Absorption of Shocks in Nonlinear Autoregressive Models

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Abstract

It generally is difficult, if not impossible, to fully understand and interpret nonlinear time series models by considering the estimated values of the model parameters only. To shed light on the characteristics and implications of a nonlinear model it can then be useful to consider the effects of shocks on the future patterns of the time series variable. Most interest in such impulse response analysis has concentrated on measuring the persistence of shocks, or the magnitude of their (ultimate) effect. A framework is developed and implemented that is useful for measuring the rate at which this final effect is attained, or the rate of absorption of shocks. It is shown that the absorption rate can be used to examine whether the propagation of different types of shocks, such as positive and negative shocks or large and small shocks follows different patterns. The nonlinear floor-and-ceiling model for US output growth is used to illustrate the various concepts. The presence of substantial asymmetries in both persistence and absorption of shocks is documented, with interesting differences arising across magnitudes of shocks and across regimes in the model. Furthermore, it appears that asymmetry became much less pronounced due to a large decline in output volatility in the 1980s.

Key words: impulse response, half-life, asymmetry, regime-switching models.

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1 Introduction

Nonlinear time series models are becoming increasingly popular in empirical macroeconomics and empirical finance for describing and forecasting variables such as output, (un)employment, stock returns and interest rates, see Granger and Teräsvirta (1993) and Franses and van Dijk (2000) for reviews. Examples of often considered models are the threshold autoregressive [TAR] model, see Tong (1990), the smooth transition autoregressive [STAR] model, see Chan and Tong (1986), Teräsvirta (1994) and van Dijk, Teräsvirta and Franses (2002), the Markov-Switching model put forward in Hamilton (1989), and the Artificial Neural Network [ANN] model advocated by Kuan and White (1994), among others. A key feature of these (and other) nonlinear time series models is that they assume that the model structure (lag length, parameters, variance) experiences occasional changes or, put differently, they assume the presence of different regimes. Hence, these models can, for example, describe asymmetric business cycle behavior as observed in output and unemployment, or different behavior in different states of the financial market (for example, in bull and bear markets) as observed in stock returns and interest rates.

A common property of many of these (univariate) nonlinear models (and this holds true even more so for their multivariate counterparts) is that only considering (estimates of) the model parameters generally is not sufficient to completely grasp the implied properties of time series generated by the model. Put differently, it is difficult to interpret a specific nonlinear model and to understand why it is useful in a particular application. Therefore, to shed light on the characteristics of a nonlinear model it often is useful to consider the effects of shocks on the future patterns of the time series variable. Impulse response functions provide a convenient tool for measuring such effects. Recent applications of impulse response analysis in nonlinear models in empirical macroeconomics and finance can be found in Weise (1999), Balke (2000), Dufour and Engle (2000), Taylor and Peel (2000), Altissimo and Violante (2001), Taylor, Peel and Sarno (2001), Balke, Brown and Yücel (2002), Skalin and Teräsvirta (2002), Atanasova (2003), Chen, Tsai and Wu (2004), Dufrenot, Mignon and Peguin-Feissolle (2004), Grier et al. (2004), and Camacho (2005), among others.

Most applications of impulse response analysis concentrate on measuring the persistence of shocks, indicated by the magnitude of their (ultimate) effect on the time series variable. Interestingly, far less attention typically is given to measuring the rate at which this final effect is attained, that is, how fast shocks are ‘absorbed’ by a time series. Due to the properties of impulse responses in linear models, they can be used straightforwardly to gain insight in this rate of absorption of shocks as well, see, for example, Lütkepohl (2005) for a discussion of impulse response functions in linear models. However, im-
pulse response analysis in nonlinear models is more complicated, as discussed at length in Gallant, Rossi and Tauchen (1993), Koop, Pesaran and Potter (1996), and Potter (2000). The complications arise because in nonlinear models (1) the effect of a shock depends on the history of the time series up to the point where the shock occurs, (2) the effect of a shock need not be proportional to its size and (3) the effect of a shock depends on shocks occurring in periods between the moment at which the impulse occurs and the moment at which the response is measured. Because of these properties of impulse responses, assessing the absorption time of shocks in nonlinear models also is more involved, as will become clear below. In this paper we develop and implement a framework that can be used for measuring and analyzing absorption of shocks in nonlinear models. Among others, we demonstrate that our absorption measure can be used to address relevant questions such as

(1) Are positive and negative shocks absorbed at the same rate? Are shocks of different magnitudes absorbed at the same rate?
(2) Does the rate of absorption of shocks depend on the initial values or the history of the time series?
(3) Are shocks absorbed at the same rate by the different components of a multivariate time series?
(4) Are shocks absorbed at the same rate by linear combinations of the components in a multivariate time series and by the individual components themselves?

It should be stressed at the outset that our absorption measure should not be considered as a substitute for conventional impulse response analysis and persistence measures but rather as a complement. Our absorption measure allows one to obtain a more complete picture of the propagation mechanism of a nonlinear model, as it can highlight interesting asymmetry properties of shocks to economic time series different from the ones that are revealed by impulse response functions as such.

Finally, an alternative approach to absorption is developed by Lee and Pesaran (1993) and Pesaran and Shin (1996). They examine the time profile of the effect of shocks by means of so-called ‘persistence profiles’, defined as the difference between the conditional variances of $n$-step and $(n-1)$-step ahead forecasts, viewed as a function of $n$. Also see Galbraith (2003) on the related concept of ‘content horizons’, defined as the ratio of the conditional variance of $n$-step ahead forecasts and the unconditional variance of the time series of interest.

Our paper proceeds as follows. In Section 2, we briefly review the main aspects of impulse response analysis in nonlinear time series models and the generalized impulse response functions introduced by Koop et al. (1996). In Section 3, we develop our measure of absorption of shocks. To facilitate the under-
standing of the concept of absorption, we concentrate on univariate models first. In this section we also demonstrate how to address the question of asymmetric absorption, that is the question whether positive and negative shocks are absorbed differently. In Section 4, we generalize our absorption measure to multivariate models. Particular attention is given to the question whether shocks are absorbed at the same rate by the different components of a multivariate time series. We also outline how to measure absorption for a linear combination of the components of a multivariate time series. In Section 5, we discuss an empirical application to quarterly US GDP growth rates using the floor-and-ceiling model of Pesaran and Potter (1997). We document the presence of substantial asymmetries in both persistence and absorption of shocks, with interesting differences arising across magnitudes of shocks and across regimes in the model. Furthermore, it appears that asymmetry became much less pronounced due to a large decline in output volatility in the 1980s. Finally, Section 6 contains some concluding remarks.

2 Preliminaries

Consider the multivariate nonlinear autoregressive time series model

\[ Y_t = F(Y_{t-1}, \ldots, Y_{t-p}; \theta) + V_t, \]

(1)

where \( Y_t = (Y_{1t}, \ldots, Y_{kt})' \) is a \((k \times 1)\) random vector, \( F(\cdot) \) is a known function that depends on the \((q \times 1)\) parameter vector \( \theta \), \( V_t = (V_{1t}, \ldots, V_{kt})' \) is a \((k \times 1)\) vector of random disturbances with \( E[V_t | \Omega_{t-1}] = 0 \) and \( E[V_t V_\ell' | \Omega_{t-1}] = H(Y_{t-1}, \ldots, Y_{t-r}; \xi) \), where the \((k \times k)\) conditional covariance matrix \( H(Y_{t-1}, \ldots, Y_{t-r}; \xi) \equiv H_t = \{H_{t,ij}, i, j = 1, \ldots, k\} \) depends on the \((s \times 1)\) parameter vector \( \xi \).

Throughout, we use upper-case letters to denote random variables and lower-case letters to denote realizations of those random variables. For example, \( y_t \) and \( v_t \) are realizations of \( Y_t \) and \( V_t \), respectively. The ‘history’ of the process up to \( t-1 \), which is the information set used to forecast future values of \( Y_t \), is denoted as \( \Omega_{t-1} \), with corresponding realizations denoted as \( \omega_{t-1} \). Because the nonlinear model (1) is Markov of order \( \max(p, r) \), it suffices to take \( \Omega_{t-1} = \{Y_{t-1}, \ldots, Y_{t-\max(p, r)}\} \).

2.1 Impulse response functions

Impulse response functions are meant to provide a measure of the effect of a shock \( v_t \) occurring at time \( t \) on the time series after \( n \) periods, that is on \( Y_{t+n} \) for \( n \geq 0 \). The impulse response measure that is commonly used in the
analysis of linear models is defined as the difference between two realizations of $Y_{t+n}$. Both realizations start from the same history $\omega_{t-1}$, but in one realization the process is hit by a shock of size $v_t$ at time $t$, while in the other realization no shock occurs at time $t$. Furthermore, all shocks in intermediate periods between $t$ and $t+n$ are set equal to zero in both realizations, such that the traditional impulse response function [TI] is given by

$$TI_Y(n, v_t, \omega_{t-1}) = E[Y_{t+n} | V_t = v_t, V_{t+1} = \ldots = V_{t+n} = 0, \omega_{t-1}] - E[Y_{t+n} | V_t = 0, V_{t+1} = \ldots = V_{t+n} = 0, \omega_{t-1}],$$

for $n = 0, 1, 2, \ldots$. The second realization usually is called the benchmark profile.

