Dynamically balanced optimal gaits of a ditch-crossing biped robot

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**Abstract**

This paper deals with the generation of dynamically balanced gaits of a ditch-crossing biped robot having seven degrees of freedom (DOFs). Three different approaches, namely analytical, neural network (NN)-based and fuzzy logic (FL)-based, have been developed to solve the said problem. The former deals with the analytical modeling of the ditch-crossing gait of a biped robot, whereas the latter two approaches aim to maximize the dynamic balance margin of the robot and minimize the power consumption during locomotion, after satisfying a constraint stating that the changes of joint torques should lie within a pre-specified value to ensure its smooth walking. It is to be noted that the power consumption and dynamic balance of the robot are also dependent on the position of the masses on various links and the trajectory followed by the hip joint. A genetic algorithm (GA) is used to provide training off-line, to the NN-based and FL-based gait planners developed. Once optimized, the planners will be able to generate the optimal gaits on line. Both the NN-based and FL-based gait planners are able to generate more balanced gaits and that, too, at the cost of lower power consumption compared to those yielded by the analytical approach. The NN-based and FL-based approaches are found to be more adaptive compared to the other approach in generating the gaits of the biped robot.

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1. Introduction

During navigation, a legged robot might have to cross ditches, move along sloping surfaces, ascend and descend staircases, avoid some obstacles, and do other things, as the situation demands. Out of all these tasks, the present study concentrates only on ditch-crossing problems. To cross a ditch, a legged robot has to plan the sequence of its leg movement in such a way that it can tackle that situation by consuming a minimum amount of energy after satisfying the conditions of dynamic balance. To the best of the authors’ knowledge, not much work has been reported in this field of research. Junshi and Junmin [1] explained the ditch-crossing process of a four-legged walking robot and proposed a transition gait to cross the ditch and developed a corresponding control program. Ditch-crossing problems of a hexapod robot were considered in [2], in detail. A genetic–fuzzy approach had been developed to generate optimal/near-optimal crab gaits of a six-legged robot crossing a ditch [3]. Ditch-crossing problems were solved using the concept of fuzzy logic controller [4], and a genetic algorithm [5] was utilized to optimize the performance of the gait planner. In the present work, optimal gait generation problems of a dynamically balanced 7-DOF biped robot have been considered.

To generate a suitable gait of a biped robot navigating on a terrain, it is required to decide the swing foot trajectory to avoid collision with the obstacles present on the terrain and the hip trajectory to fulfill the repeatability conditions. The trajectory planning and control problem of a biped robot on various maneuvers were solved by Lum et al. [6]. The biped robot also needs to identify the obstacles present in its environment. Sensory information was utilized in [7] to identify the size and shape of the obstacles. A biped robot may have to move on various types of terrain conditions, such as a flat floor, staircases, sloping surfaces, and others, to perform the task assigned to it. A biped robot with variable length legs (that is, considering a prismatic joint at knees) and translatable balance weight was designed, built and experimented to generate an efficient walking gait for ascending and descending stairs [8]. Mousavi and Bagheri [9] performed mathematical simulations of a seven-link biped robot moving on flat surface and staircase by utilizing a fixed and moving zero movement point (ZMP) [10], which is a point lying on the ground about which the movement generated by all active forces will become equal to zero. A motion planning scheme was developed for an SD-2 biped robot to climb sloping surfaces using position sensors on the joints and force sensors underneath the heel and toe. The study was considered for static walking [11], quasi-dynamic and dynamic walking [12] and the biped robot was able to detect the transition of the supporting terrain from a flat floor to a sloping surface. In addition to these studies, a walking control method consisting of position control, virtual compliance control and postural control was developed for...
The trunk motion plays a vital role in balancing the walking machine. The most popular semi-inverse method [15,16] was developed to generate the trunk motion [17] with the prior definition of ZMP location and the motion of the lower limbs [18]. The dynamic balance of a biped robot is generally measured with the help of a dynamic balance margin (DBM) [19], calculated based on the concept of the ZMP. The biped robot is said to be dynamically balanced as long as the ZMP lies inside the convex hull of the support polygon. Moreover, the joint angle control [20] with the help of indirect manipulation of the ZMP using the concept of an inverted pendulum was considered in [21]. Other control schemes, such as impedance control for the plain walking [22] of a biped robot, angular momentum control [23] for dynamic walking and integrated control [24] of a biped robot for human-like gait had also been developed.

