Dynamic System Analysis and Generalized Optimal Code Assignment of OVSF-CDMA Systems

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Abstract—OVSF-CDMA systems reallocate user codes when new calls arrive. To reduce code blocking due to new user codes, Generalized Optimal Right First Dynamic Code Assignment (GRFDCA) is brought up. GRFDCA keeps OVSF code trees in optimal right first (RF) states with respect to the nontechnology (NT) count. Dynamic model of OVSF-CDMA systems is established. Tests of hypothesis about the Markovity of arrival processes are set up. Based on the Markovity assumption, continuous time Markov model of OVSF-CDMA systems is presented. By introducing absorbing states, phase type (PH) model is applied. Dynamic properties of OVSF-CDMA systems are analyzed based on PH model. Simulations and further discussions based on the dynamic model are presented.

I. INTRODUCTION

Multimedia communication is becoming the main service of the next generation of wireless communication. In order to support various multimedia applications, a system should be able to accommodate variable user data rates. The orthogonal variable spreading factor-code division multiple access (OVSF-CDMA) system, proposed in UMTS-2000/W-CDMA supports variable data rates[1][2]. In OVSF-CDMA systems, each user is assigned one or more OVSF codes. Variable spreading factors are used to implement variable data rates.

Although OVSF-CDMA systems have many advantages, we are still facing problems. One of the problems is that in OVSF-CDMA systems without proper code assignment tend to encounter code blocking in variable data rates transmission. One of the unavoidable problems in OVSF-CDMA systems supporting variable date rate is code blocking. Since high code blocking probability leads to high dropped call ratio, discussions how to minimize code blocking probability are made in recent years. In [3], Optimal dynamic code assignment (ODCA) is proposed based on minimum-cost branch search. Another scheme is to minimize blocking probability by minimizing the number of required reassignment in the single OVSF code case [4][5]. Another scheme is multi-code reassignment, the essential idea of which is to aggregate data rate. This scheme is discussed in [6][7][8]. To optimize a system globally, right first dynamic code assignment (RFDC) is brought up in [9]. Based on the global optimization, we propose generalized right first dynamic code assignment (GRFDCA) in this paper. GRFDCA is applied in multiple code trees, which is beyond the scope of one code tree. One advantage is the ability to optimize over a multi-dimensional space. Another advantage of GRFDCA is that the normalized user data rate needs not to be of the form $2^m$. Most analysis about the dynamic properties of a system are based on the Markovity and the Poisson arrival process. We apply Kolmogorov-Smirnov statistic to the hypothesis tests. Based on the Markovity of the arrival process of new users, we set up a dynamic model for OVSF-CDMA systems. The stationary status is analyzed based on the continuous time Markov model. To analyze the average duration of heavy load states, we apply phase type (PH) model by introducing absorbing states. Statistical properties are derived in a concise matrix form.

Our contributions in this paper can be summarized as: 1. We introduce generalized right first dynamic code assignment, which is based on the view of global optimization. 2. We set up dynamic model for OVSF-CDMA systems based on the assumption of the Markovity of the arrival process. We derive the stationary properties of the dynamic model, where phase type model is applied. Conclusions of the dynamic model can be applied to system design.

II. OVSF-CDMA SYSTEM AND GRFDCA

A. Model of OVSF-CDMA System

In OVSF-CDMA systems, each data bit is mapped into an assigned OVSF code sequence. By transmitting the code sequence in the same time duration, the spectrum of signal is spreaded. The length of the code sequence is called the spreading factor. The OVSF codes can be designed from Walsh codes, as illustrated in fig.1. Each OVSF code corresponds a layer number and a branch number and is generated recursively. For example, two codes of layer 3 can be generated recursively from their mother code of layer 4, as fig.1. In the description of a code tree, we denotes a binary sequence and its inverse as $[b]$ and $[-b]$ respectively. If the normalized data rate of code in layer 1 is 1, then every code in layer $M$ has the normalized data rate of $2^M$. The maximum spreading factor $N_{max}$ is the total number of layer 1, which is called the capacity of a system. Each code in the OVSF code tree is denoted according to its code-id, which is $(layer\ number,\ branch\ number)[3]$. In W-CDMA systems, OVSF codes are designed and controlled by the radio network controller (RNC)[2].

Codes in the same layer are orthogonal. Two codes of different layers are also orthogonal unless one code is the
mother code of the other code. Since in OVSF-CDMA systems, all assigned codes are orthogonal, a code can not be used together with its mother or descendant codes. For example, after (2, 1), (1, 3) and (1, 5) are assigned, the residual capacity is \((8 - 2 - 1 - 1) = 4\), which means that the system supports normalized data rate 4. Mean while codes \((3, 1), (3, 2), (2, 2)\) and \((2, 3)\) are blocked. In other words, the new call requesting normalized user data rate 4 is blocked in OVSF code tree. Therefore, a dynamic code assignment is required.

