Abstract—This work primary objective is to develop an evaluation framework for studying bidimensional Fitts’ law applied to constant and variable gain mice. The authors, following the ISO9241 recommendations, will introduce an implementation of that framework. Suggestions about interesting tests to be performed and studies will be discussed (e.g. limited footprints tests, modification of the Fitts’ law including the variable gain factor). A complete introduction about Fitts’ law and related studies will be also presented.

Index Terms—Fitts, MacKenzie, Duncan, Tukey, Neuman-Keuls, Mouse, Windows, Pointer, ISO9241-9

I. INTRODUCTION

This project has been developed to allow researchers to study and collect data on pointer devices, especially the mouse. It was engineered to test subject on the use of mice with different behavior. In particular, so called accelerated mouse versus standard mouse, and to test the different type of acceleration curves.

Mice can have a so called acceleration. This term is used in this context to describe the velocity associated to the mice movements (it is not an acceleration as usually defined in physical science). In nowadays computers, acceleration is controlled by the operative system, so can be switched on, off and modified via software. Variable gain means that the transduction from physical movement of the mouse to movement of the pointer on the screen is variable and depend upon the instantaneous velocity of the mouse. The faster is moved the mouse the greater is the gain factor for the screen pointer (the pointer travels faster).

This project was born to give a tool useful in the analysis of a possible upgrade of the Fitts’ law (this modified law will be usable only with accelerated mouse), in which the acceleration (as a function of the velocity) is included in the law [1]. Another interesting analysis that can be done using this tool is the study of efficiency of accelerated mouse versus standard mouse. Which one is the best, and in which conditions. And the question about the possibility that the mouse working area (footprint) can affect the result (overturning the answer) of the latter point (which one is the best) can be an expansion to theses studies.

The main description of the built evaluation framework can be found in sec. III. In few worlds, the proposed tool implements the ISO9241-9 specifications [2] to build up an environment for bidimensional Fitts’ test for mice (and similar input devices). Subjects can be undergo to tests and data are collected for further statistical analysis.

II. LITERATURE

We will split this section in four parts. The first three collect the most important papers on which are based our work.

A. Fitts

Fitts’ law is a robust model of human psychomotor behavior developed in 1954. The first Fitts’ paper was reprinted in 1992 [3]. The model is based on time and distance. It enables the prediction of human movement and human motion based on rapid aimed movement.

It seems intuitive that movement time would be affected by the distance covered and the precision demanded by the size of the target to which one is moving. Fitts discovered that movement time was a logarithmic function of distance when target size was held constant, and that movement time was also a logarithmic function of target size when distance was held constant.

Fitts’ law can be applied by HCI researchers in two main ways:

- As a predictive model.
- As a means to derive the throughput.

In the former case it is possible to use Fitts’ law to predict the time needed by the user of a graphical interface to move the mouse to a button and click on it. It also possible to evaluate more complex tasks breaking complex interactions into appropriate sub-actions and then summing the time of each one.

In the second case Fitts’ law is used as part of comparison and evaluation of new pointing devices. Instead of predict movement times, now we measure several movement times and then study how different conditions can affect the throughput computed using Fitts’ law directives.

Although Fitts only published two articles on his law there are hundreds of subsequent studies related to it in the human-computer interaction (HCI) literature, and quite possibly thousands of studies published in the larger psychomovement literature [4]. Fitts’ law’s first HCI application was by Card, English, and Burr [5], who used the index of performance IP to compare performance of different input devices, with the mouse coming out on top (this early work, according to Stuart Card’s biography, “was a major factor leading to the mouse’s commercial introduction by Xerox”). Fitts’ law has been shown to apply under a variety of conditions, with many different limbs (hands, feet, head-mounted sights, eye gaze), physical environments, and user populations.

Since the advent of graphical user interfaces, Fitts’ law has been applied to tasks where the user must position the mouse cursor over an on-screen target, such as a button or other widget. Fitts’ law can model both point-and-click and drag-and-drop actions.
Extending Fitts’ law where proposed also in control theory field [6] to model the human behavior inside, for example, a force feedback teleoperation system.

B. MacKenzie

We have based our work on several articles of Scott MacKenzie. Lot of his works were about the evaluation of device performance. He also have contributed to develop the ISO 9241-9 standard (section II-C).

