On the Fundamental Performance Limits of Peer-to-Peer Data Replication in Wireless Ad hoc Networks

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Abstract—Wireless ad hoc networks are drawing increasing attention from the research community because of their potential applications. However, the fundamental capacity limits of these networks pose various technological challenges to designers of network protocols. In this paper, we attempt to capture the inherent constraints on information dissemination in a mobile wireless environment, with the emphasis on peer-to-peer (P2P) communications. More specifically, we introduce the notion of "replication-induced gain" to quantify the impact of data replication under the paradigm of P2P query-response mechanisms. Our major contribution lies in presenting several preliminary results with respect to the complexities and trade-offs involved in enhancing data availability. To the best of our knowledge, the data replication problems that arise because of scarce system resources in wireless ad hoc networks have not been investigated from this perspective. We believe that our results could provide additional insights and practical implications for P2P system designers.

Index Terms—Ad hoc networks, peer-to-peer system, scaling laws, wireless communications.

I. INTRODUCTION

A wireless ad hoc network is an autonomous system that consists of mobile nodes capable of wireless communication. These networks are drawing increasing attention from the research community because of their potential applications, such as rescue missions and battlefield deployments (see e.g., [1]–[3]). Nonetheless, the networks’ fundamental capacity limits pose various technological challenges to the design of large-scale applications. In this paper, we address the inherent constraints on information dissemination multi-hop wireless relaying, with the emphasis on peer-to-peer (P2P) computing. More specifically, we focus on the fundamental limits and scaling laws of data replication under the paradigm of P2P query-response mechanisms. Our work is motivated by the pressing need for effective and efficient information exchange in a wireless environment, where each node has a number of its own objects and issues queries to search for objects of interest. In the real world, objects are diverse in type, e.g., an object may be a numerical dataset, a certain phenomenon, or a duplicable service code. Likewise, queries may consist of file names, serial numbers, or an elaborate Boolean predicate [3]. In recent years, a great deal of effort has been devoted to searching for and replicating data in the context of P2P applications over the Internet (see e.g., [4]–[6]). Unfortunately, P2P applications in wireless ad hoc networks are rather disparate in nature (see [2]–[7] and the references therein). For example, the impact of interference and channel assignment is crucial to the efficiency of the network; moreover, most communications between source-destination pairs are carried out via multi-hop relays. In addition, a network’s traffic pattern depends on the purpose of the application (e.g., monitoring, data gathering, etc.). In view of these limitations, the above-mentioned techniques cannot be adopted directly.

For communication functionality, a fundamental question is how much information a wireless network can transport. The seminal work of Gupta and Kumar [7] proposed the concept of transport capacity. Informally, it correlates to the sum of the products of bits and the distances covered by message transmissions; the unit of measurement is bit-meters per timeslot. The finite queue lengths of the network nodes restrict the maximum throughput that can be achieved by each mobile node. Note that, as shown in [7]–[9], the transport capacity is a hard constraint on the amount of data a wireless ad hoc network can deliver. The studies of Grossglauser, Tse, and Diggavi [25] [26] showed that if mobility is allowed, the throughput capacity grows linearly with the number of users, which represents a very substantial increase. The trade-off between throughput and delay in wireless ad hoc networks has been studied extensively in recent years (see e.g., [27]–[29]). We are particularly interested in the work of Herdtner and Chong [29], who explored the scaling law of throughput capacity with constraints on the buffer size. In addition, based on the theory of continuum percolation, Dousse et al. [21] noted that full connectivity would be compromised if the whole network is operating at a given rate. Capacity degradation and interface switching delay in multi-channel networks have also been addressed in [31].

The prime objective of this paper is to explore several fundamental bounds on the gain of data availability induced by replication, rather than propose new replication strategies or a new P2P overlay design for multi-hop networks. Our main contribution lies in presenting several preliminary results with respect to wireless ad hoc networks. To the best of our
knowledge, the problems that arise due to the scarce resources of P2P applications in wireless ad hoc networks (e.g., transport capacity, memory, and energy) have not been investigated, particularly in the form of theoretical expressions. We believe that the results could provide additional insights and practical implications for P2P system designers.

The remainder of the paper is organized as follows. The system model, terminology, and problem formulation are presented in Section II. In Section III, we make some basic observations about the trade-off between communication costs and query responsiveness. Then, in Section IV, we explore the intrinsic difficulties of several replication-related performance issues, particularly in the context of stochastic analysis. Finally, we present our conclusions and discuss the direction of our future work in Section V.

