AS scheme, we also include the case with $N_r = 1$ receive antenna for comparison, which is proved to have a diversity order of 4 in [12]. In Fig. 1, we observe that due to the increased diversity order (i.e., 8) of the AS scheme, it shows a larger slope and significantly outperforms the system with $N_r = 1$. It is also shown that a larger number of subcarriers result in better performance for the ML receiver due to the cross terms over the groups in (15). However, the benefit of increasing the number of subcarriers (groups) gives diminishing returns, as shown in Fig. 1. A good performance–complexity tradeoff can be obtained using the suboptimal decoder in (20).

For systems with more realistic settings (e.g., $N_c = 48$ as in IEEE 802.11a), we consider the low-complexity suboptimal decoder (20) in Fig. 2. It is shown that when AS is employed, the BER curve of the suboptimal decoder has the same slope as that of the ML decoder. The suboptimal decoder with $N_c = 6$ and $N_c = 48$ has the same performance as the case with $N_c = 2$. Note that with $N_c = 2$, the suboptimal decoder is ML, which achieves full diversity. Unlike the case for the ML decoder, the performance of the suboptimal decoder is independent of the number of subcarriers $N_c$. Compared to the ML decoder with $N_r = 6$, there is about a 2-DB coding gain loss for the suboptimal decoder.

For full-complexity systems, nondiagonal constellations are proposed in [12], which are block diagonal and have superior performance to the diagonal codes. We finally compare the performance of the nondiagonal code versus the diagonal code. We plot the block error rate, which is the probability of incorrectly decoding a matrix symbol. Fig. 3 displays the simulated performance of the block diagonal code $BDL_{48}$ of [12] with rate $R = 1.40$ on the channels with a uniform delay profile. We also compare the best diagonal code with the same data rate. We set $N_c = 48$ and apply the low-complexity suboptimal decoder (20). It is shown that, as in the case of full-complexity systems, the nondiagonal code further improves the performance with a larger coding gain compared to the diagonal code, even when AS is used. The performance of the nondiagonal and diagonal codes in the channels with an exponential delay profile is shown in Fig. 4, where, again, the nondiagonal code outperforms the diagonal one. Figs. 3 and 4 demonstrate that the nondiagonal code would still be a better choice for noncoherent STF-coded OFDM systems, when AS is employed.

VI. Conclusion

Without CSI at the receiver, we have investigated AS for a multiantenna OFDM system using unitary signals that operate over frequency-selective channels. We have proposed a pragmatic maximum signal-power selection rule, where the receive antenna with the largest received signal power averaged over all subcarriers is chosen. We simplify code design and decoding using subcarrier grouping. Based on analysis and simulations, we have shown that using AS, the maximum spatial and multipath diversity can still be obtained without CSI at the receiver that operates over frequency-selective channels, which is the same as the full-complexity multiantenna OFDM systems. For practical applications, we also present a suboptimal decoder that is much less complex. As in the case of full-complexity systems, the block diagonal codes further improve the performance compared to the diagonal ones, even when AS is employed.

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Space–Time Code Selection for Transmit Antenna Diversity Systems

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Abstract—In this correspondence, a space–time code selection approach for transmit antenna diversity (TAD) systems is proposed. The receiver compares the equivalent single-input–single-output channel envelopes with a set of predetermined threshold levels, decides the antennas to be used, and sends this information to the transmitter, which adjusts the space–time code to the number of selected antennas using a small amount of feedback information. Threshold levels are found out-of-line based on the Doppler frequency in such a way that channel state information outdate at the transmitter is taken into account. Analysis and simulation results show that the system performs well for a wide range of Doppler frequencies, outperforming transmit antenna selection techniques and adaptive TAD systems.

Index Terms—Adaptive coding, adaptive transmission, diversity methods, multiple-input–multiple-output (MIMO) systems, space-time coding, transmit antenna diversity (TAD), Wireless local area network (LAN).

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I. INTRODUCTION

Transmit antenna diversity (TAD) is expected to contribute to providing the high data rates needed by third- and fourth-generation wireless systems. To use several transmitter antennas at the same time, different schemes have been proposed; among them, Tarokh et al. proposed the space–time trellis coding [1]. Later, Alamouti [2] proposed the space–time block code (STBC) for two-transmitter antennas, which was generalized by Tarokh et al. to an arbitrary number of antennas [3]. These space–time codes were designed under the assumption that channel state information (CSI) was not available at the transmitter end.

One of the main drawbacks of using multiple antennas in the transmitter and/or the receiver is the increase in hardware complexity and cost. To reduce the hardware cost through the use of fewer RF chains and to take advantage of the CSI at the transmitter, several closed-loop schemes based on antenna selection have been proposed in recent years [4]–[11]. In [4]–[6], the advantages and performance of hybrid selection/maximum-ratio-combining techniques were analyzed. In [7]–[10], the attainable capacity of multiple-input–multiple-output (MIMO) systems with antenna selection was studied. In the context of spatial multiplexing, the advantages of using selection techniques have been considered [11]. The use of the antenna selection technique in conjunction with space–time codes has also been of great interest [12]–[16]. In [16], it was demonstrated that antenna subset selection in conjunction with space–time codes leads to an increase in the average SNR when exact channel knowledge is available, maintaining the diversity order equal to that with all available antennas.

