Fuzzy Logic for Cooperative Robots Communication

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Except where otherwise indicated, this thesis is my own original work.

Dingyun Zhu
20 June 2007
To my parents.
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Abstract

Human-beings communicate in abbreviated ways dependent on prior interactions and shared knowledge. Much current human-machine communication is tedious and repetitive for humans. Our solution is context dependent reconstructive communication. We build up codebook based on prior communication and select from appropriate behaviors from the codebook based on current interactions.

This study reviews the basic concepts of Fuzzy Logic, concerning the main problem (rule explosion) of developing fuzzy systems, and the solutions to this problem: Hierarchical Fuzzy Rule Bases and Fuzzy Signatures, which will be the key techniques to model cooperative robots communication. In this report, we also introduce several different kinds of communications which are relevant to this project, discussing their advantages and disadvantages.

Further research is suggested to construct the context dependent communication between those robots and modeling the communication by using Fuzzy Signatures, which will be the significant part of research work in next semester as well.
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Introduction

This chapter provides a brief introduction to this project. The first part is a basic introduction to the background and recent international progress which relates to this project. The next section gives a rough description of the techniques which will be used in this project.

1.1 Motivation

Fuzzy logic is derived from fuzzy set theory, which has been widely applied in both academic research and industrial areas: Control engineering, Computer science, Medical diagnosis, and Management, etc. In the past twenty years, robotics became a really active research field, lots of new techniques have been developed in modeling different kinds of robots. In fact, the most significant motivation for investigating multi-robot communication is that it enables teams of robots to maximize their utility. So the main purpose of this project is to model the cooperative robots communication by our fuzzy techniques, which can be further extended to a higher level approach for modeling human-machine communication.

Much current human-machine communication is tedious and repetitive for humans. In order to improve this situation, our technique provides a more effective and efficient way which is to build up codebooks based on prior communication and select appropriate behaviors from the codebook according to current interactions. Therefore, the eventual success of this project will be in significantly enhanced human-machine communication in the sense of enhanced human experience due to intelligent context, which will have potentially very substantial benefits. A further instance could be web search where human-machine communication via keywords could be substantially improved by a hierarchical structuring of a context codebook in the style we use in this project.

1.2 Background

In order to model the communication between human-beings or human-machine, especially for Multi-Robot system, a lot of research work has been done. In recent years, there are four related strands of work:

- Adaptive communication [Yanco and Stein 1993]
1.2.1 Adaptive Communication

The research done at the Artificial Intelligence Laboratory of the Massachusetts Institute of Technology by Holly Yanco and Lynn Stein in 1993 describes mobile robots engaged in a cooperative task that requires communication. They used an "Adaptive Communication" protocol to model the communication between those robots. In their community, "Adaptive Communication" means robots communicate with a fixed uninterpreted symbolic vocabulary and the robots learn a private language during interaction. So adaptive communication builds a shared knowledge which can be accessed by multi-robots, in order to accomplish their cooperative tasks.

However, the adaptive communication is only on explicit communications and the language created for the robots may not provide an optimal solution to a particular task. Also, the private language may not be natural to the mobile robots and the tasks as well. In addition, this kind of fixed communication language is less able to handle complicated circumstances when the robots are working in a changing environment. So the risk of adaptive communication may be trapped in private universes of discourse.

1.2.2 Implicit Communication

"Implicit Communication" is another field of research in cooperative robots communication. It means the transmission of information via the environment. Here is a good example of implicit communication:

Consider two robots R1 and R2 mowing a lawn. Assume that both of them are able to detect mowed grass versus un-mowed grass. R2 does not need to ask R1 what part of lawn R2 has mowed. In fact, neither R1 nor R2 need to keep track of such information, it is stored in the environment. R1 and R2 communicate cutting progress through the environment. This is implicit communication. [Zebrowski 2004]

In fact, there is an interesting feature of implicit communication, which we can find out in the previous mowing robots instance. When mowing robots are working on their tasks, they are not able to stop transferring information to other robots, because they can not stop "telling" other robots that which parts of the lawn they have already finished. In other words, implicit communication is unavoidable, we can not cut it off.

According to the description of implicit communication, it is not hard for us to discover the benefits of implicit communication [Zebrowski 2004]:

- Simplicity. Often times, implicit communication occurs automatically.
- Robustness.
• Lack of dependence on underlying communication network or other communication mechanisms.

On the other hand, the limitations of implicit communication are also obvious, which include the storage capacity of the environment and the lack of intentional communication. In addition, implicit communication may be limited by any limitations in the robot’s perceptual ability. Also, only simple format information can be transmitted by implicit communication, many kinds of information can not be transmitted through the environment.

1.2.3 Fuzzy Communication

The third strand is a diffuse literature styled “fuzzy communication” which refers to various kinds of imprecise communication, some of which use fuzzy logic concepts. The most significant advantage of fuzzy communication is that it can be used to model linguistic terms rather than numerical inputs, which proved to be highly challenging after investigation. Actually, linguistic messages exist in everybody’s daily life. For instance:

Message: Today’s temperature is high, it is likely that there will be hot tomorrow.

We can see that this form of linguistic messages can be easily comprehended by another human being based on the clear description of linguistic terms. A general scheme of fuzzy communication is shown in Figure 1.1.

![Figure 1.1: Fuzzy Communication Channel - a general scheme](Pedrycz and Roventa 1999)

According to Figure 1.1, the communication between two human-beings can be modeled as the transmission of information based on the basic concepts in the codebook. In fact, the codebook is an important component which can be regarded as a shared repository for the suitable interpretation of linguistic message. We can take the two human-beings as a transmitter and a receiver respectively. Both of them are equipped with a codebook composed of different categories of genetic terms with which each linguistic message can be encoded or decoded. Therefore, fuzzy communication is composed of two essential processes [Pedrycz and Roventa 1999]:
Introduction

- Fuzzy Encoder: Encode input information
- Fuzzy Decoder: Decode the transmitted message

The mechanisms of encoding and decoding of fuzzy information are commonly referred to as fuzzification and defuzzification which are two important steps in the fuzzy inference process.