II. PRELIMINARIES

A. System Model

We assume that the whole network is synchronized in rounds,\(^1\) each of which is expressed by successive, discrete time-slots; and an integer number of rounds comprises a communication session. The Physical model [28], [31] is used to describe the interference scheme, i.e., the relation between packet transmission and the combination of available channel(s) and the time-slot arrangement. We assume that node \(i\) can transmit a packet to node \(j\) if the signal received by \(j\) is strong enough. Formally, the emitting power of node \(i\) (denoted as \(P_i\)) is within the range \([0, P_{\text{max}}]\); note that \(P_{\text{max}}\) determines the maximum transmission radius (denoted as \(r_m\)). Let \(\Gamma_i\) denote the set of other nodes transmitting in the same time-slot, and let the signal to interference and noise ratio (SINR) at \(j\) be sufficient for successful decoding, i.e., the following inequality holds:

\[
\text{SINR}_i \equiv \frac{P_i G(i, j)}{N_0 + \varphi \sum_{k \in \Gamma_i} P_k G(k, j)} \geq \beta,
\]

where \(G(\cdot)\) is the attenuation function associated with the wireless medium; and \(N_0\) is the power of the thermal background noise, which is assumed to be the same for all nodes. The coefficient \(\varphi\), which indicates the effect of interference, is dependent on the orthogonality between the codes adopted during simultaneous transmissions [30].

The limited power and signal loss during propagation over the wireless medium impose fundamental constraints on the capacity of data transmissions. Specifically, each node is capable of transmitting at most \(W\) bits per second through a wireless channel.\(^2\) Half-duplex transmission is assumed, i.e., nodes cannot transmit and receive simultaneously. Based on one of the most widely-used radio models, the energy costs of packet transmission over the Euclidean distance \(d\) is: \(E(d) = c_1 + c_2 d^\alpha\), where \(c_1\) and \(c_2\) are constant model parameters, and \(\alpha\) is the path loss exponent determined by the specific propagation environment [34]. For simplicity, we do not consider the effect of wireless fading channels in this paper.

B. Terminology

Throughout the paper we adopt the following notations. For two functions \(f, g\) defined on natural numbers: (i) If \(\lim_{n \to \infty} \inf f(n)/g(n) < \infty\), we have \(f(n) = \Theta(g(n));\) (ii) if \(\lim_{n \to \infty} \inf f(n)/g(n) > 0\), we have \(f(n) = \Omega(g(n));\) (iii) if \(\lim_{n \to \infty} \inf f(n)/g(n) = 0\), we have \(f(n) = o(g(n));\) and (iv) if \(f(n) = O(g(n))\) and \(f(n) = \Omega(g(n))\), we have \(f(n) = \Theta(g(n))\). All logarithms, unless otherwise specified, are to the base 2. In addition, the expression \(w.h.p.\) (with high probability) is used to qualify an event whose probability approaches 1 when \(n\) approaches infinity. We use the terms link and edge, as well as data and object, interchangeably.

Generally, nodes in a wireless ad hoc network have restricted memory. Let the memory capacity \(\Phi(u)\) for each node \(u\) be quantified by the maximum amount of memory available for data replication. The considered scenario assumes that each node possesses some exclusive objects and requests extra objects from other nodes in a communication session. The former are called the innate objects of \(u\), and denoted as \(\text{INNATE}(u)\). In the sequel, we assume that the content to be replicated can be divided into small packets of the same size; thus, \(\Phi(u)\) can be defined by an integer.\(^3\) The replicated objects in \(u\)’s memory are denoted as \(\text{REP}(u)\). When a node \(u\) starts searching for an object \(o\), a query \(q(u, o)\) is issued by \(u\) within the current time-slot. Note that we do not assume any a priori knowledge about objects, such as data types and their correlation characteristics. Since the content of objects may change over time, asynchronous updates of object replicas can cause data consistency problems. Furthermore, incorporating distributed data synchronization is very challenging and could be the subject of research in itself. For the sake of clarity, we assume that the packet size is small enough so that the packet delay is essentially equal to the number of hops taken by the packet. Moreover, as assumed in most previous works, each peer can only serve an object after it has been fully downloaded. However, as the issue of data coherence is beyond the scope of this paper, we assume that all objects remain valid for a “sufficiently long” period.\(^4\) We discuss this point further in Section V.

III. PROBLEM DESCRIPTION

A. Basic Observations

In this section, we use a simplified model for clarity. Specifically, the network topology is represented by a graph \(G = (V, E)\), where \(V\) denotes a set of nodes. We assume that all nodes have equal transmission radius \(r_m\) and, for each edge \((u, v) \in E\), the Euclidean distance between \(u\) and \(v\) \(\|u, v\|\) is no larger than \(r_m\). Let \(|u, v|\) denote the hop

\(^1\)As noted in the literature, rational synchronization can be achieved via extra facilities; e.g., with the assistance of GPS signals or other types of beacon. In addition, some works (e.g., [10]) propose ways to design and analyze round-based protocols for wireless ad hoc networks.

\(^2\)Note that this capability still holds in the case where a channel can be split into several subchannels [7].

\(^3\)The assumption of equal-sized objects, albeit very idealistic, can serve as a basis for understanding the intrinsic tradeoffs involved in this multi-objective optimization framework. Moreover, in this work we address the problem from the viewpoint of the finest granularity, i.e., the bit-wise information transfer.

