Complete Sharing Dynamic Spectrum Allocation for Two Cellular Radio Systems

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Abstract – An analytical formulation of the dynamic spectrum allocation problem for handling multiclass services in two cellular radio systems using a complete sharing (CS) scheme is presented. A CS scheme with multiple guard channels for giving prioritization of radio system 1’s new and handoff call connection over those of radio system 2 and giving prioritization of handoff connections over new connections in each radio system is used for the resource sharing policy at the connection level. The CS model is solved using a $K$-dimensional Markov chain. Numerical results are illustrated to show the performance of this scheme. The performance metrics include new and handoff connection blocking probabilities and system utilization.

Index Terms – analytical formulation, complete sharing, dynamic spectrum allocation, cellular radio systems

I. INTRODUCTION

Dynamic spectrum allocation (DSA) is important in the future as it can overcome the problem that is faced by current fixed spectrum allocation. Fixed spectrum allocation faces the problem of spectrum white spaces or spectrum holes. Thus the spectrum resources are not effectively and efficiently utilized. DSA can overcome this problem by dynamically allocating spectrum as and when it is needed.

One common scenario is that of a primary network and a secondary network where the primary network has priority for the usage of the spectrum band over the secondary network. Another scenario envisaged is two cellular radio networks in the same overlaying coverage area vying the usage of the spectrum band. They can share the spectrum band for transmissions [1, Fig. 8]. There are no analytical or numerical results for the second scenario at the connection level except [2]. At the connection level, new or handoff connections can arrive to a cell. Reference [2] analyzes one type of DSA for two cellular radio networks sharing the spectrum band. Two traffic classes are considered in the paper. Within each traffic class, a fixed bandwidth is allocated and shared among the new and handoff connections of both cellular networks. Handoff connections of network 1 has priority over new connections of network 1, while new connections of network 1 has priority over new and handoff connections of network 2 as well. However, handoff connections of network 2 have the priority over new connections of network 2. The contributions of this paper are twofold. Firstly, we provide a new analytical model and numerical results for the second scenario with two cellular radio networks sharing a spectrum band with multiclass traffic using CS. Furthermore, the connection handoff rates for radio networks 1 and 2 are obtained iteratively using an iteration procedure rather than assuming fixed values for all ranges of the new connection arrival rates for radio networks 1 and 2.

The rest of the paper is organized as follows. Section II describes the system model. The problem statement is presented in Section III. Section IV presents an analytical model for the proposed scheme. Numerical results for the proposed scheme are presented in Section V. Finally, concluding remarks are made in Section VI.

II. SYSTEM MODEL

We consider a typical generic radio cell with physical capacity $N$ for two radio systems in a cellular arrangement. The two base stations for these radio systems sharing the spectrum band are the traffic distribution hubs which handle transmissions by multiple users in the wireless domain and forwards traffic to/from a wired backbone network. For easy reference we define the basic unit of capacity as a channel. A user of some traffic class may transmit at a rate achievable by one channel, while other transmission rates may require multiple number of channels. The two base stations both support $K$ classes of services that can originate from at most $N$ mobile users. The generic cell is characterized by the following system level parameters used throughout the paper.

$N$: total physical capacity in a cell  
$K$: total number of traffic classes  
$r_k$: number of basic channels (units) required by each class $k$ connection  
$N_C$: guard capacity for handoff connections of radio system 1  
$N_E$: guard capacity for new connections of radio system 1 plus $N_C$  
$N_D$: guard capacity for handoff connection of radio system 2 plus $N_E$

The dynamics of a radio cell shared by radio systems 1 and 2 is driven by new connection requests, connection terminations, and handoffs induced by user mobility. Handoff connections should be given a higher access priority, or a lower blocking probability than new connections as
maintaining an ongoing connection is more important than admitting a new connection. Let $B_{k,l}$ and $B_{k,l}$ denote respectively the blocking probabilities of new and handoff connections of radio system 1, and $B_{n,2}$ and $B_{h,2}$ denote respectively the blocking probabilities of new and handoff connections of radio system 2, with $B_{k,l} \leq B_{n,2} \leq B_{h,2} \leq B_{n,2}$. One way to facilitate this is to reserve capacity for admitting some types of connections, which is not accessible by other types of requests. The reserved capacity is sometimes referred to as guard capacity.