As the interest in antenna selection in MIMO was primarily motivated by the need to diminish the RF hardware, antenna selection algorithms keep the number of selected antennas and the space–time code fixed. However, transmit antenna selection offers additional advantages, such as the minimization of the capacity and diversity losses due to channel correlation [17], [18]. Additionally, the aforementioned saving in hardware may vanish when channel selection is used in orthogonal frequency-division multiplexing (OFDM) systems with a subcarrier or subband (subcarrier group) selection approach. In the context of spatial multiplexing systems, a number of transmit adaptation schemes that select the transmitter antennas based on the CSI were proposed in [18]–[21]. Particularly, in [22] and [23], a modified version of linear precoding, which is called multimode precoding, varies the number of substreams and, hence, the number of transmitter antennas using the instantaneous CSI.

As closed-loop systems are limited by the need to maintain the amount of data in the feedback channel as low as possible, channel feedback information is one important issue when designing transmit systems. Different forms of channel information quantization have been proposed; a review of this subject can be found in [24]. Even when a number of schemes using limited feedback have been proposed, the amount of CSI at the feedback instant, as well as the feedback period (the time difference between feedback instants), would affect system performance.

In this correspondence, a space–time code selection (STCS) for TAD systems is proposed, in which the receiver decides on the antennas to be used through a comparison of the equivalent single-input–single-output (SISO) channel envelopes with a set of predetermined threshold levels and sends this information to the transmitter, which adjusts the space–time code to the number of selected antennas. The proposed selection algorithm allows the transmission to be adapted to the CSI, using 4 bits for feedback information, by selecting from one- to four-transmitter antennas and switching to the space–time code that is appropriated to this selection. Threshold levels are found out-of-line based on the maximum Doppler frequency in such a way that CSI outdate is taken into account. The inclusion of a no-transmission mode, whenever channels do not satisfy predetermined conditions, produces a throughput penalty. As the adaptation is done by varying the number of transmit antennas, the proposed system has some similarities with the multimode antenna selection [22], [23] but with several differences. First, in our proposal, a space–time encoding makes the technique suitable for systems with a single receiver antenna. Second, we propose a variable-rate system in which a bit error rate (BER) performance-throughput tradeoff is controlled by a set of threshold levels, with the inclusion of a no-transmission mode when none of the equivalent SISO channels fulfill the predetermined conditions. Third, different from most selection algorithms, the feedback period–coherence time relationship is taken into account in the proposed selection criterion; applying a selection criterion without considering the CSI outdate (such as a capacity or singular value criterion) with a single receiver antenna would lead to a single antenna selection technique.

Monte Carlo simulations show that the system performs well for a wide range of Doppler frequencies, outperforming both transmit antenna selection and adaptive transmit antenna techniques. Additionally, the system shows robustness against Doppler frequency estimation errors. Four-transmitter and one-receiver antennas were used in conjunction with the orthogonal STBCs (OSTBCs) proposed in [2] and [3], but the proposal can be used with other space–time codes. Narrowband channels were assumed to test the proposed scheme, but it could be extended to OFDM systems in a subcarrier or a subband selection basis in a similar way as in [25].

This correspondence is organized as follows. In Section II, the channel model and the system scheme are outlined. In Section III, an antenna selection algorithm is introduced in which different threshold levels are used for different equivalent SISO channels. In Section IV, the advantage of using the equivalent SISO channel will be addressed, and knowledge on Nakagami distribution is exploited for searching threshold levels. BER and throughput system analyses are performed in Section V, whereas some simulation results and performance discussion are presented in Section VI. Section VII concludes this correspondence.

II. SYSTEM MODEL

Fig. 1 illustrates the communication system model, in which four-transmitter and one-receiver antennas are employed. Initially, the transmitter sends pilot symbols to perform the vector channel estimation at the receiver. The equivalent SISO channel envelopes are compared with a given set of threshold levels, which are previously chosen according to the mobile speed. Afterward, the receiver decides on the space–time code and the transmitter antennas to use and sends this decision to the transmitter. The transmission is adapted to that
decision, which is maintained until the next decision instant, when the process is repeated.

The transmission is assumed to occur in flat frequency channels. Orthogonal space–time block encoding and decoding transform a MIMO fading channel with channel matrix $\Theta$ into an equivalent Gaussian SISO channel with a channel gain of $\|\Theta\|_F$, and the square envelope of the equivalent SISO channels using an OSTBC is given by

$$\alpha_{nT}^2 = \sum_{k=1}^{nT} r_k^2$$

where $n_T$ and $r_k^2$ represent the number of transmitter antennas and the square of Rayleigh distributed channel envelopes, respectively [26]. It is well known that $\alpha_{nT}$ follows a Nakagami-$n_T$ distribution [26] with a mean value of $\Omega_{nT} = 2n_T$. Its probability density function (pdf) is given by

$$p_N(x) = \left( \frac{n_T}{\Omega_p} \right)^{n_T} \frac{e^{-\frac{n_T}{\Omega_p}x}}{x^{\frac{n_T}{\Omega_p}+1}}$$

and the average fade duration (AFD) $\tilde{\tau}$ below $\rho = \rho_i$ is given by [27]

$$\tilde{\tau} = \frac{\Gamma(n_T+n_T\rho_i^2)}{\sqrt{2\pi f_m n_T^2 \rho_i^2}} e^{-\frac{n_T}{\Omega_p} \rho_i}$$

where $f_m$ represents the maximum Doppler shift (in hertz).