Although the above-discussed methods laid the foundation for solving the problems related to walking control of biped robots, the solutions obtained were not optimal in any sense. Moreover, they may not be suitable for on-line implementations because of their computational complexity. In order to have better mechanical design and gait planners, which will increase the dynamic balance, reduce the power consumption and increase the possibility of using them for on-line control, it is necessary to develop simultaneous optimization schemes. Soft computing-based approaches (that is, the neural network (NN), fuzzy logic (FL), genetic algorithm (GA) and their various combinations) may be suitable for handling complex, uncertain, imprecise, real-world problems [25]. Attempts were also made to solve the optimization problems of a biped robot related to mechanical design, controller design and gait generation using the concept of soft computing. A steady-state GA [26] was used for the optimized control of the biped locomotion, in which each individual GA solution consists of the required joint angles to follow a desired trajectory for the center of mass of the system. In [27], a GA was utilized for generation of the optimal walking trajectory of a biped robot. The optimal via points, which minimize the sum of deviation of velocities and accelerations, were determined using the GA. Another method of generating natural motion of a two-legged robot, utilizing a hierarchical evolution algorithm comprising a GA and an evolutionary programming (EP) algorithm, was proposed in [28]. The GA and EP were used to minimize the total energy of all the actuators and optimize the configuration of the biped robot, respectively. Moreover, a GA was utilized to generate the optimal trajectory for a 8-DOF biped robot that could walk on a flat terrain and climb stairs with deformation at the sole [29]. The trajectory was optimized after correcting the computed angle with the help of a control method. Various combinations of NN, FL and GA have also been tried by some researchers to solve similar problems. A hierarchical control system consisting of walking planning, gait generation and joint control levels was developed with the help of an adaptive network-based fuzzy inference system (ANFIS) [30], which enhances a Sugeno-type fuzzy logic controller (FLC) with a self-learning capability. The coupled evolution of morphology and control of a biped robot was proposed by Paul and Bongard [31], in a simulated environment with the help of a GA and closed-loop recurrent NN. The GA was utilized to optimize the morphology and the latter was used to control the robot. A gait synthesis method using a GA and a radial basis function neural network (RBFNN) was developed by Capi et al. [32]. The GA was utilized to generate the joint angle trajectories based on the minimum consumed energy and change in joint torques. The minimum consumed energy gaits were used to teach the RBFNN. This method was verified both in computer simulations [32] and on a real robot [33]. An on-line fuzzy adaptation scheme was proposed in [34] to tune the biped robot trajectory parameters. Gu and Hu [35] used a GA to evolve the membership function distributions of a fuzzy logic controller.
for SONY legged robots. Moreover, a method for on-line stable gait generation of a biped robot was proposed in [36] utilizing a genetic–fuzzy system, in which a GA was used to optimize the knowledge base of the FL-C, off-line.

Vundavalli et al. [37] solved the problems related to dynamically balanced ascending and descending gait generation of a biped robot moving on a staircase. The gait of the lower limbs and trunk were generated using the concept of inverse kinematics and static balance, respectively. The generated gait was also verified for its dynamic balance. The same authors were also able to develop an appropriate on-line gait planner for the above task utilizing the principle of soft computing [38]. Moreover, Vundavalli and Pratihar [39] optimized the position of mass centers of the links, hip trajectory and gaits of a biped robot negotiating various terrains, such as a staircase and a sloping surface, by using soft computing-based approaches. A GA had been used to optimize the position of mass centers and hip trajectory and GA-tuned NN and FL gait planners were developed to generate optimized gaits of the biped robot, on-line.

In the present work, an attempt has been made to generate optimal ditch-crossing gaits of a dynamically balanced 7-DOF biped robot. Three approaches, namely analytical, NN-based and FL-based, have been developed. In the analytical approach, the gaits of the lower limbs have been determined based on the concept of inverse kinematics and trunk motion is generated using the concept of static balance. However, in the present study, a different locomotion trajectory is followed by the biped robot crossing the ditch, which was not tried in [37,38]. A GA has been used to optimize the position of the mass centers of various links and coefficients of the cubic polynomial of the hip trajectory. It is important to mention that almost similar approaches have been developed by the authors in [39] for solving locomotion scenarios like ascending and descending a staircase and a sloping surface. However, in the present study, a different locomotion scenario, namely ditch crossing, has been tackled. The GA-tuned NN-based and FL-based gait planners will be used to generate suitable ditch-crossing gaits of the biped robot. The performances of all three approaches will be compared in the present paper.

The rest of the paper is organized as follows. Section 2 deals with mathematical formulation of the problem. The approaches developed are explained in Section 3. The results are presented and discussed in Section 4. Some concluding remarks are made in Section 5.

2. Mathematical formulation of the problem

This paper deals with the optimal ditch-crossing gait generation problem of a dynamically balanced biped robot. The schematic view of the robot used in this study is shown in Fig. 1. The robot is assumed to have seven links connected with the help of seven rotary joints to form a 7-DOF (two at the ankles, two at the knee and three at the hip) two-legged robot. The effect of mass has been injected into the system after adding the lumped masses to the respective links. In the present study, the balance of the robot is checked only along the direction of its movement.

Step length $l$ and hip height $h_l$ (refer to Fig. 1) are two important parameters to be considered in the gait generation of a biped robot, which are determined from its foot placement information (expressed in terms of $x$, $x_h$ and $x_s$ with respect to a fixed coordinate system), joint angles $\theta_2$ and $\theta_3$ of the swing leg and length of the links $l_2$ and $l_3$ as follows:

$$l = x_3 - x_1$$

$$h_l = L_2 \cos \theta_2 + L_3 \cos \theta_3.$$  

2.1. Gait generation and balancing

The trajectory of the swing foot moving from back to the front, in the direction of motion, is assumed to follow a cubic polynomial (refer to Fig. 1), which can be expressed as follows:

$$z = c_0 + c_1 x + c_2 x^2 + c_3 x^3,$$  

where $z$ represents the height of the swing foot (that is, ankle joint) at a distance $x$ from the starting point and $c_0$, $c_1$, $c_2$ and $c_3$ are the coefficients, whose values are to be determined with the help of some boundary conditions. The hip joint is allowed to follow a straight line trajectory, whose slope is assumed to be equal to that of the surface. The maximum duration ($T$) and velocity ($V_{\text{max}}$) of the swing leg in each time step are assumed to be equal to 5.0 s and 0.056 m/s, respectively. The maximum time step and velocity have been decided based on the assumed capacity of the motors. Each time step consists of a constant velocity of the swing leg for 3 s, and acceleration and deceleration parts for 1 s each. Both the swing foot and the hip joint trajectories are divided into eight equal intervals for the purpose of study. After knowing the values of $h_l$ and distance $l_1$ of the hip joint from the swing foot following the trajectory (refer to Fig. 1), the lower limbs’ gait can be determined using the concept of inverse kinematics. For example, the joint angles $\theta_2$ and $\theta_3$ can be obtained as given below.