Our paper considers multiple traffic classes, but we first consider a single traffic class as an illustrative example to have a better understanding of the proposed scheme. Let $N, N_c$ and $N_i$ denote respectively the total capacity, the guard capacity for handoff connection of radio system 1 and the instantaneous capacity occupancy plus the new or handoff connection capacity of radio system 1 or 2, and $N_A > N_B > N_C$ such that $(N_B-N_C)$ is the guard capacity for new connection of radio system 1 and $(N_A-N_B)$ is the guard capacity for handoff connection of radio system 2. We have the following admission rule:

1. Admit both new and handoff connections from radio system 1 and 2 if $N-N_i \geq N_A$, where $(N-N_i)$ is the free capacity left after admitting a new or handoff connection from radio systems 1 and 2.
2. Admit new and handoff connections from radio system 1 and handoff connections from radio system 2 only if $N_i \leq N-N_i < N_B$, where $(N-N_i)$ is the free capacity left after admitting a new or handoff connection from radio system 1 or a handoff connection from radio system 2.
3. Admit new and handoff connections from radio system 1 only if $N_i \leq N-N_i < N_B$, where $(N-N_i)$ is the free capacity left after admitting a new or handoff connection from radio system 1 or a handoff connection from radio system 2.
4. Admit handoff connections from radio system 1 only if $0 \leq N-N_i < N_C$, where $(N-N_i)$ is the free capacity left after admitting a handoff connection from radio system 1.

Deployment of the guard capacity policy has the following ramifications.

- A handoff connection from radio system 1 is accepted as long as there are enough channels available.
- A new connection from radio system 1 is accepted as long as the number of available channels (if it is admitted) is greater than $N_c$.
- A handoff connection from radio system 2 is accepted as long as the number of available channels (if it is admitted) is greater than $N_b$.

### III. PROBLEM STATEMENT

We consider the dynamic spectrum allocation problem for handling multiclass services in two cellular radio systems using a complete sharing (CS) scheme. A CS scheme with multiple guard channels for giving prioritization of radio system 1’s new and handoff call connections over those of radio system 2 and giving prioritization of handoff connections over new connections in each radio system is used for the resource sharing policy at the connection level. We consider a class $k$ user of radio systems 1 and and a class $k$ user of radio system 2. Their Grade of Service (GoS) are specified by the new connection blocking probability of radio systems 1 and 2, $B_{n,k,1}$ and $B_{n,k,2}$, handoff connection blocking probability of radio system 1 and 2, $B_{h,k,1}$ and $B_{h,k,2}$, and class $k$ system utilization, $N_{sk}$, at the connection level. The connection level parameters used throughout the paper are listed below.

$B_{n,k,j}$: new connection blocking probability for class $k$ of radio system $j, j=1,2$

$B_{h,k,j}$: handoff connection blocking probability for class $k$ of radio system $j, j=1,2$

$n_c$: number of class $k$ sources (users) in progress

$\lambda_{n,k,j}$: arrival rate of class $k$ new connections of radio system $j, j=1,2$

$\lambda_{h,k,j}$: arrival rate of class $k$ handoff connections of radio system $j, j=1,2$

$\mu_{ck}^{-1}$: mean connection holding time of a class $k$ connection for both radio systems 1 and 2

$\mu_{hk}^{-1}$: mean dwell time (interhandoff time) of a class $k$ connection for both radio systems 1 and 2

$\mu_k = \mu_{ck} + \mu_{hk}$: mean equivalent departure rate of a class $k$ connection for both radio systems 1 and 2

### IV. ANALYTICAL MODEL

Here we consider an analytical formulation of the dynamic spectrum allocation problem for handling multiclass services in two cellular radio systems using a complete sharing (CS) scheme. A CS scheme with multiple guard channels for giving prioritization of radio system 1’s new and handoff call connection over those of radio system 2 and giving prioritization of handoff connections over new connections in each radio system is used for the resource sharing policy at the connection level. The CS model is solved using a $K$-dimensional Markov chain.