The received signal can be expressed by the following input/output relationship:

$$Y = \frac{1}{\sqrt{nT}} S_{nT} \cdot FH + V$$

where $H$ is the channel vector, $S_{nT}$ is a space–time coding matrix, $F$ is a permutation and selection matrix, and $V$ is the noise matrix. The matrix $S_{nT}$ may take the following structures, based on the space–time codes proposed in [2] and [3], depending on the number of selected antennas.

$$S_1 = \begin{pmatrix}
  s_0 & 0 & 0 & 0 \\
  s_1 & 0 & 0 & 0 \\
  s_2 & 0 & 0 & 0 \\
  s_3 & 0 & 0 & 0 
\end{pmatrix}$$

$$S_2 = \begin{pmatrix}
  s_0 & s_1 & 0 & 0 \\
  -s_1^* & s_0 & 0 & 0 \\
  s_2 & s_3 & 0 & 0 \\
  -s_3^* & s_2^* & 0 & 0 
\end{pmatrix}$$

$$S_3 = \begin{pmatrix}
  s_0 & s_1 & \frac{s_2}{\sqrt{2}} & 0 \\
  -s_1^* & s_0^* & \frac{-s_2^*}{\sqrt{2}} & 0 \\
  \frac{s_3}{\sqrt{2}} & \frac{s_3^*}{\sqrt{2}} & \frac{(s_0-s_3^*+s_1-s_2^*)}{2} & 0 \\
  -\frac{s_3^*}{\sqrt{2}} & \frac{-s_3}{\sqrt{2}} & \frac{(s_0+s_3^*+s_1+s_2^*)}{2} & 0 
\end{pmatrix}$$

$$S_4 = \begin{pmatrix}
  s_0 & s_1 & \frac{s_2}{\sqrt{2}} & \frac{s_3}{\sqrt{2}} \\
  -s_1^* & s_0^* & \frac{-s_2^*}{\sqrt{2}} & \frac{-s_3^*}{\sqrt{2}} \\
  \frac{s_3}{\sqrt{2}} & \frac{s_3^*}{\sqrt{2}} & \frac{(s_0-s_3^*+s_1-s_2^*)}{2} & \frac{(s_0+s_3^*+s_1+s_2^*)}{2} \\
  -\frac{s_3^*}{\sqrt{2}} & \frac{-s_3}{\sqrt{2}} & \frac{(s_0+s_3^*+s_1+s_2^*)}{2} & \frac{(s_0-s_3^*+s_1-s_2^*)}{2} 
\end{pmatrix}.$$  

For example, if antennas 1 and 4 are selected, matrix $F$ should permute columns 2 and 4 of $S_2$. Then

$$F = \begin{pmatrix}
  1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 1 \\
  0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 
\end{pmatrix}. \quad (9)$$

At the transmitter end, the number of ones in $F$ specifies $n_T$, and the antenna selection into that code is directly performed through the product $S_{nT} \cdot F$. Using a bit per antenna indicating that it is selected or not, the overhead transmitting matrix $F$ is limited to $n_T$ bits.

### III. Selection Algorithm

In this section, we describe the selection algorithm carried out at the receiver baseband in the discrete time domain. It is based on the comparison of possible equivalent SISO channels with a set of four threshold levels, which are selected depending on the maximum Doppler frequency. As the decision should be maintained until a new feedback information is available, it is not sufficient to decide on the decision instant based on the instantaneous CSI only but on the rate of change of the equivalent SISO channel envelope as well.

As the rate of change of the fading envelope decreases with $n_T$, it could be established that, to select an equivalent SISO channel only when its envelope is above a predefined threshold level during a given period of time, the lower the number of antennas that comprise the channel, the higher the requirements for selecting it. For a given instantaneous BER objective, equivalent SISO channels for larger $n_T$ values will take more time to drop to the envelope level that produces such an instantaneous BER objective. Therefore, the highest threshold level, i.e., $\rho_1$, will correspond to the one-antenna equivalent SISO channels, whereas the lowest threshold level, i.e., $\rho_4$, will correspond to the four-antenna equivalent SISO channel. Then, the selection algorithm proceeds as follows.

Step 1) Once the receiver obtains the CSI (which can be estimated based on a pilot sequence sent by the transmitter using all the transmitter antennas), it individually compares the Rayleigh fading envelopes with threshold $\rho_1$. If one or more of these envelopes are detected to be over $\rho_1$, the corresponding transmitter antennas are marked as selected and the procedure jumps to Step 5.

Step 2) The receiver compares all the Nakagami-2 fading envelopes with $\rho_2$. If one or more of these envelopes are detected to be over $\rho_2$, all the transmitter antennas that compose the selected envelopes are marked as selected, and the procedure jumps to Step 5.

Step 3) The receiver compares all the Nakagami-3 fading envelopes with $\rho_3$. If one or more of these envelopes are detected to be over $\rho_3$, all the transmitter antennas that compose the selected envelopes are marked as selected and the procedure jumps to Step 5.

Step 4) The receiver compares the Nakagami-4 fading envelope with $\rho_4$. If it is detected to be over $\rho_4$, all the transmitter antennas are marked as selected. Otherwise, no channel is selected (no-transmission mode is chosen).

Step 5) The receiver constructs the permutation and selection matrix $F$ and sends it to the transmitter.

It can be seen that the procedure sequentially compares with $\rho_1$ to $\rho_4$; as soon as one or more equivalent SISO channel is detected to be over its corresponding threshold level, the antennas composing such channel(s) are marked as useful, the procedure skips the next comparison steps, and matrix $F$ is constructed and sent to the transmitter.
where, of selected antennas and the modulation system in use, the channel is available. Adopting a conservative principle, the
has been used, and therefore, no information about the future behavior of the channel is available. To maintain system simplicity, no past channel information is not possible to select \( n_T \) transmitters with \( \rho_{n_T+k} \), \( k = 1, 2, \) and 3, thus preventing the selection of faster equivalent SISO channels with lower threshold levels.