$$\theta_1 = \sin^{-1}\left(\frac{l_1 L_3 \sin \psi_3 + l_2 (l_2 + L_2 \cos \psi_3)}{(l_2 + L_2 \cos \psi_3)^2 + (l_2 \sin \psi_3)^2}\right),$$  

where $h_l = L_2 \cos \theta_3 + L_3 \cos \theta_3 + z$, $l_1 = L_2 \sin \theta_2 + L_3 \sin \theta_3 - x_1 + x$, $\psi_3 = \theta_2 - \theta_3 = \arccos((h_l^2 + L_2^2 - L_3^2 - l_1^2)/2 L_2 L_3)$. Thus, $\theta_3$ can be calculated from the expression $\theta_3 = \theta_2 - \psi_3$. Similarly, the angles $\theta_2$ and $\theta_3$ are also calculated using the mathematical expressions shown below.

$$\theta_2 = \sin^{-1}\left(\frac{l_2 l_3 \sin \psi_2 + l_2 (l_2 + L_2 \cos \psi_2)}{(l_2 + L_2 \cos \psi_2)^2 + (l_2 \sin \psi_2)^2}\right),$$  

where $h_l = L_2 \cos \theta_1 + L_3 \cos \theta_2$, $l_2 = L_2 \sin \theta_1 + L_3 \sin \theta_2 - x_1 + x$, $\psi_2 = \theta_1 - \theta_2 = \arccos((h_l^2 + L_2^2 - L_3^2 - l_2^2)/2 L_2 L_3)$. The angle $\theta_2$ can be calculated utilizing the expression $\theta_2 = \theta_2 - \psi_2$. The following repeatability conditions are to be satisfied by the lower limbs, while walking through the terrain:

$$\theta_2_{\text{initial}} = \theta_2_{\text{final}},$$

$$\theta_3_{\text{initial}} = \theta_3_{\text{final}},$$

$$\dot{\theta}_2_{\text{initial}} = \dot{\theta}_2_{\text{final}},$$

$$\dot{\theta}_3_{\text{initial}} = \dot{\theta}_3_{\text{final}}.$$  

The trunk motion can be generated utilizing the concept of static balance, which is different from the well-known semi-inverse method [15] and finally verified for its dynamic balance after determining the position of the ZMP. The mathematical expression utilized for calculating the trunk motion $\theta_4$ by using the concept of static balance is given below.

$$\theta_4 = \sin^{-1}\left(\frac{1}{m_4} \left((p \times m_{\text{sum}}) - (m_1 d_1 + m_2 d_2 + m_3 d_3 + m_4 d_5 + m_5 d_6 + m_6 d_7)\right)\right),$$  

where $p$ denotes the average distance of projected mass centers from the ankle joint and $d_1$, $d_2$, $d_3$, $d_5$, $d_6$, and $d_7$ represent the distances of masses $m_1$, $m_2$, $m_3$, $m_4$, $m_5$, and $m_6$, respectively, from the projection of the respective mass center, $r_4$ indicates the distance of $m_4$ from the hip joint and $m_{\text{sum}} = m_1 + m_2 + m_3 + m_4 + m_5 + m_6 + m_7$. The trunk motion is generated for the first half of the cycle and verified for its dynamic balance. If the system is found to be dynamically balanced for the first half of the cycle, the
Fig. 1. A schematic view of a two-legged robot (7 DOF) crossing a ditch.

trunk motion for the next half of the cycle is generated based on the repeatability conditions as given below.

\[
\begin{align*}
\theta_{4, \text{init}} &= \theta_{4, \text{final}}, \\
\dot{\theta}_{4, \text{init}} &= \dot{\theta}_{4, \text{final}}.
\end{align*}
\]

Once the entire gait is generated, it has to be verified for its dynamic balance. The generated gait is said to be dynamically balanced, as long as the ZMP lies inside the foot support polygon.

The coefficient of friction between the ground and foot is assumed to be sufficient enough to prevent slipping. The distance of the ZMP from the ankle joint of the supporting foot can be measured in the direction of motion as follows:

\[
x_{ZMP} = \frac{\sum_{i=1}^{7} \left( I_i \ddot{\omega}_i + m_i \dot{x}_i (\ddot{z}_i - g) - m_i \ddot{x}_i \right)}{\sum_{i=1}^{7} m_i (\ddot{z}_i - g)},
\]

where \( I_i \) represents the moment of inertia of the \( i \)-th link (kg m\(^2\)), \( \ddot{\omega}_i \) denotes the angular acceleration of link \( i \) (rad/s\(^2\)), \( m_i \) denotes the mass of the \( i \)-th link (kg), \( (x_i, z_i) \) is the coordinate of the \( i \)-th lumped mass, \( g \) indicates the acceleration due to gravity (m/s\(^2\)), \( \ddot{z}_i \) is the acceleration of link \( i \) in the z-direction (m/s\(^2\)), and \( \ddot{x}_i \) is the acceleration of link \( i \) in the x-direction (m/s\(^2\)).

The two-legged robot is said to be dynamically balanced only when the ZMP lies inside the foot support polygon. Otherwise, the hip joint, knee, and ankle joint torques of the supporting leg must be updated to bring the ZMP back to a safe zone. The dynamic balance margin (DBM) is defined as the distance between the edge of the foot supporting polygon and the point where the ZMP acts. The DBM for the ditch-crossing gait can be obtained as follows:

\[
x_{\text{DBM}} = \left( \frac{L_7}{2} - |x_{ZMP}| \right)
\]

where \( L_7 \) is the length of the supporting foot and \( x_{ZMP} \) is the distance of the ZMP from the ankle joint of the supporting foot measured in the direction of movement.