Complete sharing allows all the channels to be shared by multiple new and handoff traffic classes in both cellular radio systems, extending the admission rule stated in Section 2 for multiple traffic classes.

Consider a $K$-class CS model. The guard channels, $C_G$, are reserved for handoff connections only. To facilitate analytical modeling, it is necessary to make certain assumptions about the traffic parameters. It is not unreasonable to assume that the holding time has a negative exponential distribution. Although a negative exponential distribution assumption may not be as
reasonable for the cell dwell time, for analytical tractability, we will make the same assumption for cell dwell time (interhandoff time) and model the channel as a Markov process under the policies in Section 3. The cell time can be modeled as a Gamma distribution. The differences in new and handoff probabilities between having a Gamma distribution and a negative exponential distribution for the cell dwell time are not large \[3\] and thus the assumption of a negative exponential distribution for the cell dwell time is acceptable. We can model the channel occupancy as a \(K\)-dimensional Markov chain with the connection level parameters in Section 3. This Markov chain can be modeled and solved using the techniques in \[4\].

Let \(n=(n_1,n_2,\ldots,n_k)\) denote the state of the system with the number of users from radio systems 1 and 2 (\(n_k\)) in each of the \(K\) classes, and let \(r=(r_1,r_2,\ldots,r_k)\) denote the number of basic connections (\(r_j\)) required for each class \(k\) connection. Let \(\lambda_k(n)\) denote the arrival rate and \(\mu_k(n)\) the departure rate in the system. With \(N\) denoting the total number of channels, the state space of the system, denoted by \(S\), is given by \(S=\{n:r\leq n\leq N\}\), where \(r\cdot n=(r_1n_1+n_2)+\ldots+r_kn_k\).

When the system is in state \(n\) and a class \(k\) connection (new or handoff) arrives, an admission policy determines whether or not the connection is admitted into the system. Here, the admission policy is a complete sharing scheme with multiple guard channels. We can specify the admission policy by mapping \(f=(f_1,\ldots,f_k)\) for new and handoff connections of radio system 1 and 2, \(f_C=(f_{A1},\ldots,f_{Ck})\) for new and handoff connections of radio system 1 and handoff connections of radio system 2, \(f_{Ak}=(f_{A1k},\ldots,f_{A2k})\) for new and handoff connections of radio system 1, where \(f_k, f_{ak}, f_{bk}\) and \(f_{ck}\) are probabilities for \(k\). We have \(f_k(n),f_{ak}(n),f_{bk}(n)\) and \(f_{ck}(n)\) each takes on the value 0 or 1 if a class \(k\) connection is rejected or admitted, respectively, when the system is in state \(n\). They are defined by the following equations:

\[
f_k(n) = \begin{cases} 1, & r\cdot n + r_k \leq N - N_A \\ 0, & \text{otherwise} \end{cases}
\]  

(1)

\[
f_{A1k}(n) = \begin{cases} 1, & N - N_A < r\cdot n + r_k \leq N - N_B \\ 0, & \text{otherwise} \end{cases}
\]  

(2)

\[
f_{B1k}(n) = \begin{cases} 1, & N - N_B < r\cdot n + r_k \leq N - N_C \\ 0, & \text{otherwise} \end{cases}
\]  

(3)

\[
f_{C1k}(n) = \begin{cases} 1, & N - N_C < r\cdot n + r_k \leq N \\ 0, & \text{otherwise} \end{cases}
\]  

(4)

for which \(S(f) + S(f_A) + S(f_B) + S(f_C) = S\) and \(N_A > N_B > N_C\).