\(^4\)Perfect data consistency in large wireless ad hoc networks is extremely expensive, if not impossible [37]. Thus, modified evaluation metrics and sophisticated treatments for the relevant issues are required.
distance between nodes $u$ and $v$ in $G$. We assume that a maximum of one packet can be transmitted over an edge in either direction. Given an integer $h$, $N^h(u)$ is defined as \{v : v \in V, |u, v| \leq h\}; and the nodes in $N^0(u)$ are called $h$-hop neighbors of $u$. Similarly, for a given node set $S \subseteq V$, we denote $\cup_{u \in S} \{v : v \in V, |u, v| \leq h\}$ as $N^h(S)$. We disregard the notation for a round to keep the formulation simple. A query $q(u, o)$ is said to be resolved if at least one route to the target has been transferred to $u$. Consequently, a query resolution $r$ for $q(u, o)$ is derived (denoted as $r \Rightarrow q(u, o)$). Note that $r$ includes the path information $v_0v_1\ldots v_{h-1}$, where $v_0 = u$ and $o \in \text{INNATE}(v_{h-1}) \cup \text{REP}(v_{h-1})$. The hop-distance between $v_0$ and $v_{h-1}$ in $G$ is denoted as $|r, q|$. In principle, efficient object replication can best be defined by the following properties: (i) replicas are relayed in a power-efficient manner; and (ii) nodes in the vicinity try to cooperate in order to share replicas and reduce memory consumption. However, there is a trade-off between the communication cost and query responsiveness. An elementary observation is shown in Fig. 1, in which for each node $u$, $\text{INNATE}(u) = \alpha_u$. The innate objects are black, while the replicated ones are white. The dashed lines indicate that the objects are replicated via wireless transmissions. Consider an extreme case in which: (i) all combinations of a query and an object are equally probable (i.e., uniformly distributed); and (ii) $\forall u, o, r \Rightarrow q(u, o)$ exists iff $|r, q| = 1$.

To outline the basic concepts, we show that, by replicating objects efficiently, more queries can be resolved within the one-hop neighborhood. Consider a simplified case in which data packages are relayed via the shortest paths (in terms of the hop-count) and no re-transmissions occur. The performance of query processing can be evaluated as follows. In Fig. 1(a), we achieve 8 query resolutions at a cost of 8 wireless transmissions (per data package); in Fig. 1(b), the numbers are 14 and 12, respectively. The matrices in the figure show object availability over $G$ after replication. Since communications are the main cause of power depletion in wireless ad hoc networks [11], the replication strategy in Fig. 1(b) is regarded as “more efficient” in the sense of more query resolutions per joule. We call the improved efficiency replication-induced gain. Further details are given in Section IV.

B. Hardness Result

Initially, we consider a very simple case — each node $u$ in $G$ possesses exactly one innate object. Then, for $\forall$ query set $Q' \subseteq Q$, if there exists a resolution set $R'$ such that $q(u, o) \in Q'$, $\exists r \in R'$, $r \Rightarrow q(u, o)$, and $|r, q| \leq \kappa$, we call $Q'$ $\kappa$-coverable and $R'$ a $\kappa$-covering resolution set of $Q'$ (denoted as $R' \Rightarrow^\kappa Q'$). Given a query set $Q' \subseteq Q$, the question arises: How can we replicate objects so that some $R' \Rightarrow^\kappa Q'$ with a small $\kappa$ can be formed? In a sense, the concept of confining communications to local regions is similar to existing “zone-based protocols” [38].\footnote{Furthermote, as noted in previous studies, localized communications are vital in wireless ad hoc networks (e.g., [12], [13] argue that the scalability of a network depends on whether network traffic can be localized). Thus, $\kappa$ is a small integer determined primarily by the application layer.} For brevity, the memory consumption of different replication schemes is quantified by the number of replicated objects located by $R'$.

\section*{IV. Stochastic Analysis}

In the following, we examine replication-induced gain from various perspectives.\footnote{Specifically, we focus on information transfer under the P2P query-response paradigm.} First, we investigate the bounds on such a gain, taking into account the communication cost subject to several fundamental limitations of wireless relay networks (see [7]–[9], [22]–[29] and the references therein). Then, we consider a wireless ad hoc network in which the nodes are uniformly and independently distributed in a disk area $A$. In the literature, the presence of nodes is typically modeled by a spatial Poisson point process. Let $\lambda$ denote the spatial density. The probability of finding $n$ nodes in area $A_x$ is given by

$$\Pr\{n_x = k\} = \frac{(\lambda A_x)^k}{k!} e^{-\lambda A_x}, k \geq 0. \quad (1)$$