1.2.4 Characterizing Expert Systems as Communication Channels

The final strand is by Whitaker and Östberg (1988) in characterizing expert systems (ES) as communication channels where we measure the incremental information loss from the expert to the system user. Although expert systems have been discussed for many years, but their definition is still somewhat vague. Here are two items about the definition of an ES [Whitaker and Ostberg 1988]:

1. An ES embodies some representation of knowledge about a given task domain; and
2. The ES emulates the capabilities of a human within the given task domain at a level of performance equivalent an 'expert'.

The following picture (Figure 1.2) is an illustration of an ES's structure which usually is composed of two discrete components:

1. A knowledge base containing the modeled expertise; and
2. An inference engine carrying out logical operations over that knowledge base.

Figure 1.2: Typical Schematic View of an Expert System [Whitaker and Ostberg 1988]
In fact, there is another alternative perspective on ES, which takes ES as communication channels for knowledge transfer:

![Figure 1.3: Expert Systems as Communication Channels](Whitaker and Ostberg 1988)

From Figure 1.3 we can see that the path from the expert to the end-user has been taken as a communication channel, and the overall flow of information can be broken down into three processes - Knowledge Acquisition, ES Construction and Consultation Session. The advantage of the ES here is obvious that the end-user can directly apply the expertise without learning and digesting it, when the human expert is not available either due to time or cost. However, there is another problem which can be derived from this communication: an incremental loss of information occurs along the path from the original source of expertise to the end-user, and such loss is inevitable.

1.3 Our Work

From the previous sections, we have already had a short introduction to recent progresses in this research area. Now, we are going to introduce the main techniques and scenario of this project.

1.3.1 Fuzzy Techniques Used

ANU "Fuzzy group" of Department of Computer Science has productively completed previous ARC funded research on sparse rule based [Chong et al. 2000], in finding hierarchical structure in dense data [Gedeon et al. 2001a], and on data with substantial omissions or complex interdependent substructure [Gedeon et al. 2001b]. This work
is part of the foundation of this project. We expect to construct hierarchical sparse rule bases, based on the signature concepts for cooperative robots communication.

1.3.2 Scenario: (Co-operating Intelligent Robots, Terano 1993)

There is a set of identical oblong shaped tables in a room. Various configurations can be built from them, such as a large U shape, a large T shape, a very large oblong, rows of tables, etc. A group of autonomous intelligent robots is supposed to build the actual configuration according to the exact instructions given to the “Robot Foreman” (R0). The other robots have no direct communication links with R0, but they are able to observe the behavior of R0 and all others, and they all posses the same codebook containing all possible table configurations. The individual tables can be shifted or rotated, but two robots are always needed to actually move a table, as they are heavy. If two robots are pushing the table in parallel, the table will be shifted according to the joint forces of the robots. If the two robots are pushing in the opposite directions positioned at the diagonally opposite ends, the table will turn around the center of gravity. If two robots are pushing in parallel, and one is pushing in the opposite direction, the table will not move. Under these conditions the task can be solved, if all robots are provided by suitable algorithms that enable “intention guessing” from the actual movements and positions, even though they might not be unambiguous.

Figure 1.4: Co-operating Intelligent Robots

This will be the application scenario used in this project.
1.4 Big Picture of This Project

Based on the context of this project, the main work can be separated into two levels:

Theoretical work (Codebook Design).

Implementation work (Cooperative Robots Simulator).

Figure 1.4 shows an overview of this project.

As we can see from Figure 1.4, apart from the main fuzzy techniques (Sparse Hierarchical Rule Bases and Fuzzy Signatures), we also need Decision Tree in our theoretical work as well, because our codebook of cooperative robots will mainly be designed as a decision tree form which will be explained in detail in later chapters.

On the implementation level, we are going to implement our codebook onto the cooperative robots simulator which is based on a Java platform. We may need pattern
matching techniques to measure whether the task has been finished, and calculating the distance between robots or robots and tables by applying some computer vision algorithms.

### 1.5 Objectives

This literature review will focus on Fuzzy Logic Theory including its recent techniques (Fuzzy Signatures, etc.), and its application to cooperative robots communication. The ultimate objective of this review will be to develop a theoretical method to model cooperative robots communication by our fuzzy techniques, which can be used in next semester’s further research in COMP6702.
Chapter 2

Literature Review

The first part of this chapter is a basic introduction to Fuzzy Logic Theory. The second part is a literature review concerning Sparse Hierarchical Fuzzy Rule Bases and Fuzzy Signatures.

2.1 Fuzzy Logic Theory

What is "Fuzzy Logic"? In 1965, Lotfi Zadeh, Professor and Head of the Electrical Engineering Department at the University of California at Berkeley, published his famous paper "Fuzzy sets". This is the definition which he gave in his paper [Zadeh 1965]:

*Fuzzy logic is determined as a set of mathematical principles for knowledge representation based on degrees of membership rather than on crisp membership of classical binary logic.*

2.1.1 Fuzzy Sets

A fuzzy set can be simply defined as a set with fuzzy boundaries. Let \( X \) be the universe of discourse and its elements be denoted as \( x \). In classical set theory, a crisp set \( A \) of \( X \) is defined as function \( f_A(x) \) called the characteristic function of \( A \)

\[
f_A(x) : X \rightarrow \{0,1\},
\]

where

\[
f_A(x) = \begin{cases} 
1, & x \in A, \\
0, & x \notin A,
\end{cases}
\]

In the fuzzy theory, fuzzy set \( A \) of universe \( X \) is defined by function \( \mu_A(x) \) called the membership function of set \( A \)

\[
\mu_A(x) : X \rightarrow [0,1],
\]

where

\[
\mu_A(x) = 1, \text{ if } x \text{ is totally in } A, \\
\mu_A(x) = 0, \text{ if } x \text{ is not in } A, \\
0 < \mu_A(x) < 1, \text{ if } x \text{ is partly in } A,
\]
This set allows a continuum of possible choices. For any element \( x \) of universe \( X \), membership function \( \mu_A(x) \) equals the degree to which \( x \) is an element of set \( A \). This degree, a value between 0 and 1, represents the degree of membership, also called membership value, of element \( x \) in set \( A \) \cite{Negnevitsky2002}. The representation of a fuzzy set can be defined as ordered pairs of \( x \) and its corresponding membership degree:

\[
A = \{(x, \mu_A(x)) \mid x \in X, \mu_A(x) : X \rightarrow [0, 1]\}
\]

Usually, we use another way which is a more convenient way to express fuzzy sets:

\[
A = (\{\mu_A(x_1)/x_1\}, \{\mu_A(x_2)/x_2\}, ..., \{\mu_A(x_n)/x_n\})
\]

Note: "/" here does not represent division, it means membership value \( \mu_A(x) \) and corresponding element \( x \) can appear in pairs.