If all the code rate factors are on the right hand side of the array of the topology count, which means that all assigned codes are on the right hand side of the code tree. We say that a code tree is in an optimal right first state if all assigned codes are on the right hand sides of the code tree.

An example of two optimal right first states is shown in fig. 2(a), their topology counts are \(X11122\) and \(X24\). To further describe optimal right first states, number topology (NT) count is introduced in[9]. For example, the NT count in fig.2(a) and 2(b) is \((111122)\) and \((224)\). Some states cannot be described by NT count, but they share some features with optimal right first states. We call such states as called false optimal right first states.

B. Generalized RFDCA

Right first dynamic code assignment (RFDCA) is based on the optimal right first state and NT count[9]. The arriving call algorithm of RFDCA algorithm is illustrated in fig.3. The code tree in fig.3(a) is of NT count 512, and the system has a new call of normalized user data rate 2. To keep the optimal right first (or false optimal) state, the tree in fig.3(a) is adjusted to be as in fig. 3(b). Therefore, it would be very easy to assign the codes to the new calls. If the code tree has 4 layers in the (false) optimal right first state, we reassign new code only if the the code tree is of state 512 or 5111 and the normalized user data rate of the new calls is 2. We do not need to reassign the codes for new calls with other user data rates. The algorithm for leaving calls is similar to that of arriving calls.

In this section, we propose a generalized RFDCA for more general applications. We call an OVSF-CDMA system with GRFDCA as a general OVSF-CDMA system. One major difference is that the normalized data rate needs not to be of the form \(2^m\). In an OVSF-CDMA system with GRFDCA, the number of code trees \(M\) can be any positive integer, which is different from the usual constraint that \(M\) is of the form \(2^m\). In RFDCA based OVSF-CDMA system, \(M\) is always of the form \(2^m\), which makes the structure of the OVSF-CDMA code tree the same as that of a single code tree with capacity partition. The capacity of a code tree group is represented by array \([2^n, 2^{n_2}, \ldots, 2^{n_M}]\). In a general OVSF-CDMA system, the system load of each code tree is detected by centralized control algorithm. The main idea of the centralized control algorithm is that once a code tree is of heavy system load, system uses another code tree with the low system load to support the new calls. For each code tree in the general OVSF-CDMA systems, RFDCA assigns or reassigns codes for new calls.

Another feature of general OVSF-CDMA system is that the user data rate can be any positive number, which means that an arbitrary arrival process of calls can be processed. By approximation, we can correspond the normalized user data rate to the capacity of a code, so that the arbitrary normalized user data rate is realized.

GRFDCA is described as follows:

1) Select a code tree to support new calls. We label the tree 1 and others 0.
2) Once a new call arrives, system maps its normalized user
data rate to capacity of code as
\[ f(R) = 2^\gamma, \]
where \(2^{\gamma-1} < R \leq 2^\gamma\). A code tree with label 1 is to be used to support the call.

3) RFDCA algorithm is applied to control the code assignment in each code tree with label 1.

4) Once the a code tree with label 1 is detected by centralized control algorithm to be of heavy system load condition, it is then labelled 2. New calls will not be assigned to the code tree until it is in a low system load.

5) If all code trees in the system are labelled 2 and there is a code tree with residual capacity larger than the normalized data rate of the new call, this code tree is to be assigned support this call.

6) System rejects the new calls if all code trees are labelled 2 and no code tree can support the new call.

Note. Heavy system load is defined according to a manually designed ratio.

III. Dynamic Markov Model of OVSF-CDMA System

In this section, we give a dynamic model of general OVSF-CDMA systems. Based on the Markovity of arrival processes, continuous time Markov model of general OVSF-CDMA systems is established. Statistical properties of general OVSF-CDMA systems are derived based on the continuous time Markov model.

Without loss of generality, we assume that there are four user data rates, 1, 2, 4 and 8 in a general OVSF-CDMA system. In this paper, we are going to derive the properties a general OVSF-CDMA system with one code tree. The multiple code tree cases are different from the single tree case only in the aspects of the system scale and the computational complexity. For instance, two OVSF code trees with capacity 8 have 1296 states. It is hard to implement both theoretical and numerical analysis.