A. From the Perspective of Network Capacity

Initially, we ignore the issues of energy and memory constraints.\footnote{In a sense, we discuss the instances where the ratio of energy/memory resources to network capacity is high.} Assume that all relay nodes make their best efforts to cooperate in transmitting information. Consider an arbitrary source-destination pair of nodes $(v_s, v_t)$ and a replicated object $o_i$ transmitted by node $v_s$, where $o_i \in \text{INNATE}(v_s)$. As mentioned in Section III, $\kappa$-covering resolutions for most queries are preferable (i.e., $\kappa$ is a “small-enough” integer), as shown by the example in Fig. 2(a). Note that: (i) $r_s \leq r_m$; and (ii) the replicated object is delivered via multi-hop relays. Let $l_s = \|v_s, v_t\|$ such that we have $l_s \geq r_m$. If full connectivity is a prerequisite, it has been shown that there is an inherent
Fig. 1. A simple example to demonstrate that different replication scenarios influence the communication cost and time of query resolution (in terms of the number of hops).

lower bound on \( r_m \) [7], [14]. Hence, a loose lower bound on \( l_s \) can be directly deduced by \( l_s \geq \sqrt{A \log n/\pi n} \). It is not hard to validate that the maximum area to benefit from replication-induced gain (denoted as \( A^*_g \)) is the gray region in Fig. 2(b). The observations in Section III imply that the ratio of the involved communication cost to the area \( A^*_g \) should be minimized. Consequently, several bounds on can be derived. First, from Fig. 2(b) we obtain the following:

\[
\pi (\kappa^2 - \kappa) r_m^2 < A^*_g \leq \pi \kappa^2 r_m^2. \tag{2}
\]

Information theoretic limits of data transmission in wireless networks have been investigated intensively in recent years. Specifically, the work of Gupta and Kumar [7] was the first to identify the scaling law of transport capacity for static multi-hop radio networks in which the nodes are randomly located. As noted in [9], transport capacity serves as an instinctive quantity of network-layer capacity, since it summarizes the information a network can deliver via a single number. Informally, transport capacity correlates to the sum of the products of bits and the distances covered by message transmissions; the unit of measurement is bit-meters per second, or bit-meters per time-slot. Note that we consider a more realistic situation than most previous studies, in which the ratio of channels to interfaces (denoted as \( \gamma_c \)) may be greater than one [31]. Although an interface can transmit or receive data on any channel at a given time, that capacity might be lost if the number of interfaces is insufficient. In contrast, since we focus on a fixed region \( A \) and the geometry of \( A \) affects the constants, but not the scaling behavior, no explicit path loss model is necessary. The commonly used Protocol model (see e.g., [28], [30], [31]) is described as follows. The transmission from a node \( i \) to a node \( j \) on a channel is deemed successful if the following condition holds for every other node \( k \) simultaneously transmitting on that channel:

\[
\| k, j \| \geq (1 + \Delta) \cdot \| i, j \|, \quad \Delta > 0,
\]

where \( \Delta \) is a parameter that ensures the concurrently transmitting nodes are sufficiently distant from the receiver to prevent excessive interference. Note that if \( \alpha \) is greater than 2 and each transmitter adopts the same power, the Physical model is equivalent to the Protocol model [7].

Consider a fixed placement of nodes and assume that the constraint on per-node throughput is \( \eta \) (bits/sec). Moreover, during a communication session, the replicated objects are delivered with the cooperation of participating nodes in the network. If the mean transmission rate of the optimal object replicating strategy is \( R^* \) (bits/sec), then, by definition we have \( R^* \leq n \eta \). Denote \( L_s \) as the sum of the Euclidean distances between all source-destination pairs involved in transferring the replicated objects. Similarly, the sum of the

\[\begin{array}{ccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 1 & 1 & 1
\end{array}\]

Miss: 8 Gain: 8

\[\begin{array}{ccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & 1 & 1 & 0 & 1 & 1
\end{array}\]

Miss: 2 Gain: 14
Fig. 2. The maximum area that may benefit from replication-induced gain.

Fig. 3. Illustration of the process addressed by Lemma 1 and Lemma 2. This is a simplified instance consisting of three source-destination pairs only. We present the difference between \( l_s \) (the solid line) and \( l_h \) (the dashed line) by labels in the 1st node pair, assuming every bit of the replicated object is routed via the same relay path.

Euclidean-distances between all corresponding relay nodes of the replicated objects is denoted by \( L_h \). Note that the values of \( L_s \) and \( L_h \) for a source-destination pair can be different under multi-hop relaying (see Fig. 3 for an illustration). Let the Euclidean distance of the \( h \)th relaying hop of bit \( b \) be \( r_h(b) \) and the mean of the sum of \( r_h(b) \) during \( \sigma \) be \( T_\sigma \). Since data replication does not necessarily comprise the total traffic during \( \sigma \), \( T_\sigma \leq T_\sigma \).