The lead work of MacKenzie is [7]. In this article the authors give seven recommendations to HCI researchers that want to construct and use Fitts’ law models for the comparison of conditions in experiments. As stated in the article these recommendations “support and in some case supplement the methods described in the ISO 9241-9 standard”. The seven recommendations are:

1) Use the Shannon formulation of the index of difficulty (see also Appendix [3]).
2) The range of movement distance and targets width should be chosen so that the resulting ID span a representative range of values (usually from 2 to 8 bits). Each ID condition must be presented to each subject many times (15-25 times). Movement time data should also be collected.
3) A measure of subject’s movement end-points must be gathered, where obvious outliers - such as double-clicks - may be removed from the data.
4) The end-points should be used to perform the adjustment for accuracy. Having these data you can define the effective target width \( w_e \) and the effective index of difficulty \( ID_e \). A large discrepancy between \( ID \) and \( ID_e \) should be investigated (see Appendix [3]).
5) A least-squares linear regression is used to find the intercept and the slope of the Fitts’ law equation. This test is needed to have a measure of the goodness of fit.
6) You can calculate the movement time prediction using the regression model and the ID value.
7) The throughput is calculated in a way that it include both the speed and accuracy of the movement performance.

For each one of the recommendations the article gives a full justification. Authors show also that the use of a standard to evaluate the performance of device improve the quality and comparability of different works. They propose the use of the ISO9241-9 standard and they verify the quality of different works that use or not this standard.

He also has done other works about the ISO standard trying to extend and improve it. In [8] he demonstrates that the ISO standard is consistent with accepted scientific theory and practice and also that the results obtained using the standard are valid in order to predict behaviors and evaluate devices. This work define in detail the experiment design so as to allow others to replicate it.

In another work [9] he compares the difference of performance between remote pointers and standard mouse. He uses the ISO procedure to evaluate accuracy and throughput showing that standard mouse perform better.

Device extensions and improvements are presented in a paper where he reviews innovations in increasing the degree of freedom of a mouse, add tactile feedback and incorporating scrolling and zooming capabilities [10].

C. ISO9241-9

The part 9 of the ISO 9241 standard is called Requirements for non-keyboard input devices [2]. At the time of writing it is at the 90.93 stage (International Standard confirmed) as defined by the ISO Organization. The standard gives us the possibility to compare different devices using consistent procedures from one study to another, and this greatly increases our ability to understand the results and to compare different works.

ISO9241-9 describes tests to evaluate the performance, comfort and effort in using computer pointing devices. This document can help to answer questions such as which of two devices is better, which is the best configuration for a specific device, is the trackball as good as the standard mouse device, and so on. The standard gives indication on how to implement quantitative tests on the devices. Users carry out a series of tasks and measurements are taken on their performance. The measurements are then used for comparison. ISO9241-9 gives also indications for qualitative tests. Users are requested to fill-in a questionnaire on their comfort and preference [8].

The proposed standard applies to the following hand-operated devices: mice, trackballs, light-pen and styli, joy-sticks, touch-sensitive screens, tablet-overlays, thumb-wheels, hand-held scanners, pucks, hand-held bar code readers, and remote-control mice. It does not cover eye-trackers, speech activators, head-mounted controllers, datagloves, devices for disabled users, or foot-controlled devices.

ISO 9241 defines evaluation procedures for measuring user performance, comfort and effort using an experimental protocol which defines subject samples, stimuli, experimental design, environmental conditions, furniture adjustments, data collection procedures, and data analysis recommendations.

Performance is measured subdividing the evaluation in six tasks: one-direction (horizontal) tapping, multi-directional tapping, dragging, free-hand tracing (drawing), free-hand input (hand-written characters or pictures) and grasp and park (homing device switching). The tasks selected for testing should be determined by the intended use of the device with a particular user population.

The document gives also hardware indications on the device: button motion, buttons and trackball size, weight, motion resistance, and other required measurements are given.