The equivalent SISO channels are compared with their corresponding threshold levels, and as a result, channels above and under the threshold level are marked as "useful" and "not useful", respectively. The procedure is not forced to discard "useful" channels or to use "nonuseful" channels by selecting a fixed number of antennas. All "useful" equivalent SISO channels are selected with the intention to gain robustness against outdates in the feedback information through the use of the maximum possible number of suitable transmitter antennas. Finally, transmission is suppressed (no-transmission mode) if channels do not fulfill the predeterminded conditions, as in the adaptive modulation systems described in [25] and [28]–[30].

### IV. Threshold Level Search

Selecting an equivalent SISO channel only when its envelope is above a certain threshold level at the decision instant could prevent its envelope from dropping a given envelope level \( \alpha_{obj} \) until a new decision is made. By establishing a correspondence between the number of selected antennas and the modulation system in use, \( \alpha_{obj} \) can be obtained from the following approximate instantaneous BER expressions for square \( m \)-ary quadrature amplitude modulation (MQAM) and \( m \)-ary phase-shift keying (MPSK) with Gray bit mapping in additive white Gaussian noise at high SNRs, as a function of the instantaneous BER objective, the modulation system, and the mean SNR [31]:

\[
\text{BER}_{\text{MQAM}} \approx \frac{2}{k} \left( 1 - \frac{1}{\sqrt[k]{k}} \right) \text{erfc} \left( \sqrt{\frac{\alpha^2}{2} \frac{k \gamma_b}{2 k - 1}} \right)
\]

\[
\text{BER}_{\text{MPSK}} \approx \frac{1}{k} \text{erfc} \left( \sqrt{\frac{\alpha^2}{2} \gamma_b \sin \frac{\pi}{2k}} \right)
\]

where \( \alpha^2 \) represents the square equivalent SISO channel envelope, and \( M = 2^k \) and \( \gamma_b \) represent the constellation size and the mean bit energy per noise, respectively.

Fig. 2 illustrates a situation in which a selected equivalent SISO channel envelope coincides with the threshold level \( \rho_{obj} \) at the decision instant. To maintain system simplicity, no past channel information has been used, and therefore, no information about the future behavior of the channel is available. Adopting a conservative principle, the channel envelope is assumed to fall into a fade. To maintain the channel envelope above \( \alpha_{obj} \) until the next decision instant

\[
\tau_r \leq \frac{\bar{t}_i}{2} - \frac{\bar{t}_{obj}}{2}
\]

where \( \tau_r \) represents the information feedback period, and \( \bar{t}_i \) represents the AFD at the channel envelope level \( \rho_i \). Then, the minimum acceptable channel envelope value may be obtained by

\[
\rho_{n_T} = \min \left\{ \rho_{n_T} \approx \bar{t}_i - \bar{t}_{obj} \geq 2 \tau_r \right\}
\]

and \( \Omega_j \) represents the mean envelope level for a Nakagami-\( j \) channel that is given by

\[
\Omega_j = E[\alpha^2].
\]

Establishing an instantaneous BER objective allows threshold levels producing mean BER values lower than the instantaneous BER objective to be obtained, because the selected equivalent SISO channel envelope is constrained to be, on average, above a given \( \alpha_{obj} \) value. Through the instantaneous BER objective, one may adjust the BER performance-throughput tradeoff. As the instantaneous BER objective becomes lower, the threshold levels rise, the BER performance improves, and the throughput value diminishes.

To obtain the same throughput (3 b/s/Hz) when using different \( S_{n_T} \) matrices, an eight-phase shift keying (8-PSK) modulation was used when selecting one or two transmitters, whereas a 16-phase quadrature amplitude modulation (16-QAM) was used when selecting three or four transmitters. Two sets of threshold levels were found for every Doppler frequency \( f_d T_s \). Table I shows the threshold levels as a function of \( f_d T_s \), which were obtained using (13), for \( E_b/N_0 = 11 \) dB and a \( 5 \times 10^{-3} \) instantaneous BER objective. Using these levels, the proposed system will offer a throughput that is somewhat lower than \( 3 \) b/s/Hz, so it is used to compare with 3 b/s/Hz systems. Table II shows the threshold levels found for a \( 1.2 \times 10^{-3} \) instantaneous BER objective. Using this set of levels, the system throughput could be considerably lower than 3 b/s/Hz but larger than 2.25 b/s/Hz; consequently, it will be used to compare with antenna selection systems with 2 b/s/Hz (using QPSK) and with a four-antenna open-loop STBC system with 2.25 b/s/Hz (using 8-PSK).

<table>
<thead>
<tr>
<th>( f_d T_s \times 10^3 )</th>
<th>1.28</th>
<th>3.84</th>
<th>6.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_1 )</td>
<td>-1.10</td>
<td>-0.71</td>
<td>-0.41</td>
</tr>
<tr>
<td>( \rho_2 )</td>
<td>-1.10</td>
<td>-0.71</td>
<td>-0.41</td>
</tr>
<tr>
<td>( \rho_3 )</td>
<td>-1.59</td>
<td>-1.02</td>
<td>-0.67</td>
</tr>
<tr>
<td>( \rho_4 )</td>
<td>-1.59</td>
<td>-1.02</td>
<td>-0.80</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( f_d T_s \times 10^3 )</th>
<th>1.28</th>
<th>3.84</th>
<th>6.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_1 )</td>
<td>-0.43</td>
<td>-0.20</td>
<td>0.00</td>
</tr>
<tr>
<td>( \rho_2 )</td>
<td>-0.43</td>
<td>-0.20</td>
<td>0.00</td>
</tr>
<tr>
<td>( \rho_3 )</td>
<td>-1.00</td>
<td>-0.63</td>
<td>-0.36</td>
</tr>
<tr>
<td>( \rho_4 )</td>
<td>-1.00</td>
<td>-0.63</td>
<td>-0.50</td>
</tr>
</tbody>
</table>