Let \(P(n)\) denote the equilibrium probability that the system is in state \(n\). The global balance equations for the Markov process under the policies \(f, f_A, f_B\) and \(f_C\) are

\[
\sum_{k=1}^{K} [\lambda_k(n)[f_k(n) + f_{A1k}(n) + f_{B1k}(n) + f_{C1k}(n)] + \mu_k(n)]P(n)
\]  

= \[
\sum_{k=1}^{K} [\lambda_k(n - e_k)[f_k(n - e_k) + f_{A1k}(n - e_k) + f_{B1k}(n - e_k)] + \mu_k(n)]P(n)
\]  

(5)

\[+ f_{C1k}(n - e_k)]P(n - e_k) + \sum_{k=1}^{K} \mu_k(n + e_k)P(n + e_k), n \in S,
\]

where \(e_k\) is a \(K\)-dimensional vector of all zeros except for a one in the \(k\)th place,

\[
\lambda_k(n) = \begin{cases} \lambda_{k,1}, & \text{if } f_k(n) = 1 \\ \lambda_{k,2}, & \text{if } f_{A1k}(n) = 1 \\ \lambda_{k,3}, & \text{if } f_{B1k}(n) = 1 \\ \lambda_{k,4}, & \text{if } f_{C1k}(n) = 1 \end{cases}
\]  

(6)

\[
\lambda_{k,1} = \lambda_{n,k,1} + \lambda_{h,k,1} + \lambda_{n,k,2} + \lambda_{h,k,2},
\]  

(7)

\[
\lambda_{k,2} = \lambda_{n,k,1} + \lambda_{h,k,1} + \lambda_{n,k,2},
\]  

(8)

\[
\lambda_{k,3} = \lambda_{n,k,1} + \lambda_{h,k,1},
\]  

(9)

\[
\lambda_{k,4} = \lambda_{h,k,1},
\]  

(10)

and

\[
\mu_k(n) = \mu_{nk} + \mu_{nk}, 0 < r\cdot n \leq N.
\]  

(11)

The first condition in equation (6) allows both new and handoff connections of both radio systems 1 and 2 to be admitted to \((N-N_A)\) channels, while the second condition allows both new and handoff connections of both radio systems 1 and only handoff connections of radio system 2 to be admitted to \((N_NA-N_B)\) channels. The third condition in equation (6) allows both new and handoff connections of radio system 2 to be admitted to \((N_NA-N_C)\) channels, while the fourth condition allows only handoff connections of radio system 1 to be admitted to \(N_C\) channels. Equation (5) allows both new and handoff connections to be serviced with the total channel occupancy to be less than or equal to \(N\) channels. Equation (3) can be solved using LU decomposition \[5\] together with the condition for the total probability of all states to obtain \(P(n)\). LU decomposition is a common numerical technique for solving linear algebraic equations. This gives exact solution. However, efficient approximate computational algorithm \[6\] could be used for a large system state space.

Let \(\theta_{n,k,j}\) denote the probability that the next arrival is a new connection from class \(k\) of radio system \(j\). Then

\[
\theta_{n,k,j} = \frac{\lambda_{n,k,j}}{\sum_{l=1}^{K} \sum_{m=1}^{2} (\lambda_{n,l,m} + \lambda_{h,l,m}), j=1,2.
\]  

(12)

Let \(\theta_{h,k,j}\) denote the probability that the next arrival is a handoff connection from class \(k\) of radio system \(j\). We have

\[
\theta_{h,k,j} = \frac{\lambda_{h,k,j}}{\sum_{l=1}^{K} \sum_{m=1}^{2} (\lambda_{n,l,m} + \lambda_{h,l,m}), j=1,2.
\]  

(13)

