1}^{N} f_i}{N}
\]
which are solved as a system of three equations and three unknowns.

As the KM2 adopts the same $\beta$ as in the KM1, the least-squares estimation for the $k_3$ parameter is the same of (34).

C. KM3’s estimator

The squared error for the attenuation constant is given by

$$E = \sum_{i=1}^{N} (k_1 \sqrt{f_i} + k_2 f_i + k_3 \frac{1}{\sqrt{f_i}} + k_4 - \alpha(f_i))^2$$

that leads to

$$k_1 = \frac{\sum_{i=1}^{N} \sqrt{f_i} \alpha(f_i) - k_2 \sum_{i=1}^{N} f_i^{1.5} - k_4 \sum_{i=1}^{N} \sqrt{f_i} - k_5 \sum_{i=1}^{N} f_i}{\sum_{i=1}^{N} \sqrt{f_i}}$$

$$k_2 = \frac{\sum_{i=1}^{N} f_i \alpha(f_i) - k_1 \sum_{i=1}^{N} f_i^{1.5} - k_4 \sum_{i=1}^{N} f_i - k_5 \sum_{i=1}^{N} \sqrt{f_i}}{\sum_{i=1}^{N} f_i^{1.5}}$$

$$k_4 = \frac{\sum_{i=1}^{N} \alpha(f_i) - k_1 \sum_{i=1}^{N} \sqrt{f_i} - k_2 \sum_{i=1}^{N} f_i - k_5 \sum_{i=1}^{N} \frac{1}{\sqrt{f_i}}}{\sum_{i=1}^{N} \sqrt{f_i}}$$

$$k_5 = \frac{\sum_{i=1}^{N} \frac{1}{\sqrt{f_i}} \alpha(f_i) - k_1 N - k_2 \sum_{i=1}^{N} \sqrt{f_i} - k_5 \sum_{i=1}^{N} \frac{1}{\sqrt{f_i}}}{\sum_{i=1}^{N} \sqrt{f_i}}$$

which is solved as a system of four equations.

The squared error for the phase constant is given by

$$E = \sum_{i=1}^{N} \left( k_1 \sqrt{f_i} - k_2 f_i - k_3 f_i^{1.5} + k_4 \frac{1}{\sqrt{f_i}} - k_5 - \beta(f_i) \right))^2$$

and the least-squares estimation for the $k_3$ parameter results in (the summations are from $i = 1$ to $N$):

$$k_3 = \frac{\sum_{i=1}^{N} f_i \beta(f_i) - k_1 \sum_{i=1}^{N} f_i^{1.5} + k_2 \sum_{i=1}^{N} \frac{2}{f_i} \ln f_i + k_5 \sum_{i=1}^{N} \sqrt{f_i}}{\sum_{i=1}^{N} f_i^{1.5}}$$

V. EXPERIMENTAL RESULTS

This section presents experimental results that show the performance of the proposed models for modeling single cable segment topologies, in both frequency and time-domains.

The KM models were fitted to copper pairs from two different cables: a typical 50 m long Cat5 cable (4x2x0.5) and a 50 m long Ericsson ELQXBE cable (1x4x0.5). The transfer function of one pair within each cable was measured in a frequency range from 100 kHz up to 200 MHz using the following: an Agilent Vector Network Analyzer (VNA) 4395A, a Transmission/Reflection Test Set 87512A and North Hills baluns 50 $\Omega$ - 100 $\Omega$, 100 kHz - 300 MHz.

The propagation constants were obtained from the measurements via Eq. (3), and they were used by estimation routines according to Section IV to obtain the values for the $k$ parameters that best fit the measured cables. Tables II and III present the $k$ parameters values for the three models obtained by fitting each cable pair.

The accuracy of the proposed models in frequency-domain can be observed in Figs. 1 and 2, which show the measured and modeled transfer functions for the Cat5 pair and the absolute error (difference) between them, respectively. Similar conclusions were derived when observing the results for the ELQXBE pair and due to limited space only the Cat5 plots are shown here.

It can be observed that, up to 100 MHz, the error is within $\pm$ 0.4 dB in all the frequency range. Above 100 MHz the impact of inhomogeneities due to the imperfect construction of the twisted-pair cable become more prominent and discrepant with respect to models that assume a perfectly homogeneous model [9] such as the proposed ones. It was noted a maximum error of approximately 1 dB when considering the frequency range up to 200 MHz. So, we predict that the proposed models can be used up to these frequencies with relatively high security.

Moreover, from Fig. 2, it is possible to observe a better accuracy of KM2 and KM3 in the low-frequency region as a consequence of their DC attenuation terms and low-frequency attenuation term.

Fig. 3 aims at verifying the accuracy of the proposed models in time-domain, showing the measured and modeled impulse responses for the Cat5 pair. Considering this specific cable, the propagation delay was estimated as $T_{pd} = 230.1$ ns. It can be noted that all the KM models obey the minimum required time imposed by the propagation delay. For comparison, Fig 3 also shows the impulse response of Chen’s model fitted to the same twisted-pair. It can be observed that the impulse response derived from Chen’s model presents considerable energy before the propagation delay.

VI. CONCLUSIONS

G.fast systems are based on the use of short twisted-pair cables and frequencies up to hundreds of MHz. One important point in the development of G.fast is the accurate modeling
Fig. 3: Measured and modeled impulse responses for the Cat5 pair.

TABLE I: Summary of existing cable models. ‘Phys.’ identifies a physical model and ‘High f’ a model designed to behave well at G.fast frequencies (f > 100 MHz).

<table>
<thead>
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<th>‘High f’</th>
<th>Year</th>
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<td>No</td>
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<td>9</td>
<td>Yes</td>
<td>RLCG</td>
<td>2011</td>
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<td>EAB/TNO [12]</td>
<td>No</td>
<td>10</td>
<td>Yes</td>
<td>RLCG</td>
<td>2012</td>
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<td>Proposed KMs</td>
<td>No</td>
<td>3, 4, 5</td>
<td>Yes</td>
<td>γ</td>
<td>2013</td>
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</table>

of the transmission channel. This paper presented three new models to characterize short twisted-pair cables operating at G.fast frequencies. The models were derived through a generalization of the BT0H model and compared to Chen’s model. Experimental results showed that the proposed models lead to good results in both time and frequency domains.

ACKNOWLEDGMENT

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REFERENCES


TABLE II: k parameters of the KM1, KM2, KM3 models fitted to the Cat5 cable measurement.

<table>
<thead>
<tr>
<th>Model/Param. (per km)</th>
<th>KM1</th>
<th>KM2</th>
<th>KM3</th>
</tr>
</thead>
<tbody>
<tr>
<td>k1</td>
<td>0.0020</td>
<td>0.0020</td>
<td>0.0020</td>
</tr>
<tr>
<td>k2</td>
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<td>3.6287e-008</td>
<td>3.6186e-008</td>
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<tr>
<td>k3</td>
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<td>3.0114e-005</td>
<td>3.0106e-005</td>
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<tr>
<td>k4</td>
<td>-</td>
<td>0.1764</td>
<td>0.1632</td>
</tr>
<tr>
<td>k5</td>
<td>-</td>
<td>-</td>
<td>17.8555</td>
</tr>
</tbody>
</table>

TABLE III: k parameters of the KM1, KM2, KM3 models fitted to the Ericsson ELQXBE cable measurement.

<table>
<thead>
<tr>
<th>Model/Param. (per km)</th>
<th>KM1</th>
<th>KM2</th>
<th>KM3</th>
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<tr>
<td>k1</td>
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