2.2 Torques and average power consumption

The dynamics of the biped robot is solved utilizing a Lagrange–Euler formulation. The D-H parameter setting and the corresponding table are shown in Fig. 2 and Table 1, respectively. The angle of twist (i.e., \( \alpha_{i-1} \)) for all the links is found to be equal to 0.0, as their z-axes are seen to be parallel to each other. As all the joints are rotary in nature, the values of link offset are found to be equal to 0.0. The angle of twist is considered to be positive when it is measured in the anti-clockwise sense with respect to the horizontal axis. Then, the relationships between the gait angles and D-H parameter angles are set as follows:

<table>
<thead>
<tr>
<th>joint</th>
<th>( \alpha_{i-1} )</th>
<th>( \alpha_i )</th>
<th>( \theta_i )</th>
<th>( d_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>( L_1 )</td>
<td>( q_1 )</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>( L_2 )</td>
<td>( q_2 )</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>( L_3 )</td>
<td>( q_3 )</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>( L_3 )</td>
<td>( q_4 )</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>( L_4 )</td>
<td>( q_5 )</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>( L_5 )</td>
<td>( q_6 )</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>( L_6 )</td>
<td>( q_7 )</td>
<td>0</td>
</tr>
</tbody>
</table>
Following expression:

\[ P = \sum_{i=1}^{n} p_i, \]  

where \( n \) is the number of joints. In this expression, power consumption during the walking of the robot has been considered, which includes inertia, gravitational, Coriolis and centrifugal effects. However, this power term does not consider the dissipation of power for static postures, to simplify the model.

### 3. Developed approaches

Three different approaches, namely an analytical approach, an NN-based approach and an FL-based approach, are developed and tested for the generation of optimal ditch-crossing gaits of a dynamically balanced biped robot, as explained below.

#### 3.1. Approach 1: Analytical approach

A systematic procedure has been followed to generate the said gait of a biped robot, as discussed below.

**Step 1:** Assume cubic polynomial and straight line trajectories for the swing foot and hip joint (refer to Fig. 1), respectively, to simplify the simulated model of the biped robot.

**Step 2:** Generate the gait of the lower limbs based on the concept of inverse kinematics and trunk motion with the help of the static balance criterion. The method of trunk motion generation has been explained in Section 2.1.

**Step 3:** The generated motion/gait is verified for its dynamic balance by determining the position of the ZMP (refer to Eq. (9)). The following boundary conditions will have to be used to determine the coefficients \((c_0, c_1, c_2 \text{ and } c_3)\) of the swing foot trajectory given in Eq. (3).

- at \( x = x_1, z = 0 \)
- at \( x = d_l - \frac{b_1}{2}, z = \frac{l_2}{2} \)
- at \( x = d_l + d_0 - \frac{b_1}{2}, z = \frac{l_2}{2} \)
- at \( x = x_2, z = 0 \)

where \( d_l \) represents the distance of the ditch from a fixed reference point, \( d_0 \) indicates the width of the ditch and \( f_s \) denotes the length of the foot. The hip joint is allowed to follow a straight line trajectory. After knowing both these trajectories and considering a hypothetical double support phase at different instants of time in a cycle, the lower limbs’ gaits have been generated with the help of Eqs. (4) and (5). It is to be noted that the lower limbs’ gaits will have to follow the repeatability conditions as given in Eq. (6). The trunk motion is generated using the concept of static balance given in Eq. (7). It is also necessary to ensure that the trunk motion follows the repeatability conditions given in Eq. (8). The ankle joint angles for both the swing foot and the supporting foot have been assumed to be equal to 0.0 in this approach, for simplicity. The generated gait is verified for its dynamic balance through the calculation of the DBM (refer to Eq. (10)).

The torques required at various joints of the walking machine are calculated using the Lagrange–Euler formulation. The joint angles are assumed to vary by following the fifth-order polynomial given in Eq. (11). Once the joint angle variation is obtained, its first and second derivatives with respect to time can be determined. This information is then used to calculate the torques required at various joints (refer to Eq. (12)) of the two-legged robot. Finally, the amount of power requirement is calculated by utilizing the expression given in Eq. (13).

#### 3.2. Approach 2: Neural network-based approach

The optimized ditch-crossing gait generation problems of a 7-DOF biped robot have been solved utilizing an NN-based approach. An attempt has been made to optimize the position of the mass centers of the links, hip trajectory and gaits of the biped robot, while crossing a ditch. Fig. 3 shows the flowchart for the proposed algorithm, the principles of which are explained in the subsequent subsections. This approach differs from Approach 1 in the following ways.

- Here, the positions of the ankle joints of the swing foot and supporting foot are given as inputs to the gait planner to predict \( h_1 \) and \( l_1 \), whereas, in Approach 1, the initial posture of the robot (expressed in terms of \( \theta_2 \) and \( \theta_3 \)) is supplied to calculate the height \( h_1 \) and distance \( l_1 \) of the hip joint from the swing foot to start the algorithm.

- The variations of joint angles for both the trunk and the swing foot are predicted by the gait planner in this approach, whereas, in Approach 1, the trunk motion is generated using the concept of static balance and the joint angle variation of the swing foot is assumed to be equal to zero.
In this approach, the hip joint is allowed to follow a cubic polynomial trajectory and the positions of lumped masses on the links are relaxed to vary within a certain range on them. This has been done as they might have a significant influence on the DBM and power consumption of the biped robot. In the present study, the mass of each link is assumed to be concentrated at a point known as the mass center, whose position indicates the distribution of the same along that link. Thus, an attempt has been made here to determine the optimal position of the mass center. A GA is used to optimize the coefficients of the cubic polynomial of the hip trajectory and the positions of the lumped masses. A GA-trained NN gait planner is developed to maximize the DBM and minimize the power consumption subject to a constraint, such that the change in joint torque becomes less than a pre-specified value to obtain a smooth walking cycle. On the other hand, in Approach 1, the hip trajectory is assumed to be a straight line and the positions of the mass centers are kept fixed on the respective links.