We assume that the arrival process of new calls is a Poisson process and durations of calls are exponentially distributed. The processing time needed to process new arriving calls and leaving calls is neglected if no reassignment of new code is needed. Arrivals of new calls are assumed to be independent. Based on the above assumptions, states of a system with capacity 8 is described as tab.1.

We represent a stochastic process of the system as \(X(t) = [X_1(t), X_2(t), X_3(t), X_4(t)], X(t) \in N^4\), where \(X_i(t)\) is the number of the call with normalized user data rate of \(2^{i-1}\) in the system. This model is a multi-dimensional birth and death process. To analyze the process, we convert the multi-dimensional process into a one-dimensional process. Then the support of the vector \(X(t)\) is mapped to an integer number from 1 to 36. For example, if a system is in state 12 at time \(t\), 12 is assigned to \(X(t)\). Table I illustrates all 36 states. From Table I, we can see that \((i, j, k, l) = (0001)\) corresponds to 1,
The average time for a system to transit into heavy load states is a phase type random variable, whose probability distribution function (PDF)

\[ F_{T_{ave}}(t) = \alpha \exp \{ -Tt \} e, \]

where \( e = [1, 1, \ldots, 1]^T \).

For example, in a system with one code tree and capacity 8, the event space of the Markov chain contains 15 states, which is tabulated in Table II.

We denote the average time for a system to transit into heavy load states as \( T_{ave} \). To analyze the statistical properties of \( T_{ave} \), we first derive some related results about positive random variables.

Integrating on both sides of eq.4, we have

\[ \int_0^{+\infty} \exp (-Tx) \, dx = T. \] (5)

Combining eq.4 and eq. 5, we have

\[ E\{X\} = \alpha T^{-1} e. \] (6)

This result is similar to the scalar case that expectation of an exponential random variable with parameter \( \lambda \) is \( \lambda^{-1} \). For higher order of moment, we have

\[ E\{X^k\} = \Gamma (k + 1) \alpha T^{-k} e, \]

where \( k \) can be any positive number, \( \Gamma (k) \) is the Gamma function of \( k \).

### B. Statistical Properties for Stationary Analysis

A stochastic process is said to be stationary, the probability to be in each state is time invariant[12]. Since the continuous time Markov process in our model is measurable and conservative[13], the distribution of \( X(t) \) converges to a stationary distribution as \( t \) goes to infinite. The portion of time to be in a state is independent of the initial state. Then the statistical properties of the OVSF-CDMA systems can be derived from the stationary distribution.

Transition matrix \( Q \) is determined from the system model described in sec.III. The probability of being in state \( i \) is \( P_i \). The stationary distribution can be computed as follows:

\[ \sum_{j \neq i} q_{ji} P_j - q_{ii} P_i = 0 \quad (i = 1, 2, \ldots, 36), \] (7)

The performance of a system is evaluated from the three aspects, namely, number of rejected calls, code blocking probability and number of reassigned codes. When an OVSF-CDMA system processes new calls and leaving calls, it needs to reassign OVSF codes, which is time consuming. The number of reassigned codes by GRFDCA algorithm is discussed as follows.

According to the RFDCA algorithm, the two states in which OVSF code need to be reassigned are \((5, 1, 2)\) and \((5, 1, 1, 1)\) when new calls are of normalized user data rate 2. It means that when the system is in the state 26 or 33, and the new call is of normalized user data rate 2, we need reassign the OVSF codes. Because the arrival process of new calls of normalized user data rate 2 is a Poisson process with parameter \( \lambda e_2 \), we have

\[ N_{RC} = T_{ST} \lambda e_2, \]

where \( N_{RC} \) is the number of reassigned codes and \( T_{ST} \) is the staying time in state 26 or 33. Since the portion of time in a state converges to the stationary distribution as \( t \) goes to infinite, we have

\[ N_{RC} = (P_{26} + P_{33}) T \lambda e_2 \] (8)

The RHS of eq.8 is the average number of reassigned codes in first duration \( T \). Thus, eq.8 can be used to calculate number of reassigned codes.
System Parameters Design

In wireless communication systems, the major challenge is to meet the increasing user requirement by making use of limited system resources. Thus, system optimization and parameter design are important tasks. Without proper theoretical model, system parameters can only be determined based on numerical results. In this paper, we presented the the dynamic Markov of OVSF-CDMA systems. Then we could design the system parameters to meet specific requirements based on the dynamic model.

For example, the average system load (ASL) of OVSF-CDMA systems is important in evaluating the performance of a system. Based on the stationary distribution, we have the system parameters to meet specific requirements based on the Markov model. When the traffic load is as low as 20%, can support more user calls. Equivalently, we can say that some parameters, such as number of layers in code tree, are not optimized.