Consider all the data transmitted over the whole area in this time period. Then, we have:

\[
\sum_{b=1}^{\sigma n \eta} \sum_{h=1}^{hc(b)} r_h(b) \geq \sigma n \eta T_\sigma.
\]  

(3)

The total number of hops taken by all the transmitted data packets is denoted by \( H_r = \sum_{b=1}^{\sigma n \eta} \sum_{h=1}^{hc(b)} r_h(b) \) Hence, by convexity and Jensen’s inequality, the following expression holds:

\[
\left( \sum_{b=1}^{\sigma n \eta} \sum_{h=1}^{hc(b)} \frac{1}{H_r} r_h(b) \right)^2 \leq \sum_{b=1}^{\sigma n \eta} \sum_{h=1}^{hc(b)} \frac{1}{H_r} r_h^2(b).
\]  

(4)

Summing all channels, we obtain the following inequality according to the transport capacity of the entire network:

\[
\sum_{b=1}^{\sigma n \eta} \sum_{h=1}^{hc(b)} r_h^2(b) \leq \sigma \cdot \left( \frac{4WA}{\pi \triangle^2} \right).
\]  

(5)

Then, substituting (4) into (5), we derive the following inequality:

\[
\sum_{b=1}^{\sigma n \eta} \sum_{h=1}^{hc(b)} r_h^2(b) \leq \frac{4\sigma WAH_r}{\pi \triangle^2}.
\]

Unlike the proofs in most previous works, we exploit a modification of \( H_r \) in [31], which addresses the impact of an odd number of channels and interfaces. More precisely,
\[ H_r \leq \sigma n W/2\gamma_c. \]  

Combining (3)-(6) gives the inequality \( n\gamma_{\sigma} \leq W \sqrt{\frac{2An}{\Delta^2 \gamma_c}} \); thus, we have

\[ n\gamma_{\sigma} \leq W \sqrt{\frac{An}{\Delta^2 \gamma_c}}. \]  

Since \( R^* \leq n\gamma_{\sigma} \) in our scenario, we obtain the optimal bit-meters/sec in \( A \):

\[ R^* \gamma_h \leq R^* \gamma_{\sigma} \leq n\gamma_{\sigma} \leq W \sqrt{\frac{An}{\Delta^2 \gamma_c}}. \]  

Recall that \( \Delta \) is a constant that is independent of \( A, n, \) and \( W \). This is consistent with the results in [31], namely, the network capacity is \( \Theta \left( W \sqrt{\frac{An}{\gamma_c}} \right) \). The formulation can be concluded as:

\[ R^* \leq \Theta \left( \sqrt{\frac{An}{\gamma_c}} \right). \]

Provided that the number of channels is not too large, i.e., \( \gamma_c = O(n) \), we have \( R^* \gamma_h \leq \Theta(\sqrt{A}) \). Below, we address the problem of bounding \( l_s \) from a different perspective: Given the number of hops traversed by the transmitted data, we can characterize its relationship with the corresponding Euclidean distance? In [35] Vural and Ekici provided an insightful stochastic study of relating Euclidean distances to hop distances in a uniformly distributed sensor network. More specifically, the Euclidean distance of a hop (i.e., the single-hop-distance) is regarded as a random variable \( r \). The maximum possible distance covered in multiple hops (i.e., the multi-hop-distance), is of particular interest. A statistical measure, kurtosis, is used to verify the Gaussianity of the multi-hop-distance. Although most given theoretical expressions are nested integrals, several approximations are presented for computational efficiency. Let \( \tau \) be the expected single-hop-distance traversed over \( A \). The value of \( \tau \) in our model can be obtained numerically as follows (Further details about the numerical results can be found in [35]):

\[ \lambda \tau = \ln \left( 1 - \frac{\lambda \tau}{\lambda r_m - \lambda \tau - 1} \right). \]

The results in [35] show that, as the number of hops is not too small, the distribution of the multi-hop-distance will approximate a Gaussian distribution. In other words, its value can be totally specified by its mean and variance. Consider the replication scenario described above, where \( R^* \) is the optimal average transmission rate of all replicated objects. We denote the mean number of hops over these \( \sigma R^* \) bits by \( \overline{h}_g \). Then, by the linearity of expectation, \( \overline{t}_s \) can be derived as follows:

\[ \overline{t}_s = E[h_g^*] \cdot \tau = \overline{h}_g \cdot \tau. \]

**Lemma 2:** Let \( \delta_s^2 \) be the variance of \( l_s \); then, \( (\overline{h}_g)^2 \geq \left( \frac{1 - \delta_s \sqrt{2 \ln n}}{\tilde{t}_s} \right)^2 \) w.h.p.

**Proof:** Given a normalized Gaussian random variable \( n_s \) with zero mean and unit variance, it follows that

\[ l_s = \delta_s n_s + \overline{t}_s. \]
a wireless ad hoc network, the value of $A_g$ might be further constrained if the amount of traffic induced by delivering these replicated objects is too large within a given communication session. Let $m_g$ be the random variable representing the total number of nodes inside $A_g$ according to the mentioned spatial Poisson process (i.e., with intensity $\lambda$). Clearly $A_g < A$. Let $m_g^*$ denote the number of nodes that benefit from replication-induced gain via the optimal object replication strategy. Using the Chernoff’s bounds again, we obtain the following result (see, e.g., [33]).