In the proposed NN-based approach, the coefficients of hip trajectory, positions of the lumped masses and connecting weights of the fully connected feed-forward NNs are optimized off-line, utilizing a GA. The trajectory of the hip joint is assumed to follow a cubic polynomial, as given below.

\[ z_h = k_0 + k_1 x + k_2 x^2 + k_3 x^3, \]  

where \( z_h \) represents the height of the hip joint at a distance \( x \) from the starting point and \( k_0, k_1, k_2 \) and \( k_3 \) are the coefficients. The hip height at the start and end of a cycle is assumed to be the same, to fulfill the repeatability conditions for the lower limbs. These two boundary conditions are used to determine the coefficients \( k_2 \) and \( k_3 \), after extracting the other two coefficient values, such as \( k_0 \) and \( k_1 \), from the GA, which will be explained later in this section. The swing foot trajectory has been determined using the information related to the lifting and landing positions of the swing foot (i.e., \( x_1 \) and \( x_2 \)) measured from a fixed reference system (refer to Fig. 1). Two GA-trained NNs have been utilized to solve the problem. Fig. 4 shows the architecture of the proposed algorithm. Two neurons are present in the input and output layers of each of the NNs. The inputs to the first NN are the positions of the feet placement, that is, \( x_1 \) and \( x_2 \) (refer to Fig. 1). The two outputs predicted from this NN are the hip height \( h_1 \) and projected distance \( l_1 \) of the hip joint from the swing foot. With the help of the swing foot and hip trajectories, the lower limbs' gaits have been generated after utilizing the concept of inverse kinematics (refer to Eqs. (4) and (5)). The changes in joint angles of the swing leg (that is, \( \delta \theta_2 \) and \( \delta \theta_3 \)) have been calculated from the generated gait and supplied as inputs to the second NN. The two outputs predicted by this NN are the changes in angles of the swing foot and trunk (that is, \( \delta \theta_1 \) and \( \delta \theta_4 \)) of the two-legged robot. The generated gait is then checked for its dynamic balance after calculating the position of the ZMP (refer to Eq. (9)). The required joint torques and power consumption at various joints are then determined by using Eqs. (12) and (13).

The information related to the NN, such as weight values \([V], [W]\), bias values and the variables to be optimized, such as coefficients \( k_0 \) and \( k_1 \) of the hip trajectory and positions of lumped masses on the corresponding limbs of the biped robot (that is, \( r_1, r_2, r_3 \) and \( r_4 \)), are coded in the GA string. The remaining positions \( r_5 \), \( r_6 \) and \( r_7 \) are taken to be equal to \( r_1, r_2 \) and \( r_3 \), respectively, to make the structure of the biped robot symmetric about the central axis. Let us assume that there are \( M \) and \( N \) hidden neurons in the first and second NNs, respectively. The GA string can be represented as follows:

\[ \text{GA string} = [V], [W], [k_0, k_1, \ldots, k_M, \ldots, k_N], [r_1, r_2, \ldots, r_M, \ldots, r_N] \]
3.2.1. Formulation as an optimization problem

The problem considered in this study may be stated as follows: A biped robot will have to cross a ditch with the maximum dynamic balance margin by consuming minimum power and after satisfying the condition that the changes in joint torque values should be less than a pre-specified value. Thus, it may be posed as an optimization problem as given below:

Maximize \( \frac{L_f}{2} - \sum_{i=1}^{8} x_{\text{DBM}} \) + 1/P, \( \sum_{i=1}^{8} x_{\text{DBM}} + 1/P \)
subject to \( \Delta \tau_g \leq \Delta \tau_{\text{specified}} \),

where \( i \) and \( j \) represent the joint and time instant, respectively; \( L_f \) is the length of the supporting foot; \( x_{\text{ZMP}} \) is the distance of the ZMP from the ankle joint measured in the direction of motion; \( P \) is the average power consumption and \( \Delta \tau \) represents the change in joint torque value.

Fitness calculation. A batch mode of training has been employed with the help of 216 cases to train the neural networks for generating the ditch-crossing gait. The fitness \( f \) of a GA string is calculated as the average of all objective function values, as given below.

\[
f = \frac{\sum_{i=1}^{8} x_{\text{DBM}} + 1/P}{S},
\]

where \( S \) represents the number of training cases considered. A high penalty equal to -100 is added to the fitness value, if either the NNs represented by the GA string are unable to generate the dynamically balanced gait or the change in torque for a joint exceeds the pre-defined value.

3.3. Approach 3: Fuzzy logic-based approach

The fuzzy logic technique can be used to determine the input-output relationships of real-world complex systems. The ditch-crossing gait generation problem of a two-legged robot has been modeled using the Mamdani approach of a fuzzy logic controller (FLC) [4]. The working principle of this approach is also similar to that of the NN-based approach (refer to Fig. 3), in which two NNs of the latter are replaced by two FLCs. In the FL-based approach, a GA will be used to optimize the knowledge base (that is, data base and rule base) of the FLC off-line.