Figure 2.1 is a good example of a fuzzy sets (membership function).

![Fuzzy sets of short, average and tall men](Crnkovic-Dodig2006)

From this graph, there are three linguistic terms (Short, Average and Tall) which are used to describe a man’s height. So, in fuzzy logic, as we can see, if a man is 182cm tall, he is in the “Average” set with a degree of membership of 0.4, at the same time, “182cm” is also a member of the “Tall” men set with a degree of membership of 0.08 and 0 for the “Short” set. This means the value of a man’s height has partial membership in multiple sets. Note that the membership values listed here are not normalized, so the sum of membership degrees does not add up to 1.
2.1.2 Operations on Fuzzy Sets

Comparing with classical sets operations, the operations of fuzzy sets are similar. We mainly look at four operations on fuzzy sets:

1. Complement

The Complement of a set means opposition of this set. Usually, we use linguistic term "NOT" or mathematical symbol "⇁" to present the meaning of complement. In fuzzy sets, the definition of Complement is:

$$\mu_{⇁A}(x) = 1 - \mu_A(x)$$

For instance, there are five men with heights: 170cm, 172.5cm, 175cm, 177.5cm, 180cm respectively, so according to Figure 2.1, the fuzzy set of "average men" is:

$$\text{Average Men} = (0.7/170, 0.85/172.5, 1.0/175, 0.7/177.5, 0.6/180).$$

Now, we can easily calculate the fuzzy set of "NOT average men":

$$\text{NOT Average Men} = (0.3/170, 0.15/172.5, 0.0/175, 0.3/177.5, 0.4/180).$$

2. Containment

The definition of Containment is that a set can contain other sets. Generally, the larger set is called superset, and the smaller one is called subset. Suppose $A$ and $B$ are two different fuzzy sets, $\mu_A(x)$ and $\mu_B(x)$ represent the corresponding membership values, so "fuzzy set $A$ is contained in fuzzy set $B$" means:

$$\mu_A(x) \leq \mu_B(x)$$

3. Intersection

The intersection operation means the overlap of two sets, which contains the elements shared by these sets. However, in fuzzy sets, an element may partly belong to different sets with different membership degrees. The instance described before, a man is 182cm tall, which either belongs to "Average" (membership value: 0.4) or "Tall" (membership value: 0.08). Therefore, the fuzzy intersection is the lower membership value in both sets of each element.

$$\mu_{A \cap B}(x) = \mu_A(x) \cap \mu_B(x) = \min[\mu_A(x), \mu_B(x)]$$

For example, here we have two fuzzy sets of "Tall men" and "Average men":

$$\text{Tall men} = (0.0/170, 0.0/175, 0.0/180, 0.25/185, 0.6/190).$$

$$\text{Average men} = (0.7/170, 1.0/175, 0.6/180, 0.25/185, 0.0/190).$$
Consequently, the intersection of these two fuzzy sets:

\[ \text{Tall men } \cap \text{Average men} = (0.0/170, 0.0/175, 0.0/180, 0.25/185, 0.0/190). \]

4. Union

The union of two sets consists of every element which is in either set. In fuzzy operations, the union is the reverse of intersection, which means the largest membership value of element in either set.

\[ \mu_{A \cup B}(x) = \mu_A(x) \cup \mu_B(x) = \max[\mu_A(x), \mu_B(x)] \]

Consider the fuzzy sets of “Tall men” and “Average men” again:

\[ \text{Tall men} = (0.0/170, 0.0/175, 0.0/180, 0.25/185, 0.6/190). \]
\[ \text{Average men} = (0.7/170, 1.0/175, 0.6/180, 0.25/185, 0.0/190). \]

The union of tall men and average men is:

\[ \text{Tall men } \cup \text{Average men} = (0.7/170, 1.0/175, 0.6/180, 0.25/185, 0.6/190). \]

2.1.3 Fuzzy Rules

A fuzzy rule can be defined as a conditional statement in the form:

\[ \text{If } X \text{ is } A \text{ Then } Y \text{ is } B \]

where \( X \) is the linguistic input and \( Y \) is the output, \( A \) and \( B \) are fuzzy values of antecedent and consequent which are determined by corresponding fuzzy sets. For example, the relationship between a man’s height and his weight may be described in the following fuzzy rule:

\[ \text{If HEIGHT is Tall Then WEIGHT is Heavy}. \]

where \( \text{Tall} \) and \( \text{Heavy} \) are both linguistic terms defined in fuzzy sets \( \text{HEIGHT} \) and \( \text{WEIGHT} \) respectively.