**Lemma 3:** $\Pr\{m_g \geq m_g^*\} \leq \exp\{\lambda \cdot (e^{A_g} - 1) - m_g^* \cdot A_g\}$.

**Lemma 4:** $\forall m_g^*$ such that if $m_g^* \cdot (\ln m_g^* - 1) + 1 \geq \ln n$ holds, we have $m_g \geq m_g$ w.h.p.

**Proof:** Based on the argument in [33], to minimize the right-hand side of Lemma 3, we have to minimize $\lambda \cdot (e^{A_g} - 1) - m_g^* \cdot A_g$. In other words, $m_g^*$ and $\lambda$ should satisfy $m_g^* = \lambda \cdot e^{A_g}$. We rescale the graph such that $A = n$ (with the constraint on $m_g \geq \lambda$); consequently, $\lambda = 1$. In this case, even under the network capacity constraint, the probability that more than $m_g^*$ nodes exist inside $A_g$ is minimized, since the optimal object replicating strategy implies that nodes not in need of this object-replication gain are rarely found inside $A_g$.

It is not hard to validate that, with the proper choice of value for $m_g^*$ that satisfies $m_g^* \cdot (\ln m_g^* - \ln \lambda) \geq \ln n + (m_g^* - \lambda)$, we can obtain the minimized probability expression below. Note that under our rescaling, $m_g^* \cdot (\ln m_g^* - 1) + 1 \geq \ln n$. Therefore, $\Pr\{m_g \geq m_g^*\} \leq \frac{e^{-\lambda(\lambda e)^{m_g^*}}}{(m_g^*)^{m_g^*}} \leq \frac{1}{n}$.

i.e., we have

$\Pr\{m_g \geq m_g^*\} \geq 1 - \frac{1}{n} = 1 - o(n)$ w.h.p.

**Proposition 2:** Provided that $m_g^* \cdot (\ln m_g^* - \ln \lambda) \geq \ln n + (m_g^* - \lambda)$, and given the network diameter $dr_m$, $d \in \mathbb{R}^+$, we have $\kappa/d = \Theta(\sqrt{\ln \ln n/n})$ w.h.p.

**Proof:** From Lemma 3 and Lemma 4 we obtain the relations between $m_g^*$, $m_g$, and $\lambda$ under the optimal scheme of node placement and object replication. Implicitly, nodes that do not issue any query for replicated objects in this time period are outside $A_g$ w.h.p. It is not hard to validate that $\exists u_1 \in \mathbb{R}^+$ such that $A_g \cdot e^{A_g} = (1 + u_1) \ln n$

Moreover, the following result can be derived:

$$\frac{\pi \kappa^2 r_m^2}{A} \geq \ln n + \ln (1 + u_1) - \ln A_g.$$  \hspace{1cm} (16)

Therefore, $\exists u_2 \in \mathbb{R}^+$ such that

$$\frac{\kappa}{\sqrt{A}} = (1 + u_2) \cdot \sqrt{\frac{\ln \ln n - \ln A_g}{n \pi r_m^2}}$$

Since $\lim_{n \to \infty} \left(\frac{\ln A_g}{\pi r_m^2 n}\right)$ and $A = \pi (dr_m/2)^2$, with large enough $n$ we obtain:

$$\kappa/d = \Theta\left(\sqrt{\frac{\ln \ln n}{n}}\right)$$ w.h.p.

**Remark 1:** Although the derived bounds are not tight, we have established a preliminary relation between $\kappa$ and $A$. The scaling laws are events that almost certainly occur asymptotically. Moreover, under the inherent energy constraint, new problems emerge when optimizing the balance between conflicting requirements. For instance, in the illustration in Fig. 3, if most optimal relay paths pass through the same region (i.e., the shadowed rectangle), some tradeoff between replication efficiency and network lifetime is inevitable. Network lifetime will be addressed further in a future study.

**B. From the Perspective of Resource Management**

In real-life applications, because of the inherent constraints on the memory and energy resources of mobile nodes, requested objects are not usually replicated and delivered with maximum efficiency. From the recent work in [29], it can be deduced that the buffer size should be scaled in order to sustain a certain throughput capacity. Here, we present some preliminary observations and an analysis of inter-node resource management. For example, as objects are replicated and transmitted in the application-layer, consider the events illustrated in Fig. 5. Once an object $o_i$ has been replicated in node $u$ during a communication session, there is a (perhaps high) probability that more than one query for $o_i$ will be issued before the communication session ends. The available memory of $u$ is reduced by one object if another query for object $o_j$, $o_j \neq o_i$, is issued after the observation point and before object $o_i$ is removed from the memory. We denote such a case as a event induced by the incoming request for object $j$ (or $\downarrow j$ in brief). However, the memory capacity of $u$ may be insufficient if $\downarrow j$ events occur frequently and only a few objects can be removed from its memory in subsequent time-slots.
we show that even with full knowledge of an object’s request and observation point the event of memory overflow. The following formulation is derived based largely on the results given in [15]:

\[
\Pr\{T_o > \rho(\ln m + d)\} = \Pr\{\cup_{j=1}^{m} \psi_j^\tau \}
\]
\[
= 1 - \Pr\{\cap_{j=1}^{m} \psi_j^\tau\}
\]
\[
\approx 1 - e^{-e^{-\tau}}. \tag{19}
\]