Our work focuses on the Annex B of the ISO document. This annex provides performance test methods for evaluating the efficiency of input devices. Several test procedures are explained in the annex. What we are interesting in are the tests on pointing task. A throughput measurement is defined:

\[
Throughput = \frac{\text{Effective index of difficulty}}{\text{Movement time}} = \frac{ID_e}{t_m}
\]

(1)

where \( ID_e \) is the effective index of difficulty for a movement task and \( t_m \) is the movement time, calculated from the initiation of movement start of the input device to target selection.
The task we implemented is the Multi-directional pointing task. This test can be used to evaluate pointing movements in many different directions. As described in the ISO, the subject is required to move the cursors across a circle to sequentially hit numbered targets. This test should be conducted with a range of difficulties, varying the size of the circles and the distance between them (see figure 1). In case the subject is not familiar with the device, the standard suggests a special training session. Each subject should be given a practice session. Several statistical procedures can be used to evaluate if a person has learned how to use the device. In Appendix ?? some of these procedure are explained.

D. Other

There are several works in literature, other than MacKenzie about device performance evaluation.

In [11] an experiment is conducted to find out whether significant throughput differences exist between different mice. They use the first formulation of the Fitts’ experiment (see figure 2).

In [12] they use the MacKenzie and ISO experiment procedures to evaluate the performance of motion-impaired users, using also force feedback devices. They evaluate the “point and click” task revealing a number of significant differences between motion-impaired and able-bodied users.

Some authors have attempted to find the right curve of acceleration for a mouse [13]. They found that changing the gain does not speed mouse performance. Also a switch that change gains at an automatic velocity threshold does not help either. This work is quite interesting because the authors found that, even if there are no improvements in users performance using accelerated mouse, people prefer them. They show that the problem was the dimension of the mousepad. Gain has no effect on performance if the user is able to confine mouse movement to a small enough footprint.

One of the way taken by researchers to improve performance is to changing the Fitts’ parameters. In [14] the author propose these ways:

- Reduce the distance $D$: for example using “pie menus” or bringing potential target near the mouse cursor;
- Increase the target width $w$: expanding the target when over with the pointer or as suggested increasing the area cursors;
- Both decreasing $D$ and increasing $w$: using a dynamical changing control-display gain (i.e. “mouse acceleration” or “sticky” target).

In next section we will describe our evaluation framework for mouse acceleration curves.

III. Evaluation Framework

The main goal of the proposed evaluation framework is to test different mouse gain curves. All computers “translate” physical mouse movement in a screen pointer movement. This transduction follow different rules in different OSes and computers. Old OSes used a fixed gain, newer use variable gain, and often the choice is up to the user (via Control Panels, System Preferences, etc.). Fixed gain is a very simple mechanism: for each millimeter covered by the mouse a fixed distance is covered by the pointer on the screen. For example, 1 mm of the mouse corresponds to 2 pixels on the screen.

A more complex transduction is the variable gain. The gain is no more a constant but depends upon the velocity of the mouse. The more speedy is the mouse, the more greater will be the gain. So the gain can be graphically represented as a function of the velocity. The fixed gain can be represented always with a straight line, while the variable gain can be represented with different curves (see figure 3).

Is the variable gain system better then constant/fixed gain system? This is a question that has been arisen and discussed, and a true answer has not been found [15]. Some authors [13] evaluated the systems using the Fitts’ law and they reach the conclusion that fixed gain is better (or at least equal) in performance the variable gain. This is supported by the fact that the velocity factor introduces a “peggiorative” coefficient

\[^1\text{Velocity is often called acceleration and, although is wrong from physical point of view, this word is widely used.}\]
in the Fitts’ law. So the best solution should be a mouse with high resolution capabilities (dpi) and a fixed gain [13].

So, why all the recent OSes have accelerated (accelerated means with variable gain) mouse? Users seem to appreciate the variable gain. The answer maybe cannot be found performing tests on subjects in ideal ISO9241-9 ambient situations. People do not work in ideal conditions and environment. An hypothesis that should be tested is the footprint limitation [13]. In fact, a lot of users do not have a free space for their mouse. The mouse can run only on a small patch on their desktop (e.g. figure 4). So, maybe, this is the key for understanding the reason why people prefer to use accelerated mouse. In fact, having acceleration allows to move the pointer on the whole screen keeping a good precision of the pointer at low speed. On the contrary, with a fixed gain mouse, the user would be forced to continuously raise the mouse and repositioning it on the desk loosing a lot of time (the precision is not affected). This is only a theory that maybe can be proved or refuse in further tests. Another question can be posed: which gain curve that uses a straight line for the mouse to pointer transduction. Different curves should be tested starting from existing ones (for example the one used by Microsoft Windows XP [16]) and creating new ones using the idea that the more is the speed of the mouse the more should be the gain. As already said, the proposed framework has been built for this kind of tests.