This traditional impulse response function has several convenient properties in case the specification for the conditional mean $F(Y_{t-1}, \ldots, Y_{t-p}; \theta)$ in (1) is linear. First, it is symmetric in the sense that a shock of magnitude $-v_t$ has exactly the opposite effect as a shock of magnitude $+v_t$, that is $TI_Y(n, v_t, \omega_{t-1}) = -TI_Y(n, -v_t, \omega_{t-1})$. Furthermore, it might be called linear, in the sense that the impulse response is proportional to the magnitude of the shock, that is $TI_Y(n, a v_t, \omega_{t-1}) = a TI_Y(n, v_t, \omega_{t-1})$ for all $a \neq 0$. Third, the impulse response is history independent as it does not depend on the particular history $\omega_{t-1}$, that is $TI_Y(n, v_t, \omega'_{t-1}) = TI_Y(n, v_t, \omega_{t-1})$ for all histories $\omega_{t-1}$ and $\omega'_{t-1}$. Finally, the impulse response is independent of the shocks that occur in intermediate periods $t+1, \ldots, t+n$. Even though $v_{t+1}, \ldots, v_{t+n}$ are commonly set equal to 0, as in (2), the value of $TI_Y(n, v_t, \omega_{t-1})$ does not change if other values were used instead (as long as the same values are used in the two realizations defining the impulse response function). For example, in the univariate AR(1) model $Y_t = \phi Y_{t-1} + V_t$, it holds that $TI_Y(n, v_t, \omega_{t-1}) = \phi^n v_t$, from which the properties mentioned above are immediately obvious.

In general, none of these properties continue to hold in nonlinear time series models. In nonlinear models, the impact at $t+n$ of a shock that occurs at time $t$ typically depends (1) on the history of the process up to the time the shock occurs, (2) on the sign and the size of the shock, and (3) on the shocks that occur in intermediate periods $t+1, \ldots, t+n$. The first property is closely related to the observation that the accuracy of forecasts from nonlinear models depends on the initial value, see Yao and Tong (1994) and Fan, Yao and Tong (1996), among others. Concerning the third property, the assumption that no shocks occur in intermediate periods, as in the TI in (2), might give rise to quite misleading inference concerning the propagation mechanism of the nonlinear model, see Pesaran and Potter (1997) for an example.

The Generalized Impulse Response Function [GI], introduced by Koop et al. (1996), provides a natural and elegant solution to the complications involved in impulse response analysis in nonlinear models. The GI for a specific shock
and history $\omega_{t-1}$ is defined as

$$
\text{GI}_Y(n, v_t, \omega_{t-1}) = \mathbb{E}[Y_{t+n}|V_t = v_t, \omega_{t-1}] - \mathbb{E}[Y_{t+n}|\omega_{t-1}],
$$

(3)

for $n = 0, 1, 2, \ldots$. In the GI, the expectation of $Y_{t+n}$ given that a shock $v_t$ occurs at time $t$ is conditioned only on the history and on this shock. Put differently, the problem of handling shocks occurring in intermediate time periods is dealt with by averaging them out. Given this choice, the natural benchmark profile for the impulse response is the expectation of $Y_{t+n}$ conditional only on the history of the process $\omega_{t-1}$. Thus, in the benchmark profile the current shock is averaged out as well. It is straightforward to show that for linear models the GI in (3) is equivalent to the traditional impulse response in (2).

The GI as defined in (3) is a function of $v_t$ and $\omega_{t-1}$, which are realizations of the random variables $V_t$ and $\Omega_{t-1}$. Koop et al. (1996) stress that hence $\text{GI}_Y(n, v_t, \omega_{t-1})$ itself is a realization of the random variable given by

$$
\text{GI}_Y(n, V_t, \Omega_{t-1}) = \mathbb{E}[Y_{t+n}|V_t, \Omega_{t-1}] - \mathbb{E}[Y_{t+n}|\Omega_{t-1}],
$$

(4)

Using this interpretation of the GI as a random variable, various conditional versions can be defined that are of potential interest. For example, one might consider a particular history $\omega_{t-1}$ and treat the GI as a random variable in terms of the shock $V_t$ only, that is,

$$
\text{GI}_Y(n, V_t, \omega_{t-1}) = \mathbb{E}[Y_{t+n}|V_t, \omega_{t-1}] - \mathbb{E}[Y_{t+n}|\omega_{t-1}],
$$

(5)

Alternatively, one could reverse the role of the shock and the history by fixing the shock at $V_t = v_t$ and consider the GI as a random variable in terms of the history $\Omega_{t-1}$ only. In general, one might examine the GI conditional on a subset $S$ of shocks and a subset $H$ of histories, that is,

$$
\text{GI}_Y(n, S, H) = \mathbb{E}[Y_{t+n}|V_t \in S, \Omega_{t-1} \in H] - \mathbb{E}[Y_{t+n}|\Omega_{t-1} \in H].
$$

(6)

For example, one might condition on all histories such that $Y_{t-1} \leq 0$ and consider only negative shocks.

Note that as for nonlinear models analytic expressions for the conditional expectations involved in the GI in (3) and (4) usually are not available, stochastic simulation should be used to obtain estimates of the impulse response measures. See Koop et al. (1996) for a detailed description of the relevant techniques. Finally, it is useful to note that the shock- and history-specific impulse response measure $\text{GI}_Y(n, v_t, \omega_{t-1})$ in (3) can still be interpreted as a random variable if parameter uncertainty is taken into account as an additional source of randomness. In that case, Bayesian techniques are convenient to construct the distribution of the impulse response, as discussed in Koop (1996).

The two aspects of impulse responses that appear to be of most interest are (1) the final response to an impulse, and (2) the rate at which this final response
is attained. Traditionally, most attention has been given to the first element, usually referred to as persistence. In the present paper we focus on the second aspect, which we call absorption. Before we proceed to discuss how absorption can be measured in the next section, we summarize how persistence of shocks can be assessed by means of the GI. This section then closes with some remarks on how to determine whether the response to positive and negative shocks is asymmetric.

2.2 Measuring persistence of shocks

A shock $v_t$ is said to be transient at history $\omega_{t-1}$ if in the long run the shock does not affect the pattern of the time series, that is, if $G_Y(n, v_t, \omega_{t-1})$ becomes equal to 0 as the horizon $n$ goes to infinity. If this is not the case, the shock is said to be persistent. The final impulse response for a specific shock and history can be obtained as

$$G_Y^\infty(v_t, \omega_{t-1}) = \lim_{n \to \infty} G_Y(n, v_t, \omega_{t-1}),$$

if this limit exists. In practice, the final impulse response $G_Y^\infty(v_t, \omega_{t-1})$ can be estimated by $G_Y(n_{\text{max}}, v_t, \omega_{t-1})$ for certain large $n_{\text{max}}$.

Potter (1995a) and Koop et al. (1996) suggest that the dispersion of the distribution of $G_Y(n, V_t, \Omega_{t-1})$ at finite horizons $n$ can be interpreted as a measure of persistence of shocks. It is intuitively clear that if a time series process is stationary and ergodic, the effect of all shocks eventually becomes zero for all possible histories of the process. Hence, $G_Y^\infty(v_t, \omega_{t-1})$ in (7) is equal to zero for all choices of $v_t$ and $\omega_{t-1}$ or, put differently, the distribution of $G_Y(n, V_t, \Omega_{t-1})$ collapses to a spike at 0 as $n \to \infty$. By contrast, for nonstationary time series the dispersion of the distribution of $G_Y(n, V_t, \Omega_{t-1})$ is positive for all $n$. Conditional versions of the GI are particularly suited to assess the persistence of shocks. For example, one might compare the dispersion of the distributions of GIs conditional on positive and negative shocks to determine whether negative shocks are more persistent than positive ones, or vice versa. A potential problem with this approach is that no unambiguous measure of dispersion exists, although the notion of second-order stochastic dominance might be useful in this context, see Potter (2000).

2.3 Measuring asymmetric impulse response

One possible use of the GI is to examine asymmetry in the effects of positive and negative shocks. Potter (1994) defines a measure of asymmetric response to a particular (positive) shock $V_t = v_t$ given a particular history $\omega_{t-1}$ as the
sum of the GI for this particular shock and the GI for the shock of the same magnitude but with opposite sign, that is,
\[ \text{ASY}_Y(n, v_t, \omega_{t-1}) = \text{GI}_Y(n, v_t, \omega_{t-1}) + \text{GI}_Y(n, -v_t, \omega_{t-1}). \] (8)

As noted before, by taking into account parameter uncertainty as an additional source of randomness, GI\(_Y(n, v_t, \omega_{t-1})\), and hence ASY\(_Y(n, v_t, \omega_{t-1})\), can still be interpreted as a random variable. Potter (1995b) uses a straightforward simulation procedure to assess whether this shock- and history-specific asymmetry measure is significantly different from zero or not.

Alternatively, one could consider the distribution of the random asymmetry measure
\[ \text{ASY}_Y(n, V_t^+, \Omega_{t-1}) = \text{GI}_Y(n, V_t^+, \Omega_{t-1}) + \text{GI}_Y(n, -V_t^+, \Omega_{t-1}), \] (9)
where \( V_t^+ = \{v_t|v_t > 0\} \) indicates the set of all positive shocks. If positive and negative shocks induce the same response (but with opposite sign), \( \text{ASY}_Y(n, V_t^+, \Omega_{t-1}) \) should be equal to zero almost surely. More generally, the distribution of \( \text{ASY}_Y(n, V_t^+, \Omega_{t-1}) \) may be used to infer the asymmetry properties of the impulse response function \( \text{GI}_Y(n, V_t, \Omega_{t-1}) \). When this distribution has mean (or median) equal to zero, shocks may be said to have a symmetric effect on average. At the same time, the dispersion and skewness of this distribution might be interpreted as measures of the asymmetry in the effects of positive and negative shocks.