**Fig. 2.** Relationship between \( \rho_i \) and \( \rho_{obj} \).
V. THROUGHPUT AND BER ANALYSES

A. Throughput Analysis

The size of the set of possible Nakagami-$\nu_T$ channels $\mathbb{Q}_{\nu_T}$ will have $C_{\nu_T}^4$ elements, where

$$C_{\nu_T}^4 = \binom{b}{a} \equiv \frac{b!}{a!(b-a)!}$$

(16)

is the number of combinations of $b$ channels taking $a$ at a time. From the selection algorithm in Section III, $k$ to four antennas could be selected using threshold level $\rho_k$ (in the case of selecting more than one $k$-antenna equivalent SISO channel). Let $S_{\nu,T}$ be the event of selecting $\nu_T$ transmitter antennas and $D_i$, be the event of selecting $\nu_T$ antennas using threshold levels $\rho_k$. Then, the probabilities of selecting one-, two-, three-, or four-transmitter antenna are

$$P\{S_1\} = P\{D_1\}$$

(17)

$$P\{S_2\} = P\{D_2 \cup D_3\}$$

(18)

$$P\{S_3\} = P\{D_3 \cup D_4 \cup D_5\}$$

(19)

$$P\{S_4\} = P\{D_4 \cup D_5 \cup D_6 \cup D_1\}$$

(20)

The following paragraphs deal with the probability of the different events in the sample space when using the proposed algorithm. The probability of the event of selecting one single transmitter antenna using $\rho_1$, which is independent of any other event, is equal to

$$P\{D_1\} = P\{\text{exactly one } \alpha_1^2 \geq \Omega_1 \rho_1^2\}$$

$$= C_1^1(1 - q_1)q_1^1$$

(21)

where $q_1 = P(\alpha_{\nu_T}^2 \geq \Omega_{\nu_T}^2 \rho_k^2)$. Similarly, selecting two, three, and four transmitters using $\rho_1$ is only possible if

$$P\{D_2\} = P\{\text{exactly two } \alpha_1^2 \geq \Omega_1 \rho_1^2\}$$

$$= C_2^2(1 - q_1)^2 q_1^2$$

(22)

$$P\{D_3\} = P\{\text{exactly three } \alpha_1^2 \geq \Omega_1 \rho_1^2\}$$

$$= C_3^3(1 - q_1)^3 q_1^3$$

(23)

$$P\{D_4\} = P\{\text{exactly four } \alpha_1^2 \geq \Omega_1 \rho_1^2\}$$

$$= (1 - q_1)^4$$

(24)

and the probability of the event of selecting any number of antennas using $\rho_1$ is

$$P\{D_{\nu_1}\} = P\{D_1\} + P\{D_2\} + P\{D_3\} + P\{D_4\}$$

(25)

since $D_{\nu_1}, k = 1, \ldots, 4$, are mutually exclusive events.

The event of selecting two transmitters using $\rho_2$ is only possible if no transmitter has been selected using $\rho_1$. Then

$$D_2 = D_1 \cap \{\text{exactly one } \alpha_2^2 \geq \Omega_2 \rho_2^2\}$$

(26)

and

$$P\{D_2\} \approx [1 - P(D_1)] P\{\text{exactly one } \alpha_2^2 \geq \Omega_2 \rho_2^2\}$$

(27)

Then

$$P\{D_2\} \approx [1 - P(D_1)] C_0^0(1 - q_2)q_2^0$$

(28)

where the events $D_1$ and $\{\text{exactly one } \alpha_2^2 \geq \Omega_2 \rho_2^2\}$ are assumed to be statistically independent.

When exactly two Nakagami-$\nu_T$ channels are above $\rho_2$, either three or four transmitters could be selected. For example, when $(r_1 + r_2)^2 \geq \Omega_2 \rho_2^2$ and $(r_1 + r_3)^2 \geq \Omega_2 \rho_2^2$, three transmitters are selected. By contrast, if $(r_1 + r_2)^2 \geq \Omega_2 \rho_2^2$ and $(r_3 + r_4)^2 \geq \Omega_2 \rho_2^2$, four transmitters are selected. However, as only a small fraction of the events produces the selection of four transmitters, in the analysis and for the sake of simplicity, it has been assumed that three transmitters are selected whenever exactly two Nakagami-$\nu_T$ channels are above $\rho_2$. For the event of selecting three transmitters using $\rho_2$

$$D_3 = D_1 \cap \{\text{exactly two } \alpha_2^2 \geq \Omega_2 \rho_2^2\}$$

(29)

and

$$P\{D_3\} \approx [1 - P(D_1)] P\{\text{exactly two } \alpha_2^2 \geq \Omega_2 \rho_2^2\}$$

(30)

Then

$$P\{D_3\} \approx [1 - P(D_1)] C_0^0(1 - q_2)^2 q_2^2$$

(31)

where the events $D_1$ and $\{\text{exactly two } \alpha_2^2 \geq \Omega_2 \rho_2^2\}$ are assumed to be statistically independent.