A new class \(k\) connection from radio system 1 is blocked from entering the system (and is assumed lost) if upon arrival it finds that it cannot be accommodated due to insufficient channels (excluding the guard channels of \(N_C\) for an
additional $r_k$ channels. Therefore the blocking probability for a new class $k$ connection for radio system 1 considering all classes is given by

$$B_{n,k,1} = \frac{\left[N/n\right]^{(N-n_k)/r_k}}{\left[N-n_k\right]/r_k} \times \prod_{k=1}^{K} \left(1 - \frac{N - \sum n_k r_k}{r_k}\right) \times \left(1 - \frac{\sum n_k r_k}{r_k}\right)$$

$$\times P(n_1, n_2, n_3, ..., n_K) \theta_{n_k,1}, \text{if } \left(\sum_{k=1}^{K} n_k r_k\right) < r_k.$$  \hspace{1cm} (14)

A class $k$ handoff connection from radio system 1 is blocked from entering the system (and is assumed lost) if upon arrival it finds that it cannot be accommodated because the number of available channels is less than $r_k$ (including the guard channels of $N_C$). Therefore the blocking probability for a class $k$ handoff connection for radio system 1 considering all classes is given by

$$B_{n,k,1} = \frac{\left[N/n\right]^{(N-n_k)/r_k}}{\left[N-n_k\right]/r_k} \times \prod_{k=1}^{K} \left(1 - \frac{N - \sum n_k r_k}{r_k}\right) \times \left(1 - \frac{\sum n_k r_k}{r_k}\right)$$

$$\times P(n_1, n_2, n_3, ..., n_K) \theta_{n_k,1}, \text{if } \left(\sum_{k=1}^{K} n_k r_k\right) < r_k.$$  \hspace{1cm} (15)

A new class $k$ connection from radio system 2 is blocked from entering the system (and is assumed lost) if upon arrival it finds that it cannot be accommodated due to insufficient channels (excluding the guard channels of $N_D$) for an additional $r_k$ channels. Therefore the blocking probability for a new class $k$ connection for radio system 2 considering all classes is given by

$$B_{n,k,2} = \frac{\left[N/n\right]^{(N-n_k)/r_k}}{\left[N-n_k\right]/r_k} \times \prod_{k=1}^{K} \left(1 - \frac{N - \sum n_k r_k}{r_k}\right) \times \left(1 - \frac{\sum n_k r_k}{r_k}\right)$$

$$\times P(n_1, n_2, n_3, ..., n_K) \theta_{n_k,2}, \text{if } \left(\sum_{k=1}^{K} n_k r_k\right) < r_k.$$  \hspace{1cm} (16)

A class $k$ handoff connection from radio system 2 is blocked from entering the system (and is assumed lost) if upon arrival it finds that it cannot be accommodated because the number of available channels is less than $r_k$ (excluding the guard channels of $N_D$). Therefore the blocking probability for a class $k$ handoff connection for radio system 2 considering all classes is given by

$$B_{n,k,2} = \frac{\left[N/n\right]^{(N-n_k)/r_k}}{\left[N-n_k\right]/r_k} \times \prod_{k=1}^{K} \left(1 - \frac{N - \sum n_k r_k}{r_k}\right) \times \left(1 - \frac{\sum n_k r_k}{r_k}\right)$$

$$\times P(n_1, n_2, n_3, ..., n_K) \theta_{n_k,2}, \text{if } \left(\sum_{k=1}^{K} n_k r_k\right) < r_k.$$  \hspace{1cm} (17)

The total system utilization is

$$N_u = \sum_{k=1}^{K} N_u,$$

$$\lambda_{n,k,1}(1 - B_{n,k,1}/\theta_{n,k,1}) + \lambda_{n,k,1}(1 - B_{n,k,2}/\theta_{n,k,2}) = r_k \mu_{ck} + \mu_{hk}$$

where the system utilization of class $k$ connections for radio system $j$ is

$$N_{sk,j} = r_k \lambda_{sk,j} + \lambda_{h,k,j}(1 - B_{h,k,j}/\theta_{h,k,j}), \text{ if } j = 1, 2.$$  \hspace{1cm} (19)