3 Absorption of shocks in univariate models

Irrespective of whether shocks are persistent or not, it should be of interest to assess how fast innovations are absorbed, that is, the rate at which the GI approaches the final response \( \text{GI}_Y(\infty, v_t, \omega_{t-1}) \). In this section we discuss how absorption can be measured.

3.1 Definition of absorption

Suppose for the moment that \( Y_t \) is a univariate time series. Define the indicator function
\[ I_Y(\pi, n, v_t, \omega_{t-1}) \equiv I[|\text{GI}_Y(n, v_t, \omega_{t-1}) - \text{GI}_Y(\infty, v_t, \omega_{t-1})| \leq \pi |v_t - \text{GI}_Y(\infty, v_t, \omega_{t-1})|], \] (10)
for certain \( \pi \) such that \( 0 \leq \pi \leq 1 \), where \( I[A] = 1 \) if the event \( A \) occurs and 0 otherwise, and where it is assumed that the limit defining \( \text{GI}_Y(\infty, v_t, \omega_{t-1}) \)
(7) exists. In words, the function $I_Y(\pi, n, v_t, \omega_{t-1})$ is equal to 1 if the absolute difference between the GI at horizon $n$ and the ultimate response to the shock $v_t$, as given by $\text{GI}^\pi_Y(v_t, \omega_{t-1})$, is less than or equal to a fraction $\pi$ of the absolute difference between the shock $v_t$ (which is equal to the initial impact of the shock or the GI at horizon 0) and the ultimate response. Put differently, $I_Y(\pi, n, v_t, \omega_{t-1}) = 1$ if at least a fraction $1 - \pi$ of the difference between the initial and ultimate effects of $v_t$ has been absorbed after $n$ periods.

The ‘$\pi$-life’ or ‘$\pi$-absorption time’ of $v_t$ can now be defined as

$$N_Y(\pi, v_t, \omega_{t-1}) = \sum_{m=0}^{\infty} \left( 1 - \prod_{n=m}^{\infty} I_Y(\pi, n, v_t, \omega_{t-1}) \right).$$

(11)

In words, $N_Y(\pi, v_t, \omega_{t-1})$ is the minimum horizon beyond which the difference between the impulse responses at all longer horizons and the ultimate response is less than or equal to a fraction $\pi$ of the difference between the initial impact and the ultimate response. That is, $N_Y(\pi, v_t, \omega_{t-1}) = n^*$ if $I_Y(\pi, n, v_t, \omega_{t-1}) = 1$ for all $n \geq n^*$ and $I_Y(\pi, n^* - 1, v_t, \omega_{t-1}) = 0$. The reason for not defining $N_Y(\pi, v_t, \omega_{t-1})$ as the shortest horizon for which $I_Y(\pi, n, v_t, \omega_{t-1}) = 1$ is that the GI need not approach the limit $\text{GI}^\pi_Y(v_t, \omega_{t-1})$ monotonically. Hence, it may occur that $I_Y(\pi, n, v_t, \omega_{t-1}) = 1$ and $I_Y(\pi, n + j, v_t, \omega_{t-1}) = 0$ for certain $j > 0$.

Just like the shock- and history-specific GI in (3) can be regarded as a realization of the random variable $\text{GI}^\pi_Y(n, V_t, \Omega_{t-1})$ in (4), the $\pi$-absorption time $N_Y(\pi, v_t, \omega_{t-1})$ in (11) can be regarded as a realization of the random variable

$$N_Y(\pi, V_t, \Omega_{t-1}) = \sum_{m=0}^{\infty} \left( 1 - \prod_{n=m}^{\infty} I_Y(\pi, n, V_t, \Omega_{t-1}) \right),$$

(12)

where the random indicator function $I_Y(\pi, n, V_t, \Omega_{t-1})$ is defined as

$$I_Y(\pi, n, V_t, \Omega_{t-1}) \equiv I[|\text{GI}^\pi_Y(n, V_t, \Omega_{t-1}) - \text{GI}^\pi_Y(V_t, \Omega_{t-1})| \leq \pi |V_t - \text{GI}^\pi_Y(V_t, \Omega_{t-1})|].$$

(13)

Conditional versions $N_Y(\pi, S, H)$ for a particular subset $S$ of shocks and a subset $H$ of histories can be defined analogously in a straightforward manner.

It is useful to consider some basic properties of the absorption measure $N_Y(\pi, v_t, \omega_{t-1})$, which can conveniently be illustrated by means of the linear AR(1) model $Y_t = \phi Y_{t-1} + V_t$ with $|\phi| < 1$. In this case $\text{GI}^\pi_Y(n, v_t, \omega_{t-1}) = \phi^n v_t$, and $\text{GI}^\pi_Y(v_t, \omega_{t-1}) = 0$. Thus, $I_Y(\pi, n, v_t, \omega_{t-1}) = I[|\phi^n v_t| \leq \pi |v_t|]$, which is equal to 1 if $|\phi^n| = |\phi|^n \leq \pi$, or $n \leq \ln(\pi)/\ln(|\phi|)$. From (11) it then follows that $N_Y(\pi, v_t, \omega_{t-1}) = \lceil \ln(\pi)/\ln(|\phi|) \rceil$, where $\lceil z \rceil$ denotes the smallest integer greater than or equal to $z$. First, observe that $N_Y(\pi, v_t, \omega_{t-1})$ is independent of the (sign and size of the) shock $v_t$ and of the history $\omega_{t-1}$ in this case. Corresponding with the properties of the GI, this applies to all linear models but it
no longer holds for nonlinear models. Second, given that $I_Y(\pi, n, v_t, \omega_{t-1}) = 1$ implies $I_Y(\pi', n, v_t, \omega_{t-1}) = 1$ for all $0 \leq \pi < \pi' \leq 1$, it follows from the definition of the absorption measure in (11) that $N_Y(\pi', v_t, \omega_{t-1}) \leq N_Y(\pi, v_t, \omega_{t-1})$. That is, there is a negative relationship between the $\pi$-life and $\pi$. Third, the fact that $N_Y(\pi, v_t, \omega_{t-1})$ is discrete implies that the absorption measure has to be applied with some care. In particular, for a given single value of $\pi$, $N_Y(\pi, v_t, \omega_{t-1})$ is not necessarily able to discriminate between processes that have different persistence properties. Examining the absorption measure for a range of values for $\pi$ helps to remedy this. For example, in the AR(1) case, the $\pi$-absorption time for $\pi = 0.50$, which corresponds to the usual measure of the half-life of shocks, is equal to 2 for both $\phi = 0.6$ and $\phi = 0.7$. On the other hand, $N_Y(\pi, v_t, \omega_{t-1})$ for $\pi = 0.10$ is equal to 5 and 7 for $\phi = 0.6$ and 0.7, respectively. Finally, notice that for a random walk $Y_t = Y_{t-1} + V_t$, $G_Y(n, v_t, \omega_{t-1}) = v_t$ for all $n \geq 0$, so that $I_Y(\pi, n, v_t, \omega_{t-1}) = 1$ and $N_Y(\pi, v_t, \omega_{t-1}) = 0$ in all cases. This follows from the fact that for a random walk the final effect of a shock is reached instantaneously upon impact.

When implementing the absorption measure $N_Y(\pi, V_t, \Omega_t)$ in practice, a finite horizon $n_{\text{max}}$ has to be chosen both to approximate (11) and to approximate the ultimate impulse response $G_Y^\infty(V_t, \Omega_t)$ necessary for computing the indicator function $I_Y(\pi, n, V_t, \Omega_t)$ in (13). It is advisable to set the truncation point $n_{\text{max}}$ fairly large, in any case larger than the typical horizons of interest. In addition, it is appropriate to perform some empirical checks to ensure that $G_Y(n, V_t, \Omega_t)$ is approximately equal to $G_Y(n_{\text{max}}, V_t, \Omega_t)$ for horizons $n > n_{\text{max}}$. This would also provide a rough idea of the existence of the limit $G_Y^\infty(V_t, \Omega_t)$, and of the accuracy of $G_Y(n_{\text{max}}, V_t, \Omega_t)$ as a proxy for the ultimate response. We illustrate this and other aspects involved in the practical implementation of the absorption measure in the empirical application in Section 5.

### 3.2 Measuring asymmetric absorption

Possible asymmetry in the absorption of positive and negative shocks can be examined in a way similar to detecting asymmetry in impulse responses, as discussed in Section 2.3. For a specific shock $v_t$ and history $\omega_{t-1}$, a measure of asymmetric absorption can be defined as the difference in $\pi$-absorption times of $v_t$ and $-v_t$, that is,

$$ASYN_Y(\pi, v_t, \omega_{t-1}) = N_Y(\pi, v_t, \omega_{t-1}) - N_Y(\pi, -v_t, \omega_{t-1}). \quad (14)$$

If $v_t$ has symmetric absorption at $\omega_{t-1}$, $ASYN_Y(\pi, v_t, \omega_{t-1}) = 0$ for all values of $\pi$.