From this simple instance, the main difference between fuzzy system and classical rule-based system is the type of data used in the processes. In classical rule-based systems, we usually use numerical data but in fuzzy systems, we can use linguistic variables as inputs, which can also be the main advantage of fuzzy system. Fuzzy systems allow us to deal with linguistic labels instead of directly modeling the rule of system numerically. This feature offers a more convenient approach when modeling complex systems in both academic and industry applications.
2.2 Rule Explosion of Fuzzy Systems and Solutions

Fuzzy control systems are the most important applications of fuzzy theory. This is a generalized form of expert control using fuzzy sets with fuzzy rules in modeling a system. The basic idea of classical fuzzy approaches [Zadeh 1973] [Mamdani and Assilian 1975] [Koczy and Hajnal 1977] is to calculate the conclusion by evaluating the degree of matches from the observation then trigger one or several rules in the model. However, a problem is caused by the high computational time and space complexity of rule bases describing systems with multiple inputs with proper accuracy:

Given $k$ inputs, and at most $T$ linguistic terms per dimension of $X$, the number of fuzzy rules covering $X$ is $|R| = O(T^k)$ which is very high, unless $k$ is very small. The exponential explosion in rules is a major problem in applying fuzzy theory beyond control systems. Because of this rule explosion, a classical fuzzy system can only deal with no more than 6 to 10 inputs. So in order to solve this problem, it is essential to reduce $T, k$ or both. [Gedeon et al. 2001a]

2.2.1 Sparse Hierarchical Fuzzy Rule Bases System

Sparse hierarchical fuzzy systems reduce both $T$ and $k$ simultaneously by finding (top-down) sub-spaces in the data, which allows some dimensions and rules to be ignored. [Chong et al. 2000] The basic idea of using hierarchical fuzzy rule bases is the following [Koczy et al. 2000]:

Often the multi-dimensional input state space $X = X_1 \times X_2 \times \ldots \times X_k$ can be decomposed, so that some of its components, e.g. $Z_0 = X_1 \times X_2 \times \ldots \times X_{k_0}$ determine a subspace of $X(k_0 < k)$, so that in $Z_0$ a partition $\Pi = \{D_1, D_2, \ldots, D_n\}$ can be determined:

$$\bigcup_{i=1}^{n} D_i = Z_0$$

In each element of $\Pi$, i.e. $D_i$, a sub-rule base $R_i$ can be constructed with local validity. In the worst case, each sub-rule base refers to exactly $X/Z_0 = X_{k_0+1} \times \ldots \times X_k$ [Koczy and Hirota 1993], and so the hierarchical rule base has the following structure:

$R_0$:
- If $z_0$ is $D_1$ then use $R_1$
- If $z_0$ is $D_2$ then use $R_2$
  ...
- If $z_0$ is $D_n$ then use $R_n$
  where $z_0 \in Z_0$

$R_1$:
- If $z_1$ is $A_{11}$ then $y$ is $B_{11}$
- If $z_1$ is $A_{12}$ then $y$ is $B_{12}$
  ...
- If $z_1$ is $A_{1m_1}$ then $y$ is $B_{1m_1}$
where \( z_1 \in X / Z_0 \)

\[
R_2:
\begin{align*}
& \text{If } z_1 \text{ is } A_{21} \text{ then } y \text{ is } B_{21} \\
& \text{If } z_1 \text{ is } A_{22} \text{ then } y \text{ is } B_{22} \\
& \ldots \\
& \text{If } z_1 \text{ is } A_{2m_2} \text{ then } y \text{ is } B_{2m_2} \\
\end{align*}
\]

\[R_n: \]

\[
\begin{align*}
& \text{If } z_1 \text{ is } A_{n1} \text{ then } y \text{ is } B_{n1} \\
& \text{If } z_1 \text{ is } A_{n2} \text{ then } y \text{ is } B_{n2} \\
& \ldots \\
& \text{If } z_1 \text{ is } A_{nm_n} \text{ then } y \text{ is } B_{nm_n} \\
\end{align*}
\]

It is easy to see that this hierarchical approach does not help with the \( O(T^k) \) complexity of the whole rule base as the size of \( R_0 \) is \( O(T^{k_1}) \), and each \( R_i, i > 0 \), is of order \( O(T^{k-k_1}) \), so the resulting complexity is:

\[
O(T^{k_1}) \times O(T^{k-k_1}) = O(T^k).
\]

But on the other hand, if the number of variables in each \( Z_i \) is \( k_i < k - k_0 \) and \( \max_{k=1}^n \{k_i\} = K < k - k_0 \), then the resulting complexity will be \( O(T^{k_0+K}) < O(T^k) \), so that the application of the structured rule base leads in effect to the reduction of \( k \) to smaller exponent: \( k_0 < k + K \).

The main difficulty of applying this method is that often it is impossible to find \( \Pi \) so that \( k_i < k - k_0, i = 1, \ldots, n \), because such a partition does not exist.

### 2.2.2 Fuzzy Signatures

The second approach of solving rule explosion is fuzzy signatures - constructing characteristic fuzzy structures, modeling the complex structure of the data points (bottom up) in a hierarchical manner [Koczy et al. 1999] [Gedeon et al. 2001b] [Vamos et al. 2001]. Fuzzy signatures result in a much reduced order of complexity, at the cost of slightly more complex aggregation techniques.

The original definition of fuzzy sets was \( A : X \to [0,1] \), and was soon extended to \( L \)-fuzzy sets [Goguen 1967]

\[
A_S : \to [a_i]_{i=1}^k, a_i = \left\{ \begin{array}{c} [0,1] \\
[a_{ij}]_{j=1}^{k_i} \\
[a_{ij}]_{l=1}^{k_{ij}} \\
\end{array} \right. \\
A_L : X \to L, L \text{ being an arbitrary algebraic lattice. A practical special case, } \text{Vector Valued Fuzzy Sets} \text{ was introduced by [Koczy 1982], where } A_{V,K} : X \to [0,1]^k, \text{ and the range of membership values was the lattice of } k \text{-dimensional vectors with components in the unit interval. The general concept of fuzzy signature is a nested vector, where each vector component can be another nested vector structure. So it can be described as a generalized vectorial fuzzy set with possible recursive vectorial components, consequently, it is a generalization of valued fuzzy sets and denoted by:}
\[ A : X \rightarrow S^{(n)}, \]

where \( n \geq 1 \) and

\[ S^{(n)} = \prod_{i=1}^{n} S_i \]

\[ S = \begin{cases} [0, 1] \\ S^{(m)} \end{cases} \]

and \( \prod \) describes Cartesian product. \([Koczy et al. 1999]\)

In fact, we can consider fuzzy signature as a special kind of multi-dimensional fuzzy data. Some of the dimensions are formed as a sub-group of variables, which jointly determine some feature on a higher level. Figure 2.2 illustrates an example of fuzzy signature structure.