A. Functionalities

The evaluation framework proposed performs different tasks. First of all, the subject should fill a form with few data (see figure 5). Then the test can be started. This is based on the well known Fitts’ test but follow the new directives proposed in the ISO 9241-9 Standard [2] and the suggestion by Soukoreff and MacKenzie [7]. So, there are not two parallel rectangular targets to hit, but a set of circular targets that lie on a circumference (see figure 1, a screenshot can be seen in figure 6). The subject must hit the targets (only two circles are visible at a time) trying to be fast and accurate. When all the circles has been hit, the subject can take some time for relax. Meanwhile, a survey about the pointer movement can be addressed to the subject. See A. When the subject is ready a different gain curve can be tested. Five different gain curves are tested (in random order) by the subjects. For each curve the subject must perform trials with five ID (from 2 to 6, in random order). For each trial (the ID is always the same in the whole trial) 26 targets must be hit. A training phase should be made before using a new curve, but has not been yet implemented in our framework, see ??.

Fig. 3. Different constant and variable gain transfer curves.

Fig. 4. A messy but real work environment. Mice are trapped in small footprints.

is the best? In this question we can include the previous one. In fact, the constant gain can be interpreted as a variable gain

The footprint is the physical area in which the mouse is free to run.
Fig. 6. A screenshot taken while the evaluation framework is working.

The entire test should be remade (maybe after a day) with a fixed small footprint. This test can be useful for analyzing the hypothesized advantage of variable gain mouse in real environments.

B. The Code

In this section a description on the source code will be introduced. Windows XP is the chosen platform for this framework. This OS has been selected because, unlike other OSes, allow to change the variable gain curve of the mouse in a very simple way. In fact there are two Registry\textsuperscript{3} keys that control the gain curve. In this way no low level or driver coding has been needed. Visual Basic has been selected for implementing the framework because allow faster coding (in a graphical environment) then other languages and is quite commonly used on Windows platform.

The requested data at the begin of the experiment are immediately stored in a CSV file and will be used in further statistical analysis. In section III-C more information about the data will be given. The system randomly permute the different curves to test and save this order in the CSV file (and in a temp file). For each curve a random permutation of the ID to test is performed. The permutation for the current curve is stored again in the CSV file. For each ID to test the width (\(w\)) and the distance (\(d\)) of the circles are computed: \(w\) is computed choosing a random number in a preselected range, and \(d\) is computed using ID and \(w\).

Using \(d\) and \(w\) the circles can be plotted. Only two circles at the time are plotted on screen: the current one (yellow) and the next one (blue). Each click made by the user is stored (click coordinates with a status flag indicating if the click missed or hit the target), and this information will be used for statistical analysis. Clicks coordinates are very important not only for the Fitts’ law application but for the discrimination between “normal” clicks and unintentional clicks (the latter should be eliminated from the statistical analysis) too. After a successful hit the blue circle become yellow (this means that the subject must click on the next circle) and a new blue circle is plotted. This procedure continue until all circles (26) have been hit. The program stores the time elapsed from one click to the next one for future analysis.

A final note about the code: the system needs\textsuperscript{4} a logoff at the end of each mouse curve modification to activate the changes. The program is able to automatically logoff and to continue the experiment without the need of users intervention (if properly set, the OS are able to automatically open the program).

C. Data

During the test various data are collected from the subject. Before beginning the trials the subject must fill a small form. Requested data are:

- Name.
- The data filename (CSV) will be named with the current date appended at this string.
- Computer Experience (number of hour per week).
- User can select between this five suggestions: “< 1 hour”, “1 – 5 hours”, “5 – 15 hours”, “15 – 40 hours”, “> 40 hours”.
- The selection will be saved in the CSV as an index (1..5).
- Computer Game Experience (number of hour per week).
- Same parameters as before.
- Age.
- Gender (M, F).
- LeftHanded (L, R).
- AcceleratedUser (Y, N).
- AcceleratedUser means that the preferred mouse is the variable gain mouse.
- TypeOfInputDevice.
- User select between this five suggestions: “Mouse”, “Trackball”, “Touchpad”, “Trackpoint (IBM isometric joystick)”, “Other”.
- The selection will be saved in the CSV as an index (1..5).