Note that symmetry in $G_Y(n, v_t, \omega_{t-1})$, that is, $ASY_Y(n, v_t, \omega_{t-1}) = 0$ for all
number of combinations of shocks \( n \geq 0 \) in (8), implies symmetry in absorption, that is, \( \text{ASY}_Y(\pi, v_t, \omega_{t-1}) = 0 \) for all \( \pi \in (0, 1) \). Interestingly, the reverse does not hold, that is, a shock can have symmetric absorption but an asymmetric impulse response. Also, \( \text{ASY}_Y(\pi, v_t, \omega_{t-1}) \neq 0 \) for certain \( \pi \in (0, 1) \) implies that \( \text{ASY}_Y(n, v_t, \omega_{t-1}) \neq 0 \) for certain \( n \geq 0 \), whereas the reverse need not hold.

As before, the asymmetry measure in (14) can be regarded as a realization of the random variable

\[
\text{ASY}_Y(\pi, V_i^+, \Omega_{t-1}) = N_Y(\pi, V_i^+, \Omega_{t-1}) - N_Y(\pi, -V_i^+, \Omega_{t-1}),
\]

where \( V_i^+ \) is the set of positive shocks, as defined just below (9). Obviously, the asymmetry measure can also be defined for a subset \( \mathcal{S}^+ \) of positive shocks and a subset \( \mathcal{H} \) of histories.

By taking into account parameter uncertainty, one can consider whether a specific shock \( v_t \) has symmetric absorption at a specific history \( \omega_{t-1} \) by examining whether \( \text{ASY}_Y(\pi, v_t, \omega_{t-1}) \) is significantly different from zero. To assess whether the absorption of shocks in the set \( \mathcal{S} \) for the set of histories \( \mathcal{H} \) is symmetric on average, we may test whether the mean of the distribution of \( \text{ASY}_Y(\pi, \mathcal{S}^+, \mathcal{H}) \) is equal to zero. This is complicated by the fact that the different realizations \( \text{ASY}_Y(\pi, v_t, \omega_{t-1}) \) used to estimate this distribution are not independent across histories \( \omega_{t-1} \). Hence, the standard error for the mean of \( \text{ASY}_Y(\pi, \mathcal{S}^+, \mathcal{H}) \) is not equal to \( \sigma_{\text{ASY}_Y(\pi, \mathcal{S}^+, \mathcal{H})}/\sqrt{n_{\mathcal{S}^+\mathcal{H}}} \), where \( \sigma_{\text{ASY}_Y(\pi, \mathcal{S}^+, \mathcal{H})} \) is the standard deviation of \( \text{ASY}_Y(\pi, \mathcal{S}^+, \mathcal{H}) \) and \( n_{\mathcal{S}^+\mathcal{H}} \) is the number of combinations of shocks \( v_t \) and histories \( \omega_{t-1} \) for which \( \text{ASY}_Y(\pi, v_t, \omega_{t-1}) \) is computed. Note however that the \( \text{ASY}_Y(\pi, v_t, \omega_{t-1}) \) are independent across shocks \( v_t \). Therefore, as a conservative standard error for the mean of \( \text{ASY}_Y(\pi, \mathcal{S}^+, \mathcal{H}) \) we suggest to use \( \sigma_{\text{ASY}_Y(\pi, \mathcal{S}^+, \mathcal{H})}/\sqrt{n_{\mathcal{S}}} \), where \( n_{\mathcal{S}} \) is the number of shocks \( v_t \) for which \( \text{ASY}_Y(\pi, v_t, \omega_{t-1}) \) is computed.

Alternatively, the asymmetry of the distribution of \( \text{ASY}_Y(\pi, \mathcal{S}^+, \mathcal{H}) \) can be assessed by means of confidence regions. Following Hyndman (1995), we consider three different 100 \cdot (1 - \alpha)\% confidence regions:

(1) An interval symmetric around the mean

\[
\text{SAM}_\alpha = (\hat{\mu}_{\text{ASY}_Y(\pi, \mathcal{S}^+, \mathcal{H})} - w, \quad \hat{\mu}_{\text{ASY}_Y(\pi, \mathcal{S}^+, \mathcal{H})} + w),
\]

where \( \hat{\mu}_{\text{ASY}_Y(\pi, \mathcal{S}^+, \mathcal{H})} \) is the mean of the asymmetry measure \( \text{ASY}_Y(\pi, \mathcal{S}^+, \mathcal{H}) \) and \( w \) is such that \( P(\text{ASY}_Y(\pi, \mathcal{S}^+, \mathcal{H}) \in \text{SAM}_\alpha) = 1 - \alpha \).

(2) The interval between the \( \alpha/2 \) and \( (1 - \alpha/2) \) quantiles of the distribution of \( \text{ASY}_Y(\pi, \mathcal{S}^+, \mathcal{H}) \), denoted \( q_{\alpha/2} \) and \( q_{1-\alpha/2} \), respectively,

\[
\text{EQI}_\alpha = (q_{\alpha/2}, q_{1-\alpha/2}).
\]
(3) The highest-density region $[HDR]$

$$HDR_{\alpha} = \{\text{ASYN}_{Y}(\pi, S^{+}, H) \mid g(\text{ASYN}_{Y}(\pi, S^{+}, H)) \geq g_{\alpha}\},$$

where $g(\cdot)$ is the density of its argument and $g_{\alpha}$ is such that $P(\text{ASYN}_{Y}(\pi, S^{+}, H) \in HDR_{\alpha}) = 1 - \alpha$.

For symmetric and unimodal distributions, these three regions are identical. For asymmetric or multimodal distributions they are not, see Hyndman (1995) for discussion and Chan and Tong (2004) for recent tests for multimodality. In the applications below, we report $\alpha^*$, which is the minimum value of $\alpha \in (0, 1)$ such that 0 would not be included in the relevant confidence region and, hence, can be interpreted as the p-value for testing the hypothesis that the mean of $\text{ASYN}_{Y}(\pi, S^{+}, H) = 0$. Note that the three confidence regions all provide different information. The interval symmetric around the mean indicates the position of 0 relative to the mean of the distribution. The interval with equal quantiles in the tail indicates whether 0 is located in the tails or in the central part of the distribution. Finally, the $HDR$ indicates the probability that the asymmetry measure is equal to 0.

4 Absorption of shocks in multivariate models

The concept of absorption can also be used to investigate the properties of multivariate nonlinear models. In this section, we first define the multivariate extension of the univariate absorption measure used so far. Next, we discuss how to measure whether shocks are absorbed at the same rate by the different components of a multivariate time series, which we call common absorption.

4.1 Definition of absorption in multivariate models

Extending the $\pi$-absorption time measure to multivariate models is fairly straightforward. Following Pesaran and Shin (1998), we restrict attention to the generalized impulse response measuring the effect of a shock in the $j$-th equation only. Consistent with the idea of GIs that irrelevant shocks are dealt with by integrating out their effects, not only the shocks during intermediate periods $t + 1, \ldots, t + n$ are averaged out, but shocks to the other equations occurring at time $t$ as well. Hence, we have

$$\text{GI}_Y(n, v_{jt}, \omega_{t-1}) = E[Y_{t+n}|V_{jt} = v_{jt}, \omega_{t-1}] - E[Y_{t+n}|\omega_{t-1}].$$

The immediate effect of the shock is given by the impulse response at horizon $n = 0$, which is equal to $\text{GI}_Y(0, v_{jt}, \omega_{t-1}) = E[V_{jt}|V_{jt} = v_{jt}, \omega_{t-1}]$. If conditional
upon the history $\omega_{t-1}$, $V_t$ is normally distributed with covariance matrix $h_t$, it can be shown that

$$E[V_t|V_{jt} = v_{jt}, \omega_{t-1}] = (h_{t,1j}, h_{t,2j}, \ldots, h_{t,kj})' h_{t,1j}^{-1} v_{jt} = h_t e_j h_{t,jj}^{-1} v_{jt},$$

where $e_j$ is a $(k \times 1)$ vector with unity as its $j$-th element and zeros elsewhere, see Pesaran and Shin (1998). Thus, the indicator function $I_{Y_i}(\pi, n, v_{jt}, \omega_{t-1})$ now may be defined as

$$I_{Y_i}(\pi, n, v_{jt}, \omega_{t-1}) = \prod_{n=m}^{\infty} I_{Y_i}(\pi, n, v_{jt}, \omega_{t-1}).$$

The $'\pi$-life' or $'\pi$-absorption time' of $v_{jt}$ for $Y_i$ then can be defined as

$$N_{Y_i}(\pi, v_{jt}, \omega_{t-1}) = \sum_{m=0}^{\infty} \left( 1 - \prod_{n=m}^{\infty} I_{Y_i}(\pi, n, v_{jt}, \omega_{t-1}) \right). \quad (18)$$

As in the univariate case, $N_{Y_i}(\pi, v_{jt}, \omega_{t-1})$ can be regarded as a realization of the random variable

$$N_{Y_i}(\pi, V_{jt}, \Omega_{t-1}) = \sum_{m=0}^{\infty} \left( 1 - \prod_{n=m}^{\infty} I_{Y_i}(\pi, n, V_{jt}, \Omega_{t-1}) \right), \quad (19)$$

where the random indicator function $I_{Y_i}(\pi, n, V_{jt}, \Omega_{t-1})$ is obviously defined. Similarly, one can define the asymmetry measure

$$ASYN_{Y_i}(\pi, V_{jt}^+, \Omega_{t-1}) = N_{Y_i}(\pi, V_{jt}^+, \Omega_{t-1}) - N_{Y_i}(\pi, -V_{jt}^+, \Omega_{t-1}), \quad (20)$$

where $V_{jt}^+ = \{V_{jt}|V_{jt} > 0\}$, which can be used to assess whether positive and negative shocks are absorbed differently.