As shown in [15], the probability that all \( m \) different requested objects will arrive within \( \tau \) time-slots changes abruptly from nearly 0 to almost 1 in a small interval \( e \) centered around \( \tau \). (See [15] for more rigorous arguments and relevant details.)

Remark 2: Proposition 3 implies a “deterministic-like” (i.e., very high probability) result that can provide application designers with some constructive insights into conserving memory resources. More specifically, in the above scenario, since \( m \) different subsequent objects overflow the memory of node \( u \) within \( \rho \) time-slots, the following inequality can be given: \( m/\rho \geq \Phi(u)/\tau \). Hence, \( u \) may find an eligible node (one with sufficient memory before the communication session ends) in its vicinity to take over \( o_i \) if the remaining memory \( \Phi(u) \) is not enough, i.e., \( \Phi(u)/m \leq \tau/\rho \leq \ln m \). Furthermore, if object sizes are determined by some random distribution with a constant mean, how should a node decide to remove the inside objects? We assume that, for each node, a cyclic index indicating the available memory exists. The index advances with time; if it encounters the position of an object \( o_i \), then \( o_i \) is removed. On the other hand, if the index moves too slowly, the available memory might be insufficient if a large object is received. Intuitively, the speed of the index should be the median of the buffer I/O rates. We are working towards more sophisticated formulas and algorithms based on Stochastic Process theory, especially from an information theoretical point of view.

Finally, one of the most important challenges in the design of ad hoc network applications is to reduce the energy consumption of the network. Nodes are typically battery-powered and, hence, have a limited lifetime. The energy used in communicating a bit from a source to a destination has two components — one due to transmission, which depends on the number of hops and the distance and power used at each hop, and the other due to transceiver circuit energy, which is proportional to the number of hops. The number of hops determines the amount of power required for optimal energy scaling [36]. In fact, the trade-off between throughput and delay in wireless ad hoc networks has also been addressed in recent years (see [8], [27], [28] and the references therein). Note that the effect of node mobility is considered in most related work.

We complete our analysis by proposing the following expression based on the results of several recent works. Let \( T(n) \) and \( D(n) \) denote the scaling behavior of network throughput and packet delay, respectively. Under the time-division multiplexing (TDM) policy, the scaling quantity of dissipated energy \( E(n) \) with the optimal throughput-delay-
energy notion has been shown to be as follows [36]:
\[ E(n) = \Omega(D(n) + A^{\alpha/2} \cdot D(n)^{1-\alpha}), D(n) = \Theta(n(T(n))). \]  
(21)

Remark 3: From the above discussion, we can suggest some practical implications: (i) To keep \( D(n) \) scaling as \( \Theta(1) \), the minimum energy-per-bit result shows that: \( E(n) = \Omega(A^{\alpha/2}) \). (ii) Alternatively, if the scaling quantity of packet delay is enlarged to be proportional to \( \sqrt{A} \), it follows that \( E(n) = \Omega(\sqrt{A}) \), i.e., data transfer can be achieved in a more energy efficient manner.

Remark 4: In practice, power consumption during packet transmission depends on the number of relay nodes and the number of trials required to successfully transmit a packet in one hop. Let \( P_b \) denote the probability of successful transmission determined by the MAC protocol. If we consider a CSMA/CA scheme with RTS and CTS frames (i.e., an IEEE 802.11-like protocol), the expected number of one-hop packet transmissions is proportional to \( 1/P_b \). Readers can find a detailed analysis of \( P_b \) in [32].

V. CONCLUDING REMARKS AND FUTURE RESEARCH DIRECTIONS

The characteristics of wireless ad hoc networks pose particular challenges to P2P applications. This paper represents a step toward developing a better understanding of the fundamental performance limits of these networks, with special emphasis on data replication. More specifically, we highlight some of the intrinsic difficulties in computing and communications. We also introduce the concept of replication-induced gain and derive several preliminary results in a resource-constrained environment. To clarify the whole approach, we have made manifold assumptions in our argument; however, we believe the analysis adequately captures the intrinsic characteristics of P2P data replication in wireless ad hoc networks.

Furthermore, we suggest some ways in which the results could shed new light on handling the various complexities and trade-offs in wireless ad hoc networks. The results have several implications that P2P system designers may wish to consider.