Computer Experience and Game Experience are useful for future statistical classifications regardless the type of input device used. AcceleratedUser and TypeOfInputDevice are useful for classification on different input devices or same input devices with different behavior. Age, Gender and LeftHanded are useful for “social” classifications.

For each ID (index of difficulty) to test in each mouse (gain) curve to test the program will compute and save other data:

- Curve Index.
- Error number.
- Time used for completing the current test.
- Current ID.
- Current \(d\) (distance between targets).
- \(w\) (target width).
- Time elapsed between a click and the next one (there are \(N\) entries. \(N\) is the number of circles to click to complete a trial for each ID to test).

\textsuperscript{3}Windows’ Registry, search with google for regedit for more information. The keys used are SmoothMouseXCurve and SmoothMouseYCurve.

\textsuperscript{4}No API exists for performing an automatic reload of the mouse variable gain parameters.
A DTD like structure description follow. Square parenthesis indicates the exact repetition number of the structure (+, *, ? operators are not used because insufficient for a correct description).

K=5 (Gain curves to test).
IDN=5 (ID to test for each curve).
N=26 (Circles to hit for each ID).

D. Experiment Setup

The standard ISO9241-9 strictly define the test environment. Naturally, this conditions represent the (impossible?) ideal working environment. This can be good for having unbiased/uncontaminated results, but must be modified under certain condition (i.e., mouse footprint tests). The number of subjects to use in the experiment suggested by [2] is 25, but in [7] 12 is indicated as a sufficient number.

The following recommendation has been introduced by [7]. The number of trials in a single test (single ID in our case) should be from 15 to 25.

Training is very important and must be considered during each experiment. The subject learn during the trials and this fact must be taken into account (see ?).

Effective dimensions must be computed during the statistical analysis. Effective width (We) and effective index of difficulty (IDe) can be computed at the end of the experiments using, ID, w and the data from the trials.

Throughput can be calculated using equation proposed by [7]. A survey should be performed after each variable gain curve. See [A]

IV. Future Works

A. Training and Statistical Analysis

As suggested by the ISO standard if a subject has no familiarity with a device he/she needs a training before performance testing can be conducted. Subject should be given a sufficient practice session to learn the use of the device. To verify that the subject is enough trained the Duncan’s Range test should be used [2]. Thus we need to compare the performance during the experiment time.

When a significant effect has been found using analysis of variance, we still do not know which means differ significantly. It is therefore necessary to conduct post hoc comparisons between pairs of treatments. When repeated t-tests are used, the overall type I error rate increases with the number of pairwise comparisons. One method of keeping the overall type I error rate to 0.05 would be to use a much lower pairwise type I error rate (It is said to be a type I error when a true null hypothesis can be incorrectly rejected and a type II error when a false null hypothesis can fail to be rejected).

There are a number of specialist multiple comparison tests that maintain a low overall type I error. Tukey’s test and Duncan’s multiple-range test are two of the procedures that can be used and are found in most statistical packages.

Duncan’s multiple range test (MRT) is a multiple comparison procedure developed by David B. Duncan in 1955. Duncan’s MRT belongs to the general class of multiple comparison procedures that use the studentized range statistic, to compare sets of means. Duncan’s new multiple range test (MRT) is a variant of the Student Newman Keuls method that uses increasing alpha levels to calculate the critical values in each step of the Newman Keuls procedure.

David B. Duncan developed this test as a modification of the Student Newman Keuls method that would have greater power. Duncan’s MRT is especially protective against Type II error at the expense of having a greater risk of making Type I errors. It is commonly used in agronomy and other agricultural research.

Duncan’s test has been criticized as being too liberal by many statisticians including Henry Scheff, and John W. Tukey. Duncan argued that a more liberal procedure was appropriate because in real world practice the global null hypothesis H0 (all means are equal) is often false and thus traditional statisticians overprotect a probably false null hypothesis against type I errors. Later, Duncan developed the Duncan-Waller test which is based on Bayesian principals. It uses the obtained value of F to estimate the prior probability of the null hypothesis being true.