4.2 Measuring common absorption

In multivariate models, an additional question of interest is whether shocks are absorbed at the same rate by different variables in the system. Define the random variable $CN_{Y_i,Y_j}(\pi, V_{jt}, \Omega_{t-1})$ as the difference between the $\pi$-absorption times of $Y_i$ and of $Y_j$, that is

$$CN_{Y_i,Y_j}(\pi, V_{jt}, \Omega_{t-1}) = N_{Y_i}(\pi, V_{jt}, \Omega_{t-1}) - N_{Y_j}(\pi, V_{jt}, \Omega_{t-1}). \quad (21)$$

If shocks $V_{jt}$ are absorbed at the same rate by $Y_i$ and $Y_j$ on average, $CN_{Y_i,Y_j}(\pi, V_{jt}, \Omega_{t-1})$ should have a distribution with mean equal to zero.

Alternatively, one may ask whether there exists a linear combination $\beta'Y$, for a certain $(k \times 1)$ vector $\beta$, for which the effects of shocks die out faster than
for the component series \( Y_i, i = 1, \ldots, k \). If so, this linear combination can be viewed as a more stable variable as shocks last for a shorter period of time.

From the definition of the GI given in (4) and elementary properties of the conditional expectations operator it follows that

\[
\text{GI}_{\beta'Y}(n, V_{jt}, \Omega_{t-1}) = \beta' \text{GI}_Y(n, V_{jt}, \Omega_{t-1}).
\]

Hence, the GI for a linear combination of the elements in \( Y_t \) can be obtained directly as the same linear combination of the GI of \( Y_t \). Note that such a simple relationship does not exist between the \( \pi \)-absorption times of a linear combination and the absorption times of the elements of \( Y_t \). That is, in general

\[
N_{\beta'Y}(\pi, V_{jt}, \Omega_{t-1}) \neq \beta'N_Y(\pi, V_{jt}, \Omega_{t-1}),
\]

where \( N_Y(\pi, V_{jt}, \Omega_{t-1}) = (N_{Y_1}(\pi, V_{jt}, \Omega_{t-1}), \ldots, N_{Y_k}(\pi, V_{jt}, \Omega_{t-1}))' \). It is however straightforward to define the \( \pi \)-absorption time of \( \beta'Y_t \) as

\[
N_{\beta'Y}(\pi, V_{jt}, \Omega_{t-1}) = \sum_{m=0}^{\infty} \left( 1 - \prod_{n=m}^{\infty} I_{\beta'Y}(\pi, n, V_{jt}, \Omega_{t-1}) \right),
\]

where the indicator function \( I_{\beta'Y}(\pi, n, V_{jt}, \Omega_{t-1}) \) is defined as

\[
I_{\beta'Y}(\pi, n, V_{jt}, \Omega_{t-1}) = \mathbb{I}[||\beta'(\text{GI}_Y(n, V_{jt}, \Omega_{t-1}) - \text{GI}_{\infty}^Y(V_{jt}, \Omega_{t-1}))|| \leq \pi ||\beta'(h_t e_j h_{t,j}^{-1} V_{jt} - \text{GI}_{\infty}^Y(V_{jt}, \Omega_{t-1}))||].
\]

An alternative common absorption measure \( \text{CAN}_{Y,\beta'Y}(\pi, V_{jt}, \Omega_{t-1}) \) can then be defined as the difference of the \( \pi \)-absorption times of \( Y_i \) and \( \beta'Y \), that is

\[
\text{CAN}_{Y,\beta'Y}(\pi, V_{jt}, \Omega_{t-1}) = N_{Y_i}(\pi, V_{jt}, \Omega_{t-1}) - N_{\beta'Y}(\pi, V_{jt}, \Omega_{t-1}), \quad i = 1, \ldots, k.
\]

If shocks \( V_{jt} \) are not absorbed differently by the linear combination \( \beta'Y \) than by the individual series \( Y_i \) on average, \( \text{CAN}_{Y_i,\beta'Y}(\pi, V_{jt}, \Omega_{t-1}) \) should have a distribution with mean equal to zero for all \( i = 1, \ldots, k \).

5 Impulse response and absorption in the floor-and-ceiling model

US output growth has been by far the most popular macro-economic application of nonlinear regime-switching time series models. In particular, following Hamilton (1989) many attempts have been made to describe the apparent asymmetric behavior over the business cycle in US output by means of Markov-Switching models, see Clements and Krolzig (2002) and Kim, Morley, and Piger (2005) for recent contributions. Alternatively, threshold and smooth transition models have also been used for this purpose, see Potter (1995b) and
Teräsvirta (1995), respectively. Here we consider the floor-and-ceiling model of Pesaran and Potter (1997), which attempts to capture business cycle asymmetry by including ‘current-depth-of-recession’ and ‘overheating’ variables as additional regressors in a linear autoregressive model for the growth rate of output.

To develop the model, define the indicators $F_t$ and $C_t$ for the floor and ceiling regimes recursively as

$$
F_t = \begin{cases} 
I[\Delta Y_t < r_F] & \text{if } F_{t-1} = 0, \\
I[CDR_{t-1} + \Delta Y_t < 0] & \text{if } F_{t-1} = 1,
\end{cases}
$$

(24)

$$
C_t = I[F_t = 0]I[\Delta Y_t > r_C]I[\Delta Y_{t-1} > r_C],
$$

(25)

where the current-depth-of-recession variable $CDR_t$ is defined as

$$
CDR_t = \begin{cases} 
(\Delta Y_t - r_F)F_t & \text{if } F_{t-1} = 0, \\
(CDR_{t-1} + \Delta Y_t)F_t & \text{if } F_{t-1} = 1,
\end{cases}
$$

(26)

and the overheating variable $OH_t$ is given by

$$
OH_t = C_t(OH_{t-1} + \Delta Y_t - r_C).
$$

(27)

The above definitions imply that the floor regime is entered whenever output growth falls below the threshold $r_F$. In that case $CDR_t$ measures the accumulated growth below $r_F$ since entering the floor regime. The floor regime is left whenever this ‘growth deficit’ is eliminated and $CDR_t$ turns non-negative.

Note that in case the floor threshold $r_F = 0$, (26) with (24) reduces to the gap between the current value of output and its historical maximum value, that is $CDR_t = Y_t - \max_{j \geq 0} Y_{t-j}$, as used by Beaudry and Koop (1993). The ceiling regime becomes active when output growth exceeds the threshold $r_C$ in two consecutive quarters (and provided that the floor regime is not active). The overheating variable $OH_t$ then measures the accumulated growth over the ceiling $r_C$. The floor-and-ceiling model for output growth now is given by

$$
\phi(L)\Delta Y_t = \phi_0 + \theta_1 CDR_{t-1} + \theta_2 OH_{t-1} + V_t,
$$

(28)

where $\phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p$ is a lag polynomial of order $p$, with the lag operator defined as $L^m Y_t = Y_{t-m}$ for all $m$ and $\Delta = 1 - L$ is the first-difference operator. $E[V_t|\Omega_{t-1}] = 0$, and the conditional variance of $V_t$ is given by

$$
E[V_t^2|\Omega_{t-1}] \equiv H_t = (\sigma_F^2 F_t - 1 + \sigma_{COR}^2 COR_{t-1} + \sigma_C^2 C_{t-1})(1 - \sigma_B^2 I[t > \tau]),
$$

(29)

where the indicator for the corridor regime is defined as

$$
COR_t = I[F_t + C_t = 0].
$$
Compared to the original model of Pesaran and Potter (1997), the conditional volatility specification in (29) is augmented with the term \((1 - \sigma_B^2 I[t > \tau])\). This is included to accommodate the large decline in output volatility that occurred during the 1980s, see McConnell and Perez-Quiros (2000), Stock and Watson (2003) and Sensier and van Dijk (2004), among others. Note that we assume that this break affects volatility in all three regimes equally. Pesaran and Potter (1997) provide an extensive discussion and motivation of the floor-and-ceiling model, showing among others that it can be interpreted as a specific type of threshold model.