In the present FL-based approach, two modules of the FLC have been utilized to solve the ditch-crossing gait generation problem of a biped robot. The membership function distributions of the input and output variables for both the FLCs are shown in Figs. 5 and 6. In the proposed method, the positions of the lumped masses, coefficients of the hip trajectory and knowledge base of the FLC are optimized utilizing a GA. The number of rules in the rule base of the first FLC turns out to be equal to 16, as two inputs (that is, \( x_1 \) and \( x_2 \)) are represented using four linguistic terms (such as Low — L, Medium — M, High — H, Very High — VH). A particular rule of this FLC may be read as follows:

IF \( x_1 \) is L AND \( x_2 \) is M THEN \( h_1 \) is M and \( l_1 \) is L.

Similarly, the rule base of the second FLC also contains 16 rules, as two inputs (that is, \( \delta \theta_2 \) and \( \delta \theta_3 \)) are also represented with the help of four linguistic terms (such as Negative Large — NL, Negative Small — NS, Positive Small — PS, Positive Large — PL) each. One such rule of the second FLC may look like the following:

IF \( \delta \theta_2 \) is NL AND \( \delta \theta_3 \) is NS THEN \( \delta \theta_1 \) is NS and \( \delta \theta_4 \) is NL.

The linguistic terms of each output variable of the first and second modules of the FLC are represented using two bits. For example, L, M, H and VH are represented with the help of 00, 01, 10 and 11, respectively, for the first FLC. Similarly, for the second module, 00, 01, 10 and 11 are used to represent the angle variations NS, PS, PL and PS, respectively. Therefore, four bits are used to denote two outputs for each rule of the first and second modules of the FLC. Moreover, there are 14 real variables (that is, \( a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, k_1, k_2, r_1, r_2, r_3 \) and \( r_4 \)) and ten bits are assigned to represent each of them. Thus, the GA string is 140 + 16 + 16 × 4 + 16 + 16 × 4 =
The optimization problem is similar to that solved by the NN-based approach, and here, also, a batch mode of training has been adopted, where the fitness of the GA string is calculated with the help of Eq. (16).

4. Results and discussion

Table 2 shows the physical parameters of the biped robot considered in the present study. The results of computer simulations carried out using the above three approaches are discussed below.

4.1. Approach 1

The information related to the feet placement (that is, $x_1$, $x_2$, and $x_3$) of the biped robot from a fixed reference frame and the initial
posture of the robot (that is, $\theta_2$ and $\theta_3$ of the swing leg) was fed as input to the gait planner for testing its performance through computer simulations. Let us assume that the inputs are as follows: $x_1 = 0.08$ m, $x_2 = 0.23$ m, $x_3 = 0.44$ m and $\theta_2 = 40^\circ$ and $\theta_3 = -40^\circ$. The outputs of interest are the dynamic balance margin of the generated gait, joint torques and average power consumption.

The variations of the ZMP in one complete cycle are shown in Fig. 7(a). $x_{\text{ZMP}}$ is initially found to lie at the back of the ankle joint of the supporting foot. It can be observed that it is moving further toward the ankle joint (in the direction of motion), as the swing leg reaches the next landing position. This happens due to the moments generated by the moving masses in the direction of motion. It is to be noted that the ZMP always lies inside the convex hull of the foot support polygon for the complete motion cycle. Therefore, the generated gait is said to be dynamically balanced. The variations of joint torques at different joints in a cycle are shown in Fig. 7(b). The clockwise and anti-clockwise torques are considered to be positive and negative, respectively. The torque values of the ankle, knee and hip joints of the swing leg and trunk during the second time interval turn out to be more compared to those found during the other time intervals. This happens because the inertia and Coriolis/centrifugal effects are seen to be greater, as the robot starts walking from the rest.

### 4.2. Approach 2

A systematic study was conducted to identify the optimal number of neurons in the hidden layers of both the modules of the NN. A GA has been utilized to further optimize the performance of the NN. The following GA parameters are found to yield the best results: crossover probability $p_c = 0.5$ (uniform crossover), mutation probability $p_m = 0.001$ (bit-wise mutation), population size = 30, maximum number of generations = 30. The optimal number of neurons to be present in the hidden layers of the first and second modules of the NN are found to be equal to 6 and 4, respectively. The number of connecting weights turns out to be equal to 40 (i.e., $4 \times 6 + 4 \times 4 = 40$). The total number of variables is seen to be equal to 48, including the bias values $b_1$ and $b_2$. As ten bits are assigned to represent each variable, the GA string is found to be 480 bits long.

During optimization, the variables of the NN, such as the connecting weights and bias values, were varied in the ranges $(0.0, 1.0)$ and $(0.0, 0.0000001)$, respectively. Moreover, the other variables, such as $k_0$, $k_1$, $r_1$, $r_2$, $r_3$ and $r_4$, were also varied between $(1.0$ and $4.0)$, $(1.0$ and $4.0)$, $(0.01$ and $0.025)$, $(0.1$ and $0.32)$, $(0.1$ and $0.28)$, and $(0.1$ and $0.54)$, respectively. Linear, tan-sigmoid and log-sigmoid transfer functions were used for the input, hidden and output layers of the first and second modules of the NN, respectively. The coefficients of the linear, tan-sigmoid and log-sigmoid transfer functions were kept equal to 1.0, 1.0 and 2.0, respectively. The optimal GA parameters responsible for providing the best results of the ditch-crossing gait generation problem were found to be as follows: crossover probability $p_c = 0.5$ (uniform crossover), mutation probability $p_m = 0.002$ (bit-wise mutation), population size = 36, maximum number of generations = 140. The optimized values of the variables $k_0$, $k_1$, $r_1$, $r_2$, $r_3$ and $r_4$ are seen to be equal to 1.0000, 1.0000, 0.0203, 0.1000, 0.2800 and 0.2101, respectively.