When exactly three Nakagami-$\nu_T$ channels are above $\rho_2$, it is possible to select either three or four transmitters, depending on the combinations of the Rayleigh channels, but most of the events lead to the selection of four transmitters. Thus, the problem is simplified again assuming that when exactly three Nakagami-$\nu_T$ channels are above $\rho_2$, four transmitters are always selected, i.e.,

$$D_4 = D_1 \cap \{\text{three or more } \alpha_2^2 \geq \Omega_2 \rho_2^2\}$$

(32)

then

$$D_4 = D_1 \cap \{\text{exactly three } \alpha_2^2 \geq \Omega_2 \rho_2^2\}$$

(33)

and

$$P\{D_4\} \approx [1 - P(D_1)] \left[ P\{\text{exactly three } \alpha_2^2 \geq \Omega_2 \rho_2^2\} + P\{\text{exactly four } \alpha_2^2 \geq \Omega_2 \rho_2^2\} + P\{\text{exactly five } \alpha_2^2 \geq \Omega_2 \rho_2^2\} + P\{\text{exactly six } \alpha_2^2 \geq \Omega_2 \rho_2^2\} \right]$$

(34)

Thus

$$P\{D_4\} \approx [1 - P(D_1)] \times C_0^0(1 - q_2)^3 q_2^3 + C_0^0(1 - q_2)^4 q_2^4$$

(35)

where the events $D_1$ and $\{\text{three or more } \alpha_2^2 \geq \Omega_2 \rho_2^2\}$ are assumed to be statistically independent. Then, the event of selecting any antenna using $\rho_2$ is

$$D_2 = D_2^2 \cup D_3^2$$

(36)

and

$$P\{D_2\} = P\{D_2^2\} + P\{D_3^2\}$$

(37)

since $D_2, k = 2, \ldots, 4$, are mutually exclusive events.
As in the case of $D_2^3$, the events of selecting three or four antennas using $\rho_3$ is only possible if no transmitter has been selected using $\rho_1$ or $\rho_2$. Then, the event of selecting three antennas using $\rho_3$ is

$$D_3^3 = D_1 \cap D_2 \cap \{ \text{exactly one } \alpha_3^2 \geq \Omega_3 \rho_3^2 \}$$

thus

$$P \{ D_3^3 \} \approx [1 - P(D_1)] [1 - P(D_2)]$$

and

$$P \{ D_3^3 \} \approx [1 - P(D_1)] [1 - P(D_2)] C_4^3 (1 - q_3) q_3^3$$

assuming statistical independence between events $D_1$, $D_2$, and {exactly one $\alpha_3^2 \geq \Omega_3 \rho_3^2$}. Then, the event of selecting four transmitters using $\rho_3$ is

$$D_4^3 = D_1 \cap D_2 \cap \{ \text{two or more } \alpha_3^2 \geq \Omega_3 \rho_3^2 \}$$

thus

$$D_4^3 = C_4^2 (1 - q_3)^2 q_3^2 + C_4^3 (1 - q_3)^3 q_3 + (1 - q_3)^4$$

assuming statistical independence between events $D_1$, $D_2$, and {two or more $\alpha_3^2 \geq \Omega_3 \rho_3^2$}. The event of selecting any antenna using $\rho_3$ is

$$D_3 = D_3^3 \cup D_4^3$$

and

$$P(D_3) = P \{ D_3^3 \} + P \{ D_4^3 \}$$

since $D_3^3$ and $D_4^3$ are mutually exclusive events.

The event of selecting four transmitters using $\rho_4$ is only possible if no previous selection has been made. Thus

$$D_4^4 = D_1 \cap D_2 \cap D_3 \cap \{ \alpha_4^2 \geq \Omega_4 \rho_4^2 \}$$

then

$$P \{ D_4^4 \} \approx [1 - P(D_1)] [1 - P(D_2)] [1 - P(D_3)] [1 - q_4]$$

Finally, the no-transmission event is given by

$$NT = D_1 \cup D_2 \cup D_3 \cup D_4^4$$

thus

$$P(NT) = [1 - P(D_1)] [1 - P(D_2)] [1 - P(D_3)] [1 - P(D_4^4)]$$

and the throughput is equal to

$$T_{br} = 1 - P(NT).$$

### B. BER Analysis

The probability of error of the STCS system is given by

$$P_e = \sum_{k=1}^{4} P \left(D_k^3\right) \int_{\alpha_{br1}}^{\infty} \text{BER}_k \left(\alpha_k^2\right) \tilde{p}_k \left(\alpha_k^2\right) d\alpha_k$$

$$+ \sum_{k=2}^{4} P \left(D_k^3\right) \int_{\alpha_{br2}}^{\infty} \text{BER}_k \left(\alpha_k^2\right) \tilde{p}_k \left(\alpha_k^2\right) d\alpha_k$$

$$+ \sum_{k=3}^{4} P \left(D_k^3\right) \int_{\alpha_{br3}}^{\infty} \text{BER}_k \left(\alpha_k^2\right) \tilde{p}_k \left(\alpha_k^2\right) d\alpha_k$$

$$+ P \left(D_4^4\right) \int_{\alpha_{br4}}^{\infty} \text{BER}_4 \left(\alpha_4^2\right) \tilde{p}_4 \left(\alpha_4^2\right) d\alpha_4$$

where $\text{BER}_k(\alpha_k^2)$ is the BER expression for $n_T = k$ as given by (10) and (11), and $\tilde{p}_k(\alpha_k^2)$ is a truncated chi-square distribution with $2k$ degrees of freedom, which is given by

$$\tilde{p}_k \left(\alpha_k^2\right) \equiv \frac{p_k \left(\alpha_k^2\right)}{P_k(\infty) - P_k(\alpha_k^2)}$$

where $p_k(\alpha_k^2)$ and $P_k(\alpha_k^2)$ represent, respectively, the chi-square probability distribution and cumulative distribution function with $2k$ degrees of freedom. The first term on the right-hand side of (52) represents the case of selecting from one to four antennas using $\rho_1$, whereas the second term represents the case of selecting from two to four antennas using $\rho_2$. The use of $\rho_3$ to select three or four antennas is represented by the third term, whereas the last one considers the selection of four antennas using $\rho_4$.