![Figure 2.2: Fuzzy Signature Structure](image)

Also, the fuzzy signature structure shown in Figure 2.2 can be presented in vector form which only includes all the leaves:
As we can see in Figure 2.2 and the vector form of this fuzzy signature, every joint in the graph represents a fuzzy set, so the whole structure of fuzzy signature looks like a tree graph, but here \([X_{11} \, X_{12}]\) is actually a sub-group which relates to the higher level compound joint \(X_1\). The lowest level of this fuzzy signature has three leaves - \([X_{211} \, X_{212} \, X_{213}]\) which are composed of their higher level variable \(X_{21}\). In fact, we can find the similar formation in the sub-group of \([X_{31} \, X_{32} \, X_{33}]\). So \([[X_{211} \, X_{212} \, X_{213}] \, X_{22}]\) equals to \([X_{21} \, X_{22}]\) which can be equivalent on higher level variable \(X_2\). Finally, this fuzzy signature can be abstracted to the highest level \(X = [X_1 \, X_2 \, X_3]\).

The relationship between higher levels and lower levels is controlled by a set of fuzzy aggregations. The results of the parent signature at each level are computed from their branches with appropriate aggregation of their child signature [Wong et al. 2003]. Let \(A_1\) be the aggregation which is associating \(X_{11}\) and \(X_{12}\). From the previous introduction, we know \(X_{11}\) and \(X_{12}\) can be used to derive their higher level variable \(X_1\), so by applying aggregation \(A_1\), \(X_1 = X_{11} \cdot A_1 \cdot X_{12}\).

By referring Figure 2.2, the whole fuzzy signature structure has four aggregations:

- \(A_1 : [X_{11} \, X_{12}]\)
- \(A_{21} : [X_{211} \, X_{212} \, X_{213}]\)
- \(A_2 : [X_{21} \, X_{22}] = [[X_{211} \, X_{212} \, X_{213}] \, X_{22}]\)
- \(A_3 : [X_{31} \, X_{32} \, X_{33}]\)

Actually, aggregations for fuzzy signatures are not necessarily identical or different, it can be changed based on different circumstances. Suppose we have a fuzzy signature with fuzzy values based on previous example:

\[
X = \begin{bmatrix}
0.6 \\
0.3 \\
0.2 \\
0.5 \\
0.8 \\
0.4 \\
0.7 \\
0.4 \\
0.1
\end{bmatrix}
\]

Then we define the four aggregations as the following operations:
• $A_1 : \min(X_{11}, X_{12})$

• $A_{21} : \text{average}(X_{211}, X_{212}, X_{213})$

• $A_2 : \max(X_{21}, X_{22}) = \max(\text{average}(X_{211}, X_{212}, X_{213}), X_{22})$

• $A_3 : \text{average}(X_{31}, X_{32}, X_{33})$

By applying these aggregation operations, we can calculate the fuzzy values of this signature:

\[
X = \begin{bmatrix}
0.6 \\
0.3 \\
0.2 \\
0.5 \\
0.8 \\
0.4 \\
0.7 \\
0.4 \\
0.1 \\
\end{bmatrix} \rightarrow \begin{bmatrix}
0.3 \\
0.5 \\
0.4 \\
0.4 \\
\end{bmatrix}
\]

Finally, the fuzzy signature will be:

\[
X = \begin{bmatrix}
0.3 \\
0.5 \\
0.4 \\
\end{bmatrix}
\]

Each of these signatures contains information which is related to the particular fuzzy set. Comparing with lower level signatures, less information will be kept in higher level signatures. In some cases, in order to become more compatible with information obtained from other sources which some detail variables are missed or simply omitted, it is necessary to reduce the information by applying relevant aggregations. Usually, when interpolation within a fuzzy signature rule base has been done, however, some of them do not have the same structure, so in order to be able to interpolate between the corresponding branches or roots, all signature must be reduced to the maximal common sub-tree which is defined after the interpolation process [Wong et al. 2003].

From the previous description of fuzzy signatures and fuzzy aggregations, it is easy to see that fuzzy signatures are another kind of hierarchical fuzzy structure, which can reduce the number of inputs $k$ by applying aggregations from the bottom level to the top. That is the reason why we mentioned before it is another solution to rule explosion.

Here we just used some very simple aggregations for fuzzy signatures, such as "$\min$, $\max$, average", more complicated aggregations have been introduced in [Mendis et al. 2005]. Fuzzy signatures are introduced to handle complex structured data [Wong et al. 2004], which can be resulting in effective and efficient fuzzy inference. In the following chapter, we are going to introduce how to construct fuzzy signatures for our cooperative robots scenario.
Literature Review
Approach and Methodology

In the previous chapter, we have already introduced the scenario of cooperative robots [Terano et al. 1993] and our main fuzzy techniques: Sparse hierarchical fuzzy rule bases and Fuzzy signatures, which will be applied as the main approach to model this scenario. In the following sections, we are going to discuss our methodology on how to construct codebook by modeling context dependent reconstructive communication and the fuzzy signatures for our cooperative robots.

3.1 Context Dependent Reconstructive Communication

First of all, let us build up a simple framework where the cooperative robots scenario can be formalized. In the description, the letters N, S, E, W will be used in the usual sense for North, South, East, and West. In the following figures, all configurations face North. As a matter of course, each of the table configuration might face any other direction besides North. For simplicity, we assume that the sides of the tables are always parallel with the N-S and E-W axes. There are a few essentially different robot positions allowed, because two robots are needed to push or rotate a table, we identify two places per side for the robots manipulating tables, which we label a $P_{cc}$ (counterclockwise) or $P_{cw}$ (clockwise) for their potential rotational effect.