Table 3 shows the optimized values of the connecting weights determined by the GA. Moreover, the bias values, such as $b_1$ and $b_2$, are seen to be equal.
of the first and second modules of the NN were found to be equal to 0.000004 and 0.000070, respectively. Once the GA-based training is completed, the NN will be able to generate the dynamically balanced gaits in an optimal sense. To test the performance of this approach, the same set of inputs considered in the previous approach (that is, $x_1 = 0.08$ m, $x_2 = 0.23$ m and $x_3 = 0.44$ m) was fed to this planner. Fig. 8(a) shows the variations of ZMP during the time of a cycle. It is interesting to note that the ZMP value varies from a negative to a positive value towards the end of the cycle. This is because the ZMP moves from the back to front of the ankle joint in the direction of motion and finally moves towards the end of the supporting foot polygon. This happens because the ZMP starts from the back of ankle joint towards its front, while moving in the direction of motion. The variation of joint torques in one complete cycle are shown in Fig. 8(b). The pattern of variation was also found to be similar to that obtained in Approach 2. However, its performance is seen to be different from that of Approach 1.

4.3. Approach 3

A GA was used to optimize the knowledge base of the two modules of the FLC for handling the problem related to ditch crossing gait generation of the biped robot. The ranges of the real variables $a_1$ through $a_6$ during optimization were set equal to (0.008, 0.0167), (0.008, 0.0167), (0.05, 0.0733), (0.01, 0.023), (4.0, 8.333), (4.0, 8.333), (2.0, 5.0) and (2.0, 6.333), respectively, for the said problem. The ranges of other variables, such as $k_0$, $k_1$, $r_1$, $r_2$, $r_3$ and $r_4$, were kept the same as those mentioned in Section 4.2.

The following GA parameters were found to give the best results: crossover probability $p_c = 0.5$ (uniform crossover), mutation probability $p_m = 0.0001$ (bit-wise mutation), population size = 70, maximum number of generations = 95. The optimum values of $k_0$, $k_1$, $r_1$, $r_2$, $r_3$ and $r_4$ were seen to be equal to 1.0000, 1.0000, 0.0119, 0.1275, 0.2800 and 0.1276, respectively. The optimized values of half base-widths of the triangular membership function distributions (refer to Figs. 5 and 6), such as $a_1$, $a_2$, $a_3$, $a_4$, $a_5$, $a_6$, $a_7$, $a_8$ turn out to be equal to 0.0081, 0.0151, 0.0675, 0.0133, 7.9349, 4.7121, 2.0792 and 2.0804, respectively. Tables 4 and 5 show the optimal rule bases evolved for the first and second modules of the FLC, after the GA-based training was over. The first and second modules of FLC were found to contain 12 and 8 optimal rules, respectively.

Once the optimal FLCs were obtained, the performance of this gait planner was tested, with the help of the same set of inputs ($x_1$, $x_2$ and $x_3$) considered in Approaches 1 and 2. Fig. 9(a) shows the variations of the ZMP in a complete cycle. It is interesting to note that the ZMP values vary from negative to positive towards the end of the cycle. This happens because the ZMP starts from the back of ankle joint towards its front, while moving in the direction of motion. The variations of joint torques in one complete cycle are shown in Fig. 9(b). The pattern of variation was also found to be similar to that obtained in Approach 2. However, its performance is seen to be different from that of Approach 1.

4.4. A comparative study

The performances of the developed three approaches were compared in terms of average dynamic balance and the power consumption in a complete cycle. Fig. 10(a) shows the comparison of variations of the DBM values obtained by three approaches in one complete cycle. For a particular test case, the values of the average $x_{\text{DBM}}$ were calculated for Approaches 1, 2 and 3 and these were found to be equal to 0.023193 m, 0.024116 m and 0.025087 m, respectively. Thus, Approach 3 has performed slightly better than the other approaches. Fig. 10(b) compares the values of average power

![Fig. 8. Variations of the ZMP and joint torques, while crossing the ditch in one complete cycle — Approach 2.](image)

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Optimized rule base for the first module of the FLC.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>$x_2$</td>
</tr>
<tr>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>L</td>
<td>M</td>
</tr>
<tr>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>L</td>
<td>VH</td>
</tr>
<tr>
<td>M</td>
<td>L</td>
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<td>M</td>
<td>M</td>
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<tr>
<td>M</td>
<td>VH</td>
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<td>H</td>
<td>M</td>
</tr>
<tr>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>VH</td>
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<table>
<thead>
<tr>
<th>Table 5</th>
<th>Optimized rule base for the second module of the FLC.</th>
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<td>$\delta x_1$</td>
<td>$\delta x_2$</td>
</tr>
<tr>
<td>NL</td>
<td>NL</td>
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<tr>
<td>NL</td>
<td>NS</td>
</tr>
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<tr>
<td>PS</td>
<td>PL</td>
</tr>
<tr>
<td>PL</td>
<td>NS</td>
</tr>
</tbody>
</table>
consumed at different joints in a cycle by following the three developed approaches. The values of average power consumption were found to be equal to 13.546982 W, 5.775630 W and 8.650062 W for Approaches 1, 2 and 3, respectively. Thus, Approaches 2 and 3 are found to outperform Approach 1 in terms of both $x_{DBM}$ and power consumption.