The floor-and-ceiling model is applied to quarterly observations on seasonally adjusted real US GDP, from 1954:1-2004:4, expressed in annualized percentage points. Following Pesaran and Potter (1997), we set \(p = 2\) in (28). We fix the date of the volatility break \(\tau\) to be equal to 1984:1, as established by McConnell and Perez-Quiros (2000) and Stock and Watson (2003). We estimate the model parameters with maximum likelihood, using a grid search over the floor and ceiling thresholds \(r_F\) and \(r_C\) where we require that each of the regimes contains at least 10% of the observations. The resulting parameter estimates, with heteroskedasticity-consistent standard errors in parentheses, are given by \(\hat{\phi}_0 = 1.205(0.153), \hat{\phi}_1 = 0.310(0.006), \hat{\phi}_2 = 0.314(0.006), \hat{\theta}_1 = -0.524(0.050), \hat{\theta}_2 = -0.072(0.002), \hat{\sigma}_F = 5.280(0.645), \hat{\sigma}_{COR} = 4.095(0.113), \hat{\sigma}_C = 3.740(0.150), \hat{\sigma}_B = 0.554(0.002), \hat{r}_F = -2.775, \text{ and } \hat{r}_C = 3.297.\) In the effective estimation sample, 23, 120 and 58 observations are located in the floor, corridor and ceiling regimes, respectively. The statistically significant negative estimates of \(\theta_1\) and \(\theta_2\) indicate that the model implies the presence of ‘dampening effects’. Given that \(CDR_t\) is zero in the corridor and ceiling regimes and becomes negative in the floor regime, average growth increases sharply during recessions. Similarly, \(OH_t\) is positive in the ceiling regime and equal to zero otherwise, such that average growth falls when the economy is overheated. Finally, note that the estimate of \(\sigma_B\) indicates that volatility of GDP growth has been substantially lower after 1984 than it was before.

We use impulse response analysis to examine whether the persistence and absorption of shocks varies with the history of the time series, with the sign of the shock and with its magnitude. In addition, we consider the effects of the volatility break in 1984:1 on these impulse response properties. For this purpose, we compute two sets of impulse response functions \(G_{\Delta Y}(n, v_t, \omega_{t-1})\) for all 201 histories \(\omega_{t-1}\) in the sample, for values of the normalized shock equal to \(v_t/\sqrt{h_t} = 0, \pm 0.1, \ldots, \pm 2.9, \pm 3\), where \(h_t\) denotes a realization of the conditional variance \(H_t\). For the first set of GIs, \(H_t\) is set equal to \(\sigma_F^2 F_{t-1} + \sigma_{COR}^2 COR_{t-1} + \sigma_C^2 C_{t-1}\), corresponding with the pre-break level of the conditional variance in (29), while for the second set of GIs, this is multiplied with \((1 - \sigma_B^2)\) such that \(H_t\) corresponds with the post-break conditional variances. The relevant histories consist of the growth rate in the two previous periods and the lagged \(CDR\) and \(OH\) variables, that is \(\Omega_{t-1} = \{\Delta Y_{t-1}, \Delta Y_{t-2}, CDR_{t-1}, OH_{t-1}\}.\)
GIs are computed for horizons \( n = 0, 1, \ldots, n_{\text{max}} \) with \( n_{\text{max}} = 20 \), using the algorithm outlined in Koop et al. (1996), with 50,000 replications to average out the effects of shocks occurring in intermediate periods, which are sampled from a normal distribution. Impulse responses for the log level of GNP are obtained by accumulating the impulse responses for the growth rate, that is
\[
\text{GI}_Y(n, v_t, \omega_{t-1}) = \sum_{i=0}^{n} \text{GI}_Y(i, v_t, \omega_{t-1}).
\]
At horizons beyond 20 quarters, the impulse response \( \text{GI}_Y(n, v_t, \omega_{t-1}) \) is equal to 0 for virtually all shocks \( v_t \) and \( \omega_{t-1} \). Hence, \( \text{GI}_Y(n_{\text{max}}, v_t, \omega_{t-1}) \) forms a reliable estimate of the final impulse response \( \text{GI}_Y^\infty(v_t, \omega_{t-1}) \).

Figures 1 and 2 show distributions of \( \text{GI}_Y(n, V_t, \mathcal{H}) \) and \( \text{ASY}_Y(n, V_t^+, \mathcal{H}) \), respectively, at horizons \( n = 0, 1, \ldots, 4 \) and 20 using the pre-break and post-break levels of conditional volatility, where \( \mathcal{H} \) is the set of all histories in a particular regime. These and all subsequent distributions are obtained with a kernel density estimator, using a Gaussian kernel with automatic bandwidth selection. In addition, we set \( \phi(v_t/\sqrt{h_t}) \) as weight for \( \text{GI}_Y(n, v_t, \omega_{t-1}) \), where \( \phi(z) \) denotes the standard normal probability distribution. The reason for using this weighting scheme is that the standardized shocks \( v_t/\sqrt{h_t} \) then effectively are sampled from a discretized normal distribution and the resulting distribution of \( \text{GI}_Y(n, V_t, \mathcal{H}) \) should resemble a normal distribution if the effects of shocks are symmetric and proportional to their magnitude (as is the case in linear models). Table 1 shows the means of the impulse response distributions. In addition, means of \( \text{GI}_Y(n, S^+, \mathcal{H}) \) and \( \text{ASY}_Y(n, S^+, \mathcal{H}) \) are shown where \( S \) is taken to be the set of small \((0 < |V_t/\sqrt{H_t}| \leq 2)\), medium \((1 < |V_t/\sqrt{H_t}| \leq 2)\) or large \((2 < |V_t/\sqrt{H_t}| \leq 3)\) shocks. Table 2 contains further summary statistics for the distributions of \( \text{ASY}_Y(n, S^+, \mathcal{H}) \) at horizon \( n = 20 \). The reason for reporting results for horizons \( n = 0, 1, \ldots, 4 \) and 20 is that almost all impulse responses turn out to be hump-shaped, where the largest effect (in absolute value) is attained within the first year followed by a gradual decline or increase towards the ultimate response.

Panels (a), (c) and (e) of Figure 1 show that the \( \text{GI}_Y(n, V_t, \mathcal{H}) \) distributions are heavily skewed when using the pre-break volatility levels, especially in the floor and corridor regimes. This definitely suggests the presence of asymmetry in the impulse response, but note that it need not be the case that the (final) impulse response is larger for positive shocks than for negative ones. In fact, the distributions of the asymmetry measure \( \text{ASY}_Y(n, V_t^+, \mathcal{H}) \) shown in panels (a), (c) and (e) of Figure 2 have considerable probability mass at negative values, suggesting that for quite a few shocks and histories, the response to a negative shock actually is larger. The means of \( \text{ASY}_Y(n, V_t^+, \mathcal{H}) \), as shown in the columns headed A in Table 1 are close to zero, confirming that the asymmetry in the impulse response is not (only) related to the sign of the shocks. Distinguishing between different magnitudes of shocks is more revealing in this respect. The means of \( \text{ASY}_Y(n, S^+, \mathcal{H}) \) in Table 1 shows that small negative shocks have larger effects than small positive ones and vice versa for
Fig. 1. Distributions of impulse response functions $\text{GI}_Y(n, V_t, \mathcal{H})$ in floor-and-ceiling model, conditional on different sets of histories consisting of all observations that fall in a particular regime (floor, corridor, ceiling).
Fig. 2. Distributions of asymmetry measures $ASY_Y(n, V_t^+, \mathcal{H})$ in floor-and-ceiling model, conditional on different sets of histories consisting of all observations that fall in a particular regime (floor, corridor, ceiling).
medium and large shocks. Comparing the mean of $ASY_Y(n, S^+, \mathcal{H})$ for $n = 20$ with the conservative standard error $\sigma_{ASY_Y(n, S^+, \mathcal{H})}/\sqrt{nS}$ in Table 2, it appears that the asymmetry is significant in the floor and corridor regimes for all magnitudes of shocks, and only for large shocks in the ceiling regime. The values of $\alpha^*$ reported in the final three rows for the different confidence regions suggest that the asymmetry is most pronounced for large shocks occurring in the floor and corridor regimes, with positive shocks being more persistent. This is in contrast with Pesaran and Potter (1997), who find that negative shocks are more persistent than positive ones on average. We do confirm their finding that shocks are more persistent in the corridor regime, with the absolute

### Table 1. Impulse responses and asymmetry measures

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<td>20</td>
<td>3.14</td>
<td>0.29</td>
<td>7.51</td>
<td>16.78</td>
<td>5.07</td>
<td>2.52</td>
<td>9.15</td>
<td>16.22</td>
<td>4.17</td>
<td>2.13</td>
<td>7.47</td>
<td>12.96</td>
</tr>
</tbody>
</table>

| ASY$_Y(n, S^+, \mathcal{H})$, pre-break volatility | | | | | | | | | | | | |
| 1   | 0.01 | −0.74 | 1.17 | 3.67 | 0.00 | −0.39 | 0.45 | 2.82 | 0.00 | −0.10 | −0.01 | 1.50 |
| 2   | 0.02 | −1.36 | 2.09 | 6.93 | 0.00 | −0.71 | 0.81 | 4.97 | 0.00 | −0.16 | −0.01 | 2.43 |
| 3   | 0.02 | −1.97 | 2.98 | 10.13 | −0.01 | −1.04 | 1.20 | 7.17 | 0.00 | −0.22 | −0.02 | 3.35 |
| 4   | 0.02 | −2.31 | 3.46 | 11.92 | −0.02 | −1.25 | 1.45 | 8.48 | 0.00 | −0.27 | −0.02 | 3.89 |
| 20  | 0.02 | −2.52 | 3.76 | 13.11 | −0.04 | −1.46 | 1.68 | 9.53 | 0.00 | −0.34 | 0.04 | 4.48 |