There are three essentially different combinations:

1. Shift (labeled as $C_{SH}$), when two robots are side by side at the same side of the table (see Figure 3.1);
2. Rotation (counterclockwise $C_{RC}$ or clockwise $C_{RW}$), when two robots are taking different sides of the table (see Figure 3.2);
3. Stop (labeled as $C_{ST}$), there are two ways each of the above movements could be stopped, two examples are shown in Figure 3.3.

![Figure 3.3: Stop Combinations](image)

After having an overview of the possible robot positions and action combinations, let us build the part of the codebook that enables robots to recognize a situation and take action accordingly. Suppose a robot ($R_1$) is taking up a $P_{CC}$ position at the East side of a table, we further assume that another robot ($R_2$) is the nearest free robot that will go to the same table to help. There are three possible actions:

1. $R_2$ guesses that $R_1$ intends to shift the table westwards. Then it must take $P_{CW}$ position in East to complete the $C_{SH}$ combination (see Figure 3.4).

![Figure 3.4: $R_2$ helps $R_1$ to complete $C_{SH}$](image)

2. $R_2$ guesses that $R_1$ intends to rotate the table counterclockwise. Then it must take the $P_{CC}$ position itself in the west to complete the $C_{RC}$ combination (see Figure 3.5).
Approach and Methodology

3. $R_2$ guesses or knows that $R_1$ is in an incorrect position as neither shifting the table westwards, nor rotating it counterclockwise serves the achieving of the goal combination. Therefore, it is going to take the $P_{CW}$ or $P_{CC}$ in the West to form a $C_{ST}$ combination indicating “stop” by physically preventing both shifting the table westwards and of rotating it counterclockwise (see Figure 3.6).

Based on the above instance, we may see that it is possible to build up codebooks for the cooperative robots. The codebook will take the form of a decision tree, where the inputs are a robot’s direct observations, the first level outputs are its intention guesses and the second level outputs the concrete actions.

The pre-condition which should be mentioned before we start constructing the codebook is all possible table configurations have already been defined for robot foreman and other normal robots. And other robots will guess which configuration is the correct one according to foreman’s actions. The set of such table configurations always determines each change made by robot’s action. In fact, at the beginning, the set contains all table configurations, the guess of possible configuration is made by pattern matching.
Suppose the actual observation of robot $R_i$ is that: **Robot $R_j$ has taken up position $P_k$ at table $T_l$**

Here is the codebook relates to this observation, which is modeled as a decision tree (see Figure 3.7):

Is $P_k$ a part of $C_{ST}$?
- IF Yes THEN No action
- IF No THEN is $R_i$ nearest to $T_l$?
  - IF No THEN No action
  - IF Yes THEN Move to $T_l$
    - IF it is a contradiction of the goal configuration to Shift or Rotate $T_l$
      - THEN Move to $C_{ST}$
    - IF it is no contradiction of the goal configuration to Shift $T_l$
      - THEN Move to $C_{SH}$
    - IF it is no contradiction of the goal configuration to Rotate $T_l$ counterclockwise
      - THEN Move to $C_{RC}$
    - IF it is no contradiction of the goal configuration to Rotate $T_l$ clockwise
      - THEN Move to $C_{RW}$

In the codebook, *Move to $T_l$* is an action that may need intelligent route planning, obstacle avoidance, etc. Moreover, we also need to measure whether robot $R_i$ is the nearest one to the table on which $R_j$ has already taken up a position. So it is possible for us to calculate the distances by applying some computational intelligence computer vision algorithms. These two issues are more likely on the implementation level which will not be discussed in detail in this report, but will be explained more in next semester’s further research.

*Move to $C_{ST}$* is another action which needs the extension of another simple decision tree like the following sub-codebook:

- IF $P_k$ is in *North* THEN take the position in *South*
- IF $P_k$ is in *East* THEN take the position in *West*
- IF $P_k$ is in *South* THEN take the position in *North*
- IF $P_k$ is in *West* THEN take the position in *East*
  - IF $P_k$ is $P_{CC}$ THEN take the position $P_{CW}$
  - IF $P_k$ is $P_{CW}$ THEN take the position $P_{CC}$

*Move to $C_{SH}$* requests the extension like the sub-codebook:

- IF $P_k$ is in *North* THEN take the position in *North*
- IF $P_k$ is in *East* THEN take the position in *East*
- IF $P_k$ is in *South* THEN take the position in *South*
- IF $P_k$ is in *West* THEN take the position in *West*
  - IF $P_k$ is $P_{CC}$ THEN take the position $P_{CW}$
  - IF $P_k$ is $P_{CW}$ THEN take the position $P_{CC}$
Move to $C_{RC}$ and Move to $C_{RW}$ are actions for rotating a table, which is also executed by a similar sub-codebook:

IF $P_k$ is in *North* THEN take the position in *South*
IF $P_k$ is in *East* THEN take the position in *West*
IF $P_k$ is in *South* THEN take the position in *North*
IF $P_k$ is in *West* THEN take the position in *East*
IF $P_k$ is $P_{CC}$ THEN take the position $P_{CC}$
IF $P_k$ is $P_{CW}$ THEN take the position $P_{CW}$

**Figure 3.7**: Codebook for robot $R_i$

This sample codebook illustrates clearly that the context dependent reconstruction of the communication among intelligent robots might lead to effective cooperation, and the achievement of the final tasks can not be done without collaboration and communication. In the next section, we will propose a formal method for building the codebook using the fuzzy signature technique.
3.2 Fuzzy Signatures for Cooperative Robots

In the previous chapter, we have already introduced the basic concept of fuzzy signatures. Fuzzy signatures which structure data into vectors of fuzzy values, each of which can be a further vector, are introduced to handle complex structured data \cite{Koczy1999, Gedeon2001b, Vamos2001, Wong2003}. The process of constructing fuzzy signature has also been discussed in \cite{Wong2004}.

Let $S_0$ denote the set of all fuzzy signatures whose structure graphs are sub-trees of the structural ("stretching") tree of a given signature $S_0$. Then the signature sets introduced on $S_0$ are defined by:

$$A_{S_0} : X \rightarrow S_0$$

In this case, the prototype structure $S_0$ describes the "maximal" signature type that can be assumed by any element of $X$ in the sense that any structural graph obtained by a set of repeated omissions of leaves from the original tree of $S_0$ might be the tree stretching the signature of some $A_{S_0}$.