There are some works in literature that compare statistical tests. In [17] the authors present some multiple comparison methods and discuss the practical use of the methods.

Also in [18] the author compares different performances of various multiple comparison methods. The procedures evaluated were:

- Scheffe’s method,
- Tukey’s HSD method,
- Fisher LSD method,
- Tukey’s multiple range test,
• Waller-Duncan’s Bayesian K-Ratio rule,
• Duncan’s multiple range test,
• Newman -Keuls’s multiple range test.

It gives indication about which method to use in which situation.

It is possible to improve our framework including a training phase, based on the Duncan’s work, at the beginning of the test. In this way, we can suppose that every user has reach the trained level and we can thrown away all the results that can affect the statistical evaluation of the performance.

Once we have all the data collected from the experiments it is possible to evaluate, using statistical analysis, the performance of each mouse curve used.

V. CONCLUSIONS

This work primary objective was to develop an evaluation framework for studying bidimensional Fitts’ law applied to constant and variable gain mice. The framework can be used for further studies like tests on different input devices, modification of the Fitts’ law including the variable gain factor, experiments with mouse footprint, and so on. An important upgrade for this framework should be the training procedure to be included before testing each variable gain curve. The software can be used and modified by anyone and must be considered under GPLv3 licence (http://gplv3.fsf.org/).

ACKNOWLEDGMENT

Appendix A

Example: Survey Questions

This questions should be answer with a number from 1 (minimum) to 5 (maximum). This is a recommendation from [7]. ISO [2] recommends a number from 1 to 7. Other questions can be found on [2].

- Smoothness during operation
  1) Very rough ... Very smooth (5)
- Effort required for operation
  1) Very high ... Very low (5)
- Accuracy (small target can be hit easily?)
  1) Very inaccurate ... Very accurate (5)
- Targeting speed
  1) Unacceptable ... Acceptable (5)
- Overall operation of input device
  1) Very difficult to use ... Very easy to use (5)

B. ISO Math

The ISO 9241-9 standard defines in its Annex B the procedure to calculate performance. We will give some of the definition in this appendix.

1) Calculation of the throughput:
   • Target widths \( w \) is the width of a target presented on a display.
   • Effective target widths \( w_e \) is the width of the distribution of selection coordinates made by a subject during a pointing/tapping test. In particular:
     \[
     w_e = 4.133s_x
     \]
   where \( s_x \) is the standard deviation of the selection coordinates in the movement direction.
   • Index of difficulty is the measure, in bits, of the user precision. For selection, pointing, or dragging tasks it is defined as:
     \[
     ID = \log_2 \frac{d + w}{w}
     \]
   For tracing tasks it is defined as:
     \[
     ID = \frac{d}{w}
     \]
   where \( ID \) is the index of difficulty, \( d \) is the distance of movement to the target and \( w \) is the target width of the displayed target.
   • Effective index of difficulty is the measure, in bits, of the user precision. For selection, pointing or dragging tasks:
     \[
     ID_e = \log_2 \frac{d + w_e}{w_e}
     \]
   For tracing tasks:
     \[
     ID = \frac{d}{w_e}
     \]
   where \( ID_e \) is the effective index of difficulty, \( d \) is the distance to the target and \( w_e \) is the effective target width of the displayed target.
   • Throughput is defined in the following way:
     \[
     \text{Throughput} = \frac{ID_e}{t_m}
     \]
   where \( t_m \) is the movement time, calculate from the initiation of movement to target selection.

There are some works in literature [7], [8] where it is suggested to use the Shannon formulation of the index of difficulty:

\[
ID = \log_2 \left( \frac{d}{w} + 1 \right)
\]

Moreover if the purpose of the analysis is the comparison of several experiments condition, then the throughput is calculated first for each subject (as the mean throughput) and then these subject throughput are averaged to produce a grand throughput:

\[
TP = \frac{1}{y} \sum_{i=1}^{y} \left( \frac{1}{x} \sum_{j=1}^{x} \frac{ID_{ij}}{MT_{ij}} \right)
\]

where \( y \) is the number of subjects, and \( x \) is the number of movement conditions.

2) Tests: For the tapping test it is possible to use two kind of procedure:

- One-direction tapping test used (see figure [2]).
- Multi-directional tapping test (see figure [1]).

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