Table 6 compares the results of Approaches 2 and 3 for ten random test cases. In all cases, Approach 3 is able to perform slightly better than Approach 2. This may be because some problem information has been injected into the database of the FLCs before the commencement of its training in Approach 3, as the membership function distributions of the variables have been initially designed with the help of human expertise. On the other hand, this is not done in Approach 2. The knee torque ($\tau_6$) of the supporting leg also turns out to be more compared to other joint torques. This has happened because the entire structure of the biped robot is supported by this knee joint, when the robot moves. This exactly matches the experience of human beings. Fig. 11 shows the simulation results of the two-legged robot crossing a ditch using Approaches 1, 2 and 3, in one cycle. The results of the simulation show that the biped robot has successfully crossed the ditch with the help of the gait generated by all three approaches. In Approach 1, the hip trajectory follows a straight path, whereas it is found to follow a cubic polynomial in both Approaches 2 and 3. Moreover, it is interesting to observe from this figure that the lower limbs do not produce a periodic gait. This is because the step length is not kept the same on the two sides of the supporting leg, as the robot has to place the swing leg foot on the other side of the ditch.

4.5. Robustness test

A robustness test was conducted on Approaches 2 and 3 by allowing some variations in the values of input variables (that is, $x_1$, $x_2$ and $x_3$) and noting down the % change in $x_{DBM}$ values. The results of the robustness test are shown in Table 7. It is interesting to note that $\pm 15\%$ variations in the input variables have led to a maximum of 0.988% and $-0.551\%$ variations in the $x_{DBM}$ values for Approaches 2 and 3, respectively. Thus, both the NN-based and FL-based gait planners are found to be robust.

5. Concluding remarks

The following conclusions have been drawn from the above study.

- All the developed approaches are able to generate dynamically balanced gaits for the biped robot, while crossing a ditch.
- The NN-based and FL-based approaches have yielded dynamically more balanced gaits at the cost of lower power consumption compared to those obtained by the analytical approach. It
Table 6
Average joint torque values and $x_{DBM}$ comparisons for Approaches 2 and 3.

<table>
<thead>
<tr>
<th>Scenario number</th>
<th>Average torque values in Nm</th>
<th>Average $x_{DBM}$ in m</th>
<th>Average $x_{DBM}$ in ft</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau_1$</td>
<td>$\tau_2$</td>
<td>$\tau_3$</td>
</tr>
<tr>
<td>Approach 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>19.478</td>
<td>6.906</td>
<td>5.308</td>
</tr>
<tr>
<td>2</td>
<td>20.652</td>
<td>7.288</td>
<td>5.291</td>
</tr>
<tr>
<td>3</td>
<td>20.067</td>
<td>7.166</td>
<td>4.962</td>
</tr>
<tr>
<td>4</td>
<td>22.878</td>
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<td>5</td>
<td>19.969</td>
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</tr>
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<td>6</td>
<td>19.844</td>
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<td>5.194</td>
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<tr>
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<td>22.176</td>
<td>7.872</td>
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</tr>
<tr>
<td>8</td>
<td>20.117</td>
<td>7.159</td>
<td>5.121</td>
</tr>
<tr>
<td>9</td>
<td>19.577</td>
<td>6.989</td>
<td>5.497</td>
</tr>
<tr>
<td>10</td>
<td>21.601</td>
<td>7.678</td>
<td>5.373</td>
</tr>
<tr>
<td>Approach 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>22.846</td>
<td>8.232</td>
<td>4.644</td>
</tr>
<tr>
<td>2</td>
<td>24.066</td>
<td>8.717</td>
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</tr>
<tr>
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<td>4.795</td>
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<td>4</td>
<td>25.848</td>
<td>9.443</td>
<td>5.563</td>
</tr>
<tr>
<td>5</td>
<td>22.049</td>
<td>7.939</td>
<td>4.854</td>
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<tr>
<td>6</td>
<td>21.751</td>
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<tr>
<td>7</td>
<td>24.783</td>
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<tr>
<td>9</td>
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<td>10</td>
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</table>

Fig. 11. Results of computer simulations to show the two-legged robot crossing a ditch using (a) Approach 1, (b) Approach 2 and (c) Approach 3.

Table 7
Results of robustness test for the developed gait planners.

<table>
<thead>
<tr>
<th>% changes in inputs</th>
<th>Percent change in $x_{DBM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Approach 2</td>
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<tr>
<td>−15</td>
<td>0.988</td>
</tr>
<tr>
<td>−10</td>
<td>0.533</td>
</tr>
<tr>
<td>−5</td>
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<td>−0.190</td>
<td>−0.551</td>
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<tr>
<td>−0.277</td>
<td>−0.492</td>
</tr>
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</table>

has happened because both the NN-based and the FL-based approaches are able to generate adaptive gaits, as the situation demands. The knee torque of the supporting leg turns out to be the maximum out of all joint torques. This aspect exactly matches the experience of human beings.

- The FL-based approach has performed slightly better than the NN-based approach in terms of dynamic balance margin. This might have happened because some problem information has been injected into the database of the FL-based gait planner, prior to its GA-based optimization, which cannot be done while developing the NN-based gait planner.

- It is possible to implement the proposed algorithms on-line, as the CPU time values for solving 20 test cases by the FL-based and NN-based approaches are found to be equal to 0.018 and 0.010 s, respectively, on a P-IV machine. In the present study, the performances of the proposed approaches were tested through computer simulations. However, it will be more interesting to test their performances on a full three-dimensional (3D) biped robot model. The following issues are to be considered for carrying out experiments with a real 3D robot: on-line recognition of the environment; modeling of foot-ground interaction as a multi-DOF system; study of the movement of the robot in the lateral direction; design and development of a closed-loop control system for the same; and others. It is important to mention that in order to implement the movement of the robot in the lateral plane, a biped robot having more than seven DOFs is to be considered.

The walking cycle of a biped robot consists of single and double support phases. The present work deals with the study of single support phase only; the double support phase will be studied in the future.
References


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