### VI. Simulation Results and Discussion

For the simulations, symbol-synchronous receiver sampling and ideal timing have been assumed. Narrow-band Rayleigh channels, which were modeled as a circular complex Gaussian variable with zero mean and unitary standard deviation, were used. For simplicity, a zero delay feedback channel and a perfect channel estimation at the receiver end have been assumed.

The transmitter power was maintained constant independent of the number of transmitter antennas in use, distributing it evenly over them. The symbol time was $T_s = 3.2 \mu s$ and the carrier frequency $f_c = 3.5$ GHz. The normalized Doppler frequency of the simulated channels was $f_d T_s = 1.28 \times 10^{-4}$, $f_d T_s = 3.84 \times 10^{-4}$, and $f_d T_s = 6.4 \times 10^{-4}$, corresponding to mobile speeds of approximately $v = 12.5$ km/h, $v = 37$ km/h, and $v = 62$ km/h, respectively. Pilot symbols used for channel selection (feedforward and feedback STCS information) were inserted every 140 symbols. Then, for the three test channels, a fixed feedback spacing of 140 symbols corresponds to...
TABLE III
ANALYTIC THROUGHPUT VERSUS SIMULATED THROUGHPUT
(IN BITS PER SECOND PER HERTZ) FOR SEVERAL MOBILE
VELOCITIES AND \( E_b/N_0 = 11 \text{ dB} \)

<table>
<thead>
<tr>
<th>( v ) (km/h)</th>
<th>Analytic throughput</th>
<th>Simulated throughput</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5</td>
<td>2.88</td>
<td>2.89</td>
</tr>
<tr>
<td>37</td>
<td>2.81</td>
<td>2.80</td>
</tr>
<tr>
<td>62</td>
<td>2.74</td>
<td>2.70</td>
</tr>
</tbody>
</table>

Fig. 3. Comparison between BER analysis results and simulation results.

10%, 30%, and 50% of the channels’ coherence time \( T_C \), respectively, when using the relationship between \( T_C \) and \( f_d \) that is stated by

\[
T_C = \frac{9}{16\pi f_d}. \quad (54)
\]

Table III shows the throughput results from analysis and simulations for different mobile speeds and \( E_b/N_0 = 11 \text{ dB} \). There is a good agreement between analysis and simulations, and it can be observed that in both cases throughput decreases with mobile speed. This is because threshold levels are moved up with speed, providing protection to the system against CSI is outdated.

Fig. 3 shows a comparison between analysis and simulation BER results for different mobile speeds. Analysis results are very similar despite the mobile velocity. This is consistent with the computation of the threshold levels for the same instantaneous BER objective in both situations. The differences between simulation and analysis results can be explained by the fact that the truncated pdfs in (3) are an approximated model of the channel statistics in the proposed system, since there is a small but finite probability of the channel envelopes that is under \( \alpha_{obj} \). This has been verified in the simulations and originates from the fact that fading duration is random in nature while the calculation of \( \alpha_{obj} \) is performed using its average value. The probability of finding channel values under \( \alpha_{obj} \) increases with the feedback period, which explains the BER degradation for high mobile speeds and high SNRs that can be observed for the case of 62 km/h in Fig 3.

To present the simulation results, we compare the double antenna selection (DAS) technique proposed in [16] and the four-transmitter open-loop STBC (OLH\textsubscript{4}) proposed in [3] with the single antenna selection (SAS) technique. Using 8-PSK for SAS and DAS and 16-QAM for OLH\textsubscript{4}, systems transmitting at 3 b/s/Hz are obtained, whereas using QPSK for SAS and DAS and 8-PSK for OLH\textsubscript{4}, systems transmitting at 2 and 2.25 b/s/Hz are obtained. Similarly, to compare with four-transmitter systems, we also compare with the combination of OSTBC and adaptive subgroup antenna encoding (OSTBC-SGE) that is proposed in [32] and the combination of OSTBC and beamforming (OSTBC-BF) that is proposed in [33]. 8-PSK and 16-QAM were used for OSTBC-SGE and OSTBC-BF to obtain systems transmitting at 3 b/s/Hz; when using QPSK for OSTBC-SGE and 8-PSK for OSTBC-BF, systems transmitting at 2 and 2.25 b/s/Hz were obtained. When simulating OSTBC-SGE, a 4-bit feedback was used. With regard to beamforming systems, quantization techniques have a significant impact on their performance [34], [35]. In [34], it was shown that for a four-transmitter beamforming system with a 4-bit feedback, an optimum beamformer design leads to a system approaching the perfect CSI (PCSI) case. To compare with beamforming techniques, PCSI at the transmitter was assumed at the feedback instant, considering the CSI until the next feedback instant, when simulating the OSTBC-BF systems.

Figs. 4 and 6 show the simulated BER performance and throughput for different SNRs and mobile speeds compared with systems transmitting at 3 b/s/Hz. Fig. 4 shows the BER performance and throughput of the proposed technique compared with systems transmitting at 3 b/s/Hz for \( v = 12 \text{ km/h} \) over uncorrelated channels.