In fact, there are two approaches to construct the sub-structures of the fuzzy signature, $S_0$ \cite{Wong2004}:

1. Predetermined by a human expert in the field.
2. Determined by finding the separability from the data. \cite{Chong2002, Wong2003, Wong2004}

In our cooperative robots case, as we are handling complex circumstances and we actually do not have enough data so far, so we will only use the first approach to construct the fuzzy signatures. Based on the context of the robots scenario, we propose the use of an alternative form of fuzzy signature, which uses a better hierarchical structure where the internal nodes are simple, while the leaves are now populated with small rule bases, generally of 1 variable. The effect is to retain the much reduced order of complexity, and to also substantially reduce the complexity of aggregations to simple combinations of basic fuzzy functions \cite{Mendis2006}.

We still use the example which we also used in constructing the codebook in last section:

The actual observation of robot $R_i$ is that: **Robot $R_j$ has taken up position $P_k$ at table $T_l$**

The information communicated to robot $R_i$ is partly defined by the last movement of $R_j$ and the evaluation of the situation with the tables’ configuration. Accordingly, the membership degrees of each possible actions ($C_{ST}$, $C_{SH}$, $C_{RC}$ and $C_{RW}$) will be attached to each leaf of the signature. Therefore, $R_i$ will follow the scheme where an element of communication will active the context and reevaluate the values of these membership degrees ($\mu_{ST}$, $\mu_{SH}$ and $\mu_{RO}$), then apply the action with the maximal membership degree, ie, the most likely action according to the observations. Here we post some possible situations which may occur in our cooperative robots scenario to show how this signature works:
Figure 3.8: $R_i$ increases the membership degree of $C_{ST}$ in the fuzzy signature

Figure 3.9 illustrates a situation where $R_j$ is very likely trying to destroy an almost finished "T" shape configuration. At this stage, another robot $R_i$ will increase its membership degree of stop combination high (e.g. raise $\mu_{ST}$ to 0.8), and decrease the values of shift and rotate combinations (e.g. lower both $\mu_{SH}$ and $\mu_{RO}$ to 0.1). So $R_i$ will follow the most likely good action and take up the position $(W, P_{CC})$ to stop the $R_j$'s erroneous action.

There is another example shown in figure 3.10, $R_j$ is taking a good action of shifting the last table to finish the task. So $R_i$ will increase the membership value of shift combination ($\mu_{SH}$), then take the position $(E, P_{CC})$ to help $R_j$ to finish the last shift.

Figure 3.9: $R_i$ increases the membership degree of $C_{SH}$ in the fuzzy signature

Here we only provide some basic ideas and simple scenarios, which may illustrate the processes of the fuzzy signatures construction for our cooperative robots. Due to the time limitation in this semester, the fuzzy signatures for cooperative robots communication are still under construction currently. The final fuzzy signatures structure will be added into this chapter after we finish the construction in next semester's further research: COMP6702.
Conclusion

This literature review report begins with the basic theory of Fuzzy Logic, and it also introduces the major advantage of fuzzy systems: it is possible for us to model complex problems which may consist of precise numerical values by natural descriptions and linguistic terms. However, the classical fuzzy systems suffer from the rule explosion which can be specified as the main problem in this area. In order to solve this problem, two significant solutions developed in recent years are introduced in this report:

**Sparse Hierarchical Fuzzy Rule Bases** Reduced the complexity of fuzzy system by finding sub-spaces (top-down) in the data \[\text{[Koczy et al. 2000]}\].

**Fuzzy Signatures** Vector valued fuzzy sets \[\text{[Koczy 1982]}\], modeling the complex structure of the data points (bottom up) in a hierarchical manner \[\text{[Koczy et al. 1999]}\].

The major feature of fuzzy signature is that it allows to handle complex problems or systems in a hierarchical structure, which would be a more effective and efficient way. This is also the most important reason why we are going to use these fuzzy techniques to model our cooperative robots scenario.

In addition to these main theories, we introduced four different sorts of communications developed in recent years, which are related to our cooperative robots communication. Each of them has advantages in some cases but they all might be trapped in some complicated circumstances. All of these progresses could be recognized as the background of this project.

Furthermore, the approach of modeling cooperative robots communication has been presented in the last chapter. The major method is to build up codebook by applying context dependent reconstruction communication which combined with fuzzy signatures. By going through several instances which may occur in our scenario, it illustrates that our method could lead to effective cooperation, and the achievement of the final tasks can not be done without collaboration and communication.

### 4.1 Detailed Future Research Plan

In this semester’s study, we already specified the main approach and methodology which will be used to model the cooperative robots communication include Context Dependent Reconstructive Communication and Fuzzy signatures, etc, all of these are
essential to this project. Especially the Sparse Hierarchical Rule Bases and Fuzzy Signatures, which need more deep literature reviews and understanding. The following section is the detailed research plan for next semester:

<table>
<thead>
<tr>
<th>Tasks</th>
<th>Duration</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Theoretical work (more literature review)</strong></td>
<td><strong>Total 4 weeks</strong></td>
</tr>
<tr>
<td>Sparse Hierarchical Rule Bases Systems</td>
<td>1 week</td>
</tr>
<tr>
<td>Construct Fuzzy Signatures</td>
<td>1 week</td>
</tr>
<tr>
<td>Context Independent Reconstruction Communication Study</td>
<td>1 week</td>
</tr>
<tr>
<td>Build up codebooks combined with Fuzzy Signatures</td>
<td>1 week</td>
</tr>
<tr>
<td><strong>Implementation work</strong></td>
<td><strong>Total 5 weeks</strong></td>
</tr>
<tr>
<td>Implement codebooks for robots’ actions</td>
<td>3-4 weeks</td>
</tr>
<tr>
<td>Test and Evaluation</td>
<td>1 week</td>
</tr>
<tr>
<td><strong>Thesis writing and Presentation</strong></td>
<td><strong>Total 4 weeks</strong></td>
</tr>
<tr>
<td>Describe more literature review</td>
<td>1 week</td>
</tr>
<tr>
<td>Describe implementation work</td>
<td>1 week</td>
</tr>
<tr>
<td>Analyze test and evaluation results</td>
<td>1 week</td>
</tr>
<tr>
<td>Presentation and Finalization</td>
<td>1 week</td>
</tr>
</tbody>
</table>
Key Abstracts

The following papers are the key importance in both current literature review progress and future research plan.