| GI$_Y(n, S^+, \mathcal{H})$, post-break volatility | | | | | | | | | | | | |
| 0   | 1.90 | 1.09 | 3.20 | 5.41 | 1.47 | 0.85 | 2.48 | 4.20 | 1.35 | 0.77 | 2.27 | 3.84 |
| 1   | 1.82 | 0.93 | 3.23 | 5.88 | 1.91 | 1.10 | 3.21 | 5.41 | 1.58 | 0.87 | 2.71 | 4.65 |
| 2   | 2.00 | 0.91 | 3.72 | 7.08 | 2.46 | 1.42 | 4.13 | 6.97 | 1.91 | 1.04 | 3.31 | 5.70 |
| 3   | 1.87 | 0.72 | 3.66 | 7.31 | 2.72 | 1.58 | 4.55 | 7.65 | 1.96 | 1.06 | 3.41 | 5.86 |
| 4   | 1.83 | 0.61 | 3.71 | 7.61 | 2.93 | 1.71 | 4.89 | 8.19 | 2.02 | 1.09 | 3.51 | 6.01 |
| 20  | 1.63 | 0.34 | 3.61 | 7.83 | 3.24 | 1.92 | 5.38 | 8.90 | 2.04 | 1.10 | 3.56 | 6.05 |

| ASY$_Y(n, S^+, \mathcal{H})$, post-break volatility | | | | | | | | | | | | |
| 1   | 0.00 | −0.22 | 0.33 | 1.22 | 0.00 | 0.00 | −0.03 | 0.09 | 0.00 | −0.06 | 0.12 | 0.14 |
| 2   | 0.00 | −0.45 | 0.65 | 2.43 | 0.00 | 0.01 | −0.06 | 0.12 | 0.01 | −0.10 | 0.23 | 0.21 |
| 3   | 0.00 | −0.66 | 0.95 | 3.57 | −0.01 | 0.03 | −0.11 | 0.12 | 0.01 | −0.12 | 0.29 | 0.13 |
| 4   | 0.00 | −0.80 | 1.15 | 4.28 | −0.01 | 0.05 | −0.15 | 0.07 | 0.02 | −0.12 | 0.34 | 0.05 |
| 20  | 0.00 | −1.05 | 1.53 | 5.54 | 0.00 | 0.11 | −0.26 | −0.11 | 0.03 | −0.12 | 0.39 | −0.17 |

Note: Mean of GI$_Y(n, S, \mathcal{H})$ and ASY$_Y(n, S^+, \mathcal{H})$ in floor-and-ceiling model, conditional on different sets of shocks and histories. The sets of shocks are defined as $A[\mathcal{H}]= \{ V_1 \}, S(mall)= \{ V_1 \mid |V_1/\sqrt{\mathcal{H}}| > 0 \}, M(edium)= \{ V_2 \mid |V_2/\sqrt{\mathcal{H}}| > 1 \}, L(arge)= \{ V_3 \mid |V_3/\sqrt{\mathcal{H}}| > 2 \}$. The sets of histories consist of all observations that fall in a particular regime (floor, corridor, ceiling).
ultimate effect being equal to 5.07 for an average magnitude of the initial shock of 3.31. The difference with persistence in the ceiling regime, where the average initial shock equals 3.02 and the eventual response equals 4.17, is not large though. Note that in the floor regime, the response at $n=20$ (and beyond) of 3.26 is actually smaller than the average magnitude of the impulse at $n=0$ of 4.26.

Given the large decline in volatility in 1984, it is not surprising that the impulse response distributions shown in panels (b), (d) and (f) of Figure 1 are less dispersed than their counterparts in panels (a), (c) and (e). We do note though that the reduction in standard deviation of $\text{GI}_Y(n, V_t, \Omega_{t-1})$ is less than proportional to the reduction in the conditional standard deviation $\sqrt{\Omega_t}$, especially at longer horizons. More interestingly, we observe that the change in volatility has substantially reduced the asymmetry in the impulse responses, in particular in the corridor regime. This is confirmed in Table 2, where the means of $\text{ASY}_Y(n, S^+, \mathcal{H})$ are very close to zero for all magnitudes of shocks for the corridor regime. Asymmetry has become considerably smaller in the floor regime as well, although Table 2 shows that at $n=20$ it remains significant.

Truncating the summations in (11) at $n_{\text{max}} = 20$ and using $\text{GI}_Y(n_{\text{max}}, v_t, \omega_{t-1})$
as an estimate of the final impulse response $G_Y^n(v_t, \omega_{t-1})$, we compute $\pi$-absorption times $N_Y(\pi, v_t, \omega_{t-1})$ and asymmetry measures $ASY_Y(\pi, v_t, \omega_{t-1})$ for $\pi = 0.50, 0.40, \ldots, 0.10$, again using both pre-break and post-break volatility levels. Table 3 reports the means of $N_Y(\pi, S, H)$, while Table 4 contains summary statistics for the distribution of $ASY_Y(\pi, S^+, H)$, where $S$ and $H$ are defined as above. To save space, Table 4 only reports results for $\pi = 0.50$ and 0.10. Figures 3 and 4 show distributions of $N_Y(\pi, S, H)$ for $\pi = 0.50$ using pre-break and post-break volatility levels, respectively. Distributions of $ASY_Y(\pi, S^+, H)$ are shown in Figures 5 and 6. Additional tables and figures with detailed results for other values of $\pi$ are available upon request.

### Table 3. Absorption times

<table>
<thead>
<tr>
<th>$\pi$</th>
<th>A S M L</th>
<th>A S M L</th>
<th>A S M L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-break volatility</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>3.69</td>
<td>3.62</td>
<td>3.98</td>
</tr>
<tr>
<td>0.40</td>
<td>4.09</td>
<td>3.99</td>
<td>4.44</td>
</tr>
<tr>
<td>0.30</td>
<td>4.63</td>
<td>4.52</td>
<td>5.05</td>
</tr>
<tr>
<td>0.20</td>
<td>5.32</td>
<td>5.19</td>
<td>5.78</td>
</tr>
<tr>
<td>0.10</td>
<td>6.41</td>
<td>6.27</td>
<td>6.83</td>
</tr>
<tr>
<td>Post-break volatility</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>3.28</td>
<td>3.33</td>
<td>3.23</td>
</tr>
<tr>
<td>0.40</td>
<td>3.69</td>
<td>3.76</td>
<td>3.57</td>
</tr>
<tr>
<td>0.30</td>
<td>4.32</td>
<td>4.37</td>
<td>4.25</td>
</tr>
<tr>
<td>0.20</td>
<td>5.20</td>
<td>5.28</td>
<td>5.10</td>
</tr>
<tr>
<td>0.10</td>
<td>6.59</td>
<td>6.67</td>
<td>6.50</td>
</tr>
</tbody>
</table>

Note: Mean of $N_Y(\pi, S, H)$ in floor-and-ceiling model, conditional on different sets of shocks and histories. The sets of shocks are defined as $A(\text{II}) = \{V_t\}$, $S(\text{mall}) = \{V_t | 1 \geq |V_t/\sqrt{H_t}| > 0\}$, $M(\text{edium}) = \{V_t | 2 \geq |V_t/\sqrt{H_t}| > 1\}$, $L(\text{arge}) = \{V_t | 3 \geq |V_t/\sqrt{H_t}| > 2\}$. The sets of histories consist of all observations that fall in a particular regime (floor, corridor, ceiling).

Comparing the columns headed A in the top panel of Table 3 shows that on average shocks are absorbed fastest in the corridor regime, followed by the ceiling and floor regimes. The difference in absorption times between the corridor and ceiling regimes is quite small, but between the corridor and floor regimes it varies around one quarter for the values of $\pi$ considered. The ranking of absorption times in the different regimes critically depends on the magnitude of the shocks, as shown by the remaining columns of Table 3. For small shocks, the ordering is as just described. By contrast, medium-sized shocks are absorbed fastest in the ceiling regime, followed by the corridor and floor regimes, while the floor regime shows faster absorption for large shocks than the corridor and ceiling regimes.

The columns headed A in the first and third panel of Table 4 show that positive shocks have larger mean absorption time than negative shocks in the corridor and ceiling regimes, whereas the opposite holds in the floor regime. The
average asymmetry is not significantly different from zero though. Turning to the subsets of shocks of different magnitudes, we observe interesting differences in the absorption time for \( \pi = 0.50 \) or, put differently, the half-life of shocks. In the floor and ceiling regimes, positive large shocks are absorbed faster than negative large shocks, while there is no significant asymmetry for small and

### Table 4. Asymmetry measures for absorption times

<table>
<thead>
<tr>
<th>( \pi = 0.50 ), pre-break volatility</th>
<th>floor regime</th>
<th>corridor regime</th>
<th>ceiling regime</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>S</td>
<td>M</td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>St. dev.</td>
<td>2.52</td>
<td>2.47</td>
<td>2.73</td>
</tr>
<tr>
<td>Skewness</td>
<td>−0.68</td>
<td>−0.89</td>
<td>−0.52</td>
</tr>
<tr>
<td>( \text{SAM}_{\alpha} )</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>( \text{EQI}_{\alpha} )</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
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<tr>
<td>( \text{HDR}_{\alpha} )</td>
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<td>1.00</td>
<td>1.00</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>( \pi = 0.50 ), post-break volatility</th>
<th>floor regime</th>
<th>corridor regime</th>
<th>ceiling regime</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>S</td>
<td>M</td>
</tr>
<tr>
<td>Mean</td>
<td>−0.33</td>
<td>0.00</td>
<td>−0.96</td>
</tr>
<tr>
<td>St. dev.</td>
<td>2.43</td>
<td>2.44</td>
<td>2.38</td>
</tr>
<tr>
<td>Skewness</td>
<td>−0.10</td>
<td>0.00</td>
<td>−0.46</td>
</tr>
<tr>
<td>( \text{SAM}_{\alpha} )</td>
<td>1.00</td>
<td>1.00</td>
<td>0.90</td>
</tr>
<tr>
<td>( \text{EQI}_{\alpha} )</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>( \text{HDR}_{\alpha} )</td>
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<td>1.00</td>
<td>1.00</td>
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</tbody>
</table>