Further approximations could be derived with more realistic scenarios of the underlying replication-and-query process. In addition, the constructive forms that can generate algorithms from the above proofs are of particular interest, e.g., devising a replicating strategy that minimizes the expected memory consumption. We have not studied approximation algorithms in this paper because the traditional means of analyzing and evaluating online algorithms, namely, competitive analysis, has been criticized as being unrealistic in many real-life instances [16]. This problem tends to be exacerbated in our model due to the complications that follow from applying various constraints in practice (e.g., the dynamics of mobility, limited bandwidth, and frequent link failure). We are trying to refine competitive analysis in order to address the above concerns. As mentioned earlier, communication and computation overheads can contribute to a loss of data fidelity, and replication-induced gain also relies on the rate at which nodes lose interest in searching for certain objects. Techniques to optimize the balance between replication-induced gain and data fidelity have yet to be investigated.

Finally, the dynamics of user request traffic is an important subject that has not been fully investigated. As noted in [17], the intuitive solution of simply re-applying placement from scratch may cause several problems (e.g., sluggish reaction to system changes and substantial reconfiguration costs). Moreover, a new approach for dealing with dynamic queries, such as the use of flooding techniques in unstructured P2P networks, has been proposed by Jiang and Jin [18]. Nonetheless, the optimization of search latency and search costs often conflict with each other; hence, another interesting aspect would be to adaptively balance query execution against the routing overhead to facilitate higher data availability. Note that, in multihop wireless communication networks, it is usually necessary to support an evolving set of applications; therefore, resource-adaptive protocols are needed to improve throughput and other performance factors. Obviously, numerous technological challenges must still be resolved in the development of practical and efficient strategies that use fewer system resources and are more robust against network failures and delays.

APPENDIX

PROOF OF PROPOSITION 1

Proof: For ease of reduction, we introduce the maximal independent set (MIS) problem. Briefly, an independent set \( S \) of \( G = (V, E) \) is a subset of \( V \) such that \( \forall u, v \in S, (u, v) \notin E \). \( S \) is called a maximal independent set (MIS) if any node \( v \) not in \( S \) is adjacent to some node in \( S \). The problem of deciding whether there exists an independent set of cardinality larger than an integer \( s \) is NP-complete (see [19] for more details). The MIS problem is obviously NP-complete, since by randomly taking a resolution set \( R' \), one can easily check whether \( R' =^* Q' \) with memory consumption less than \( d \) in polynomial time. To prove the NP-completeness of EMC, we prove that MIS can be reduced to EMC in polynomial time. For brevity, we consider a special case of \( \kappa = 1 \wedge c = n \) (denoted as the EMC' problem). A reduction to EMC' can thus be performed as follows. First we construct \( G' = (V', E') \) via the following s: (i) we duplicate \( V \) and mark the copies with a prime, thus \( \forall u \in V, \exists \) a copy \( u' \in V' \); and (ii) we add an extra node \( s' \) to \( V' \), thus \( |V'| = |V'| + 1 \). A corresponding EMC' problem can be derived for \( G' \): (i) \( \forall u'_i \in V', u'_i \) possesses an innate object \( o'_i \) (the innate object of \( s' \) is denoted as \( o'_s \)); and (ii) the query set contains \( \{q(u'_i, o'_s): u_i \in V\} \). A polynomial time node clustering algorithm is then exploited on \( G' \).

We adopt the terms “cluster-head” and “border-node” used in [20]. Let \( E' = \{(u'_i, u'_j): (u_i, u_j) \in E\} \). Finally, we choose an arbitrary node \( u \in V \) and add an edge \( (s', u') \) to \( E' \). An example is given in Fig. 6, where nodes \( a \) and \( d \) are the cluster-heads; and nodes \( b \) and \( c \) are the border-nodes. Now we consider the above case with \( \kappa = 1 \). Note that all cluster-heads are separated by at least one border-node. Hence, it is easy to verify that: (i) the minimal number of

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10Considering the approaches proposed in [19], after construction of the dominating set, the resulting graph consists of two types of nodes: cluster-heads and border-nodes. Note that the cluster-heads induce an independent set (i.e., a dominating set, where each node pair is not adjacent). A node is a border-node if it is not a cluster-head and there is more than one cluster-head in its two-hop neighborhood.
object replications that satisfies the 1-coverable requirement is \( h \), where \( h \) denotes the number of cluster-heads in \( G \); and (ii) \( h \) is also equivalent to the cardinality of an independent set \( S \) over \( G \), where \( S \) is approximated by the node clustering algorithm mentioned above. As a result, any answer to this 1-coverable EMC\(^c\) problem with memory consumption \( d = h \) can be associated with an answer MIS with the same number over \( G \). Note that all data structures and computations involved can be carried out in polynomial time. Since a general problem cannot be easier than its special cases, we can conclude that EMC is NP-complete.

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**REFERENCES**


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Fig. 6. Graphs that facilitate the proof of NP-completeness.

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