Fig. 5. BER performance and throughput of the proposed technique compared with systems transmitting at 3 b/s/Hz for \( v = 37 \text{ km/h} \) over uncorrelated channels.
is recent enough during the whole time period between decisions, and selecting the best antenna provides good results. As the diversity gains are expected to appear at high SNRs, the benefits of selecting the best antenna exceed those of using STCS at low SNRs; but at high SNRs, the STCS largely outperforms SAS. It can be observed that OSTBC-BF obtains a better performance than STCS for SNRs up to 8 dB, but a higher slope in the BER curve for the STCS produces important gains for higher SNRs. Finally, the gain of the STCS w.r.t. OSTBC-SGE is significant at any SNR. The throughput for the systems of reference is, in this case, equal to 3 b/s/Hz, while using STCS has an effect on the throughput; the lower legend shows the throughput of the STCS system, which ranges from 2.8838 to 2.9137 b/s/Hz, representing an average throughput penalty of 3.5% of the maximum attainable throughput.

Figs. 7 and 9 show the simulated BER performance and throughput for different SNRs and mobile speeds compared with antenna selection systems transmitting at 2 and 2.25 b/s/Hz, respectively. In Fig. 7, it can be observed that for $v = 12.5$ km/h both SAS and DAS outperform STCS for SNRs up to 8 dB, but the STCS BER performance is equal or better than that of all other systems for higher SNRs. With regard to adaptive transmission systems, OSTBC-SGE is severely affected in this conditions, particularly by the feedback information outdate, causing its performance to be worse than that of OLH4 for high SNRs. On the other hand, the OSTBC-BF obtains an excellent performance for low SNRs, but for $E_b/N_0 \geq 14$ dB it is lower than that of STCS. Finally, the STCS throughput is 37% higher on average than that of SAS, DAS, and OSTBC-SGE and 21.5% higher than that of OLH4 and OSTBC-BF. Fig. 8 shows that for $v = 37$ km/h the STCS BER performance is equal or better than that of all other systems for the whole SNR range. The throughput decrease is around 10% on average.

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Fig. 9. BER performance and throughput of the proposed technique compared with systems transmitting at 2 and 2.25 b/s/Hz for $v = 62$ km/h over uncorrelated channels.

Fig. 10. BER performance and throughput of the proposed technique compared with systems transmitting at 2 and 2.25 b/s/Hz for $v = 12$ km/h over correlated channels with $\bar{\sigma} = 0.6$.

to satisfy this new constraint. It can be observed that the SAS and DAS BER performances were significantly affected in such a way that the STCS outperforms SAS in 4 dB for a mean BER objective of $10^{-3}$, increasing this difference for higher SNRs. Similarly, OSTBC-BF experiments a considerable BER performance diminishing due to correlation, which was outperformed by STCS for high SNRs. The effects of correlation on the STCS are noticeable in its throughput, which has dropped to 2.54 b/s/Hz; however, it is still higher than that of all systems of reference.

As stated earlier, the $\alpha_{\text{obj}}$ value (or equivalently, the predefined BER objective) controls the BER performance/throughput relationship. Nevertheless, Doppler frequency estimation errors could cause a set of threshold levels to be used for a different speed range than that for which it was found. Fig. 11 shows that in this case the BER/throughput tradeoff is altered but without causing a collapse in the system performance. It can be seen that when using a set of threshold levels found for $v = 37$ km/h, the STCS system with a mobile speed of $v = 12.5$ km/h diminishes its BER performance, with an increment in the resulting throughput (from around 93% to around 96% of the maximum throughput). A similar situation can be observed when a set found for $v = 62$ km/h is used for $v = 37$ km/h. In this case, the BER performance is improved at the expense of a lower throughput. However, the resulting throughput is between 2.7053 and 2.7173 b/s/Hz (90.1% and 90.6% of the maximum throughput), which is still close to that when using the corresponding set of threshold levels.

VII. CONCLUSION

A simple STCS scheme for TAD systems with a single receiver antenna has been proposed. The space–time code for transmission is selected by comparing the equivalent SISO channels with a set of predetermined threshold levels. A threshold level search procedure based on an instantaneous BER constraint, taking into account the feedback information outdate, is described, and two sets of threshold levels are proposed as a function of the Doppler frequency for comparing with different spectral efficiency systems. BER and throughput analyses were performed, obtaining a good agreement between analysis and simulation results. Simulation results show that the system performs well for a wide range of Doppler frequencies, outperforming both transmit antenna selection and adaptive transmission techniques at the expense of a short throughput reduction, or obtaining a moderate BER performance improvement for high SNRs but with a larger throughput. Finally, the proposed system showed robustness against Doppler frequency estimation errors.

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Comments on “Mobile Radio Power Distribution in Shadowing and Variable Range”

Saralees Nadarajah and Samuel Kotz

Abstract—It is pointed out that the mean power distribution recently introduced by McGuffin [1] derived a compound distribution to model the power fluctuations of mobile radios. The distribution is given by [1, (8) and (9)]. The paper also derived the various properties of this distribution (see [1, (10)–(14)].

I. INTRODUCTION

The recent paper by McGuffin [1] derived a compound distribution to model the power fluctuations of mobile radios. The distribution is given by [1, (8) and (9)]. It was derived as the distribution of the product of two independent random variables: one following the Pareto distribution and the other following the lognormal distribution. The properties of this distribution (including those derived by McGuffin [1]) and its generalizations have extensively been studied in the statistics literature (see [3]–[5] and the references therein).

II. MEAN POWER DISTRIBUTION

We would like to point out that the distribution given by [1, (8) and (9)] is due to Colombi [2]. It was derived as the distribution of the product of two independent random variables: one following the Pareto distribution and the other following the lognormal distribution. The properties of this distribution (including those derived by McGuffin [1]) and its generalizations have extensively been studied in the statistics literature (see [3]–[5] and the references therein).

III. DISCUSSION

The purpose of this paper is not just to provide priority statements for the results derived in [1]. We feel that the previously mentioned references can be of assistance in modeling problems of the type considered by McGuffin [1].

REFERENCES


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