<table>
<thead>
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<th>( \pi = 0.10 ), pre-break volatility</th>
<th>floor regime</th>
<th>corridor regime</th>
<th>ceiling regime</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>S</td>
<td>M</td>
</tr>
<tr>
<td>Mean</td>
<td>−0.33</td>
<td>1.00</td>
<td>1.05*</td>
</tr>
<tr>
<td>St. dev.</td>
<td>3.02</td>
<td>3.11</td>
<td>2.39</td>
</tr>
<tr>
<td>Skewness</td>
<td>−0.88</td>
<td>−1.04</td>
<td>0.37</td>
</tr>
<tr>
<td>( \text{SAM}_{\alpha} )</td>
<td>1.00</td>
<td>0.75</td>
<td>0.41</td>
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<td>0.85</td>
<td>0.63</td>
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<tr>
<td>( \text{HDR}_{\alpha} )</td>
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<table>
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<th>( \pi = 0.10 ), post-break volatility</th>
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<th>corridor regime</th>
<th>ceiling regime</th>
</tr>
</thead>
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<td>A</td>
<td>S</td>
<td>M</td>
</tr>
<tr>
<td>Mean</td>
<td>−0.41</td>
<td>0.12</td>
<td>−1.46*</td>
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<tr>
<td>St. dev.</td>
<td>2.93</td>
<td>2.82</td>
<td>2.96</td>
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<tr>
<td>Skewness</td>
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<td>−0.14</td>
<td>−0.28</td>
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<tr>
<td>( \text{SAM}_{\alpha} )</td>
<td>1.00</td>
<td>1.00</td>
<td>0.84</td>
</tr>
<tr>
<td>( \text{EQI}_{\alpha} )</td>
<td>1.00</td>
<td>1.00</td>
<td>0.99</td>
</tr>
<tr>
<td>( \text{HDR}_{\alpha} )</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

*Note: Summary statistics for asymmetry measure \( \text{ASYN}_{\pi}(\pi, S^+, H) \) in floor-and-ceiling model. The different sets of shocks are defined as \( A[0] = \{ V_i \}, \text{S(mall)} = \{ |V_i| \geq |V|/\sqrt{H} | \geq 1 \}, \text{M(edium)} = \{ |V| > |V|/\sqrt{H} | > 1 \}, \text{L(large)} = \{ |V| \leq |V|/\sqrt{H} | < 1 \} \}. \) The sets of histories consist of all observations that fall in a particular regime (floor, corridor, ceiling). Entries in rows labelled Mean which are larger than two times \( \bar{S}_{\pi} \) and \( n_{\pi} \) are marked with an asterisk, where \( \bar{S}_{\pi} \) is the standard deviation of \( \text{ASYN}_{\pi}(\pi, S^+, H) \) and \( n_{\pi} \) is the number of shocks \( n_{\pi} \) for which \( \text{ASYN}_{\pi}(\pi, S^+, H) \) is computed. Entries in rows labelled \( Z_{\alpha} \) represent the minimum value of \( \alpha \in (0, 1) \) such that \( 0 \) would not be included in the relevant confidence region \( Z_{\alpha} = \text{SAM}, \text{EQI} \) and \( \text{HDR} \).
Fig. 3. Distributions of absorption times in floor-and-ceiling model for $\pi = 0.50$ using pre-break volatility, conditional on different sets of shocks and histories. The sets of shocks are defined as $A(ll)=\{V_t\}$, $N(egative)=\{V_t|V_t < 0\}$, and $P(ositive)=\{V_t|V_t > 0\}$. The sets of histories consist of all observations that fall in a particular regime (floor, corridor, ceiling).

medium shocks in these regimes based on the standard error $\sigma_{ASYM}(\pi,S+\kappa)/\sqrt{n_S}$. In the corridor regime, large positive shocks have a shorter half-life as well, but in addition the same holds for medium-sized shocks. By contrast, small positive shocks in the corridor regime have a longer half-life than negative ones. Similar patterns emerge for $\pi = 0.10$ in the third panel of Table 4. Also note that in the floor regime there is a ‘reversal’, in the sense that positive large shocks are absorbed faster for larger values of $\pi$, while they are absorbed slower for smaller values of $\pi$. 

24
Comparing the two panels of Table 3 reveals that the decline in volatility in 1984 has substantially reduced the average absorption times for shocks occurring in the ceiling regime for all values of $\pi$ considered. Interestingly, for the floor and corridor regimes we observe that the half-life of shocks has become smaller, but that absorption times have become longer for smaller values of $\pi$, although the latter changes are fairly small. The volatility change has also had mixed effects on the asymmetry properties of $N_Y(\pi, S, H)$, which can be learned from Table 4. Asymmetry has been hardly affected for certain
Fig. 5. Distributions of asymmetry measures for absorption times in floor-and-ceiling model for $\pi = 0.50$ using pre-break volatility, conditional on different sets of shocks and histories. The sets of shocks are defined as $S(\text{mall}) = \{V_t | 1 \geq |V_t / \sqrt{H_t}| > 0\}$, $M(\text{edium}) = \{V_t | 2 \geq |V_t / \sqrt{H_t}| > 1\}$, $L(\text{arge}) = \{V_t | 3 \geq |V_t / \sqrt{H_t}| > 2\}$. The sets of histories consist of all observations that fall in a particular regime (floor, corridor, ceiling).

Subsets of shocks $S$ and histories $H$ and for certain values of $\pi$, such as large shocks occurring in the floor regime for $\pi = 0.50$ or occurring in the ceiling regime for $\pi = 0.10$. In other cases, asymmetry has disappeared, see the half-life of large shocks occurring in the ceiling regime or small shocks in the corridor regime for both $\pi = 0.50$ and 0.10. The half-life of medium shocks in the corridor regime provides the most striking example. As can be seen from panel (e) of Figures 5 and 6 the distribution of $\text{ASYN}_Y(\pi, S, H)$ has effectively collapsed to a spike at 0 in this case. In still other cases, asymmetry has been
Fig. 6. Distributions of asymmetry measures for absorption times in floor-and-ceiling model for $\pi = 0.50$ using post-break volatility, conditional on different sets of shocks and histories. The sets of shocks are defined as $A(\text{l})= \{V_t\}$, $S(\text{mall})= \{V_t | 1 \geq |V_t/\sqrt{H_t}| > 0\}$, $M(\text{edium})= \{V_t | 2 \geq |V_t/\sqrt{H_t}| > 1\}$, $L(\text{arge})= \{V_t | 3 \geq |V_t/\sqrt{H_t}| > 2\}$. The sets of histories consist of all observations that fall in a particular regime (floor, corridor, ceiling).

introduced, see large shocks in the floor regime and medium shocks in the ceiling regime at $\pi = 0.10$. Panels (d) and (f) of Figures 5 suggest that the absorption of medium-sized shocks in the floor and ceiling regimes also has become more asymmetric, but according to Table 4 this is not significant.

Finally, we note that our empirical analysis in this Section is based entirely on a univariate reduced-form model. This does not allow us to distinguish between different possible sources or explanations for the observed asymme-
tries in absorption times and changes therein. We therefore restrict ourselves
to documenting their existence. Investigating the underlying causes requires a
structural modeling approach, and is left for future research.

6 Concluding remarks

In this paper we proposed a new tool that can be used to examine the prop-
erties of univariate and multivariate nonlinear time series models. This tool,
which we called the absorption rate, can be viewed as complementary to the
familiar impulse response function, as both consider different aspects of the
propagation of shocks. The absorption rate can be used to examine whether
the propagation of different types of shocks, such as large and small shocks,
positive and negative shocks, and shocks in various regimes, follows a different
pattern. In multivariate models, the absorption rate can also reveal whether
shocks have longer lasting effects on certain variables than on others. Hence,
the absorption rate can help to interpret a possibly complicated nonlinear
model, with potentially a large number of parameters.

In a sense, the absorption rate is informative for the degree of nonlinearity a
particular model is picking up from the data. If all kinds of shocks have similar
effects on the future path of a time series variable, the nonlinear model can
be said to have linear properties, even though parameters for the nonlinear
component are highly significant. Such a finding can imply that either there is
not enough nonlinearity in the data or the model is not capturing the nonlinear
features adequately.

The above leads to the suggestion that the absorption rate can provide useful
prior information as to how successful a particular nonlinear model will be
when it comes to out-of-sample forecasting. With respect to our empirical
application to US GDP, we found considerable evidence for asymmetry in the
absorption rate of different types of shocks in the different regimes in the
floor-and-ceiling model. However, it appears that asymmetry became much
less pronounced due to a large decline in output volatility in the 1980s. Hence,
it may not come as a surprise that linear models tend to beat this nonlinear
model in terms of forecasting output growth during recent years.

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