## V. SIMULATION RESULTS

Simulations are based on the assumptions mentioned in sec.III. The default range of $\lambda$ is and $\mu$ are from 1 to 100 and from 1 to 10. $[e_1,e_2,e_3,e_4]$ is chosen from $[0.25,0.25,0.25,0.25]$, $[0.4,0.4,0.1,0.1]$, $[0.1,0.4,0.4,0.1]$ and $[0.1,0.1,0.4,0.4]$. The simulations are based on $N$ new calls or over a duration $T$ seconds, default $N$ is 1000 and $T$ is 200.

### A. Number of Reassigned Codes

For a system with GRFDCA algorithm and capacity 8, we need to solve a linear equation with 36 unknown variables to compute the stationary distribution. For ODCA algorithm under same conditions, a linear equation with 67 unknown variables need to be solved, whose computational complexity increases greatly with increasing capacity. So, in this condition, we only simulate the ODCA algorithm as a baseline of GRFDCA, which provides results of GRFDCA with the Markov model. Fig. 4 shows the number of reassigned codes of GRFDCA versus the mean arrive rate(MAR), $[e_1,e_2,e_3,e_4]$ is set as $[0.25,0.25,0.25,0.25]$. Results of ODCA algorithm are also shown in fig.4. It is clear that the performance of GRFDCA algorithm is much better than that of ODCA algorithm. When the traffic load is as low as 3 Erlang with a MAR of 12, GRFDCA algorithm may reduce number of reassigned codes of ODCA by 70%. Even the traffic load is 10 Erlang with a mean arrive rate of 40, the reduction is more than 50%.

### B. First Arriving Time

We compute moments of several different PH random variables and apply them to the finite time analysis analysis of OVSF-CDMA systems. According to the theoretical results in sec. IV, we compute average $T_{ave}$. Fig. 5 illustrates the average $T_{ave}$ versus MAR from 1 to 20, which means traffic load is from 1 to 5 ($\mu$ is set as 4). On the other hand, if we fix MAR $\lambda$ as 10, average of $T_{ave}$ versus the mean staying time $\mu$ from 0.125 to 2.5 is shown in fig. 6. Based on the above results, with the increasing MAR, $T_{ave}$ are the same with different code rate ratios.

Average system load is sensitive to MAR if MAR is small. For instance, when $\lambda$ equals to 2, code ratio $[0.4,0.4,0.1,0.1]$ corresponds to $T_{ave}$ around 4, but code ratio $[0.1,0.1,0.4,0.4]$ corresponds to $T_{ave}$ just around 1. For discussion about the inverse of mean serving time, $T_{ave}$ holds a different theory. $T_{ave}$ with different code ratios are similar when $\mu$ is small, but far different when $\mu$ is greater than 2.

### C. System Parameters Design

System parameters can be theoretically designed based on the above stationary analysis and finite time analysis. For example, if the capacity of a tree is 8, the normalized user data rates are 1, 2, 4 and 8. The average call serving time is 1/4. Fig. 7 shows the average system load(ASL) versus MAR from 1 to 100. It is clear that the MAR between 10 and 50 is a proper range for this system, neither high nor low MAR causes the system to be busy (ASL higher than 80%) or idle (ASL lower than 40%). If we know that MAR is 20, ASL versus $\mu$ from 1 to 20 is shown in fig. 8. It is clear that the inverse of mean call duration between 2 and 15 is proper for the system optimization.
only needs to maintain two arrays: one is the NT count which describes the status of the OVSF code tree, the other is the sequence of codes in layer 1. Assume that number of the OVSF code tree layer is \( n + 1 \) and the number of code trees is \( M \), then the upper bound of the storage size recorded by RNC is \( M \left( 2^{n+1} + 1 \right) \).

VI. CONCLUSION

We proposed a generalized RFDCA algorithm in OVSF-CDMA systems with stationary and finite time analysis. Compared with other algorithms, the proposed generalized RFDCA algorithm reduces number of reassigned codes in a system. In addition, this algorithm is designed for general systems without constraint of the number of layers and normalized user data rates. GRFDCA algorithm assigns OVSF codes rapidly, which has wide applications due to its excellent performance. Dynamic system model is proposed in this paper. Based on the Markovity, continuous time Markov model is used to describe dynamic behaviors of OVSF-CDMA systems. Phase Type Model is applied to the analysis of the statistical properties of finite time behaviors. Simulation results are presented to support advantages of GRDCA algorithm. One distinct feature is the low number of reassigned codes. Application of dynamic model in designing system parameters is discussed.

REFERENCES