• *Interpolation in Hierarchical Fuzzy Rule Bases* [Koczy et al. 2000]
  
  **Abstract:** A major issue in the field of fuzzy applications is the complexity of the algorithms used. In order to obtain efficient methods, it is necessary to reduce complexity without losing the easy interpretability of the components. One of the possibilities to achieve complexity reduction is to combine fuzzy rule interpolation with the use of hierarchical structured fuzzy rule bases, as proposed by Sugeno. As an interpolation method the KH interpolation is used, but other techniques are also suggested. The difficulty of applying this method is that it is often impossible to determine a partition of any subspace of the original state space so that in all elements of the partition the number of variables can be locally reduced. Instead of this, a sparse fuzzy partition is searched for and so the local reduction of dimensions will be usually possible. In this case however, interpolation in the sparse partition itself, i.e. interpolation in the meta-rule level is necessary. This paper describes a method how such a multi-level interpolation is possible.

• *Constructing Hierarchical Fuzzy Rule Bases for Classification* [Gedeon et al. 2001a]
  
  **Abstract:** Fuzzy rule based systems have been very popular in many control applications. However, when fuzzy control systems are used in real problems, many rules may be required. The number of rules required depends on the number of inputs and the number of fuzzy linguistic terms used. This exponential explosion of fuzzy rules can take too much computing time to solve any but the simplest problems. This paper proposes a hierarchical fuzzy system that partitions a problem for more efficient computation. The hierarchical fuzzy rule base algorithm constructs rules from data for the purpose of performing fuzzy classification. Illustration examples are also generated and the results show that this hierarchical fuzzy system can be successfully used for classification applications.

• *Sparse Fuzzy Systems Generation and Fuzzy Rule Interpolation: A Practical Approach* [Chong et al. 2000]
Abstract: In this paper, we explore the use of a sparse fuzzy system generation technique in conjunction with simple projection-based fuzzy rule interpolation, to generate sparse fuzzy systems with relatively few rules whilst still achieving reasonable system accuracy. Through setting a parameter value, the user is able to control, to some extent, the number of rules generated by the rule extraction technique. The rule interpolation approach enables the sparse fuzzy system to maintain a reasonable accuracy. The effectiveness of this approach is validated experimentally.

• Fuzzy Signatures [Koczy et al. 1999]

Abstract: There are many areas where objects with very complex and sometimes interdependent features are to be classified, similarities and dissimilarities are to be evaluated, etc. Two very obvious examples where such a need emerges are the economy and the medical field. In these cases traditionally subjective evaluation is applied with all its disadvantages (irreproducibility, lack of formal models, etc.). In this paper a novel approach will be introduced: the use of fuzzy signatures as a generalization of the concepts of fuzzy sets and vector valued fuzzy sets.

Fuzzy signatures describe objects with help of a set of (not necessarily homogeneous) interpretations qualitative measures, which are also arranged in hierarchical structure expressing interconnectedness and interdependence with a way modelling the human approach to the problem.

This study will introduce and discuss in details the concepts of fuzzy signatures and operations on them, finally, it will present simple examples.

• Hierarchical fuzzy signature structure for complex structured data [Wong et al. 2003]

Abstract: This paper presents an algorithm for constructing a hierarchical fuzzy signature structure by using concepts from interclass separability and hierarchical fuzzy systems. Fuzzy signature is introduced to handle complex structured data and interdependent features problems. Fuzzy signature can also used in cases where data is missing. However, when dealing with a very large data set, it is possible that they hide hierarchical structure that appears in the sub-variable structures. The proposed hierarchical fuzzy signature structure will be used in problems that fall into this category. In the end, reduced hierarchical fuzzy signature structures can assist human experts better by removing unnecessary information when making decisions.

• Construction of Fuzzy Signature from Data: An Example of SARS Pre-clinical Diagnosis System [Wong et al. 2004]

Abstract: There are many areas where objects with very complex and sometimes interdependent features are to be classified; similarities and dissimilarities are to be evaluated. This makes a complex decision model difficult to construct effectively. Fuzzy signatures are introduced to handle complex structured data and interdependent feature problems. Fuzzy signatures can also used in cases
where data is missing. This paper presents the concept of a fuzzy signature and how its flexibility can be used to quickly construct a medical pre-clinical diagnosis system. A Severe Acute Respiratory Syndrome (SARS) pre-clinical diagnosis system using fuzzy signatures is constructed as an example to show many advantages of the fuzzy signature. With the use of this fuzzy signature structure, complex decision models in the medical field should be able to be constructed more effectively.

- **Investigation of Aggregation in Fuzzy Signatures** [Mendis et al. 2005]

  **Abstract:** The hierarchical fuzzy signatures structure is a novel concept that can be used to find the degree of similarity or dissimilarity of objects which contain complex structured data, for classification or decision making. Fuzzy signatures are vector valued fuzzy sets, where each vector component can be a further vector valued fuzzy set. Thus, it differs from sparse hierarchical fuzzy rule based systems. Medical and economic diagnoses are the obvious applications of the fuzzy signatures. In this paper we present results of three experiments, which were carried out to find the applicability of different aggregation functions, the relationship between the fuzzy signature structure and aggregation functions, and applicability of the fuzzy signatures method for different real world problems. Also, a new method of aggregating fuzzy signatures using weights called the weighted aggregation method has been proposed. Experiments show that the weighted aggregation method provides better results for fuzzy signatures.
Bibliography