Equating the class $k$ handoff connections arrival rate for radio system $j$ to the product of the average handoff rate for a class $k$ connections and the average number of class $k$ connections for radio system $j$, we can get an approximate class $k$ handoff connections arrival rate for radio system $j$ as follows:

$$\lambda_{h,k,j} = \frac{N_{sk,j}}{r_k} = \frac{\mu_{ck} + (\widehat{B}_{h,k,j}/\theta_{h,k,j})\mu_{hk}}{\mu_{ck} + \mu_{hk}}.$$  \hspace{1cm} (20)

Thus, the class $k$ handoff connections arrival rate for radio system $j$ can be approximated under low blocking probabilities as follows:

$$\lambda_{h,k,j} = \frac{\mu_{hk} \lambda_{n,k,j}}{\mu_{ck}}, \text{ if } j = 1, 2,$$  \hspace{1cm} (21)

where $\mu_{hk} = v_k / s$, $v_k$ is the speed of the class $k$ mobile and $s$ is the cell length of a square cell. We consider a cellular system consisting of square cells without loss of generality.

Note that the state steady probabilities, $P(n)$’s, are functions of the handoff call arrival rates of radio systems 1 and 2, $\lambda_{h,k,j}$ and $\lambda_{h,k,j}$, and they are functions of the system utilizations of radio system 1 and 2, $N_{sk,j}$ and $N_{sk,j}$, which are functions of $P(n)$’s. We use an iterative method to determine the $\lambda_{h,k,j}$ and $\lambda_{h,k,j}$ and the $P(n)$’s as follows.

1. Initialize the handoff call arrival rate, $\lambda_{h,k,j}(0) = \mu_{h,k,i}/\mu_{c}$, $\lambda_{h,k,j}(0) = \lambda_{h,k,j}/\mu_{c}$, and, where the zero in parentheses means step 0.
2. Iterate between $P(n)$’s as in (5), $N_{sk,j}$ and $N_{sk,j}$ as in (20) and $\lambda_{h,k,j}$ and $\lambda_{h,k,j}$ as in (21) until $\lambda_{h,k,j}(m - \lambda_{h,k,j}(m - 1) < \varepsilon_{\lambda_{h,k,j}},$  \hspace{1cm} (23)

$$\lambda_{h,k,j}(m) - \lambda_{h,k,j}(m - 1) < \varepsilon_{\lambda_{h,k,j}},$$  \hspace{1cm} (24)

where the variable $m$ in the parentheses denotes the $m$th iteration step, and $\varepsilon_{\lambda_{h,k,j}}$ and $\varepsilon_{\lambda_{h,k,j}}$ are the error thresholds for $\lambda_{h,k,j}$ and $\lambda_{h,k,j}$, respectively. Using this iterative method, the handoff call arrival rates for radio system 1 and 2, $\lambda_{h,k,j}$ and $\lambda_{h,k,j}$, respectively, are obtained iteratively. Note that the initial handoff call arrival rate of radio systems 1 and 2, $\lambda_{h,k,j}(0)$ and $\lambda_{h,k,j}(0)$, are set to the approximate their handoff call arrival rates under low blocking probabilities as in (22).
V. NUMERICAL RESULTS

Numerical results for two traffic classes ($K=2$) are presented here as illustrative examples. The parameters used in the numerical example are in Table I.

<table>
<thead>
<tr>
<th>TABLE I. PARAMETERS USED IN THE PROPOSED CS SCHEME</th>
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<tbody>
<tr>
<td>Symbols</td>
</tr>
<tr>
<td>$N$</td>
</tr>
<tr>
<td>$N_A$</td>
</tr>
<tr>
<td>$N_B$</td>
</tr>
<tr>
<td>$N_C$</td>
</tr>
<tr>
<td>$r_1$</td>
</tr>
<tr>
<td>$r_2$</td>
</tr>
<tr>
<td>$\theta_{n,1,1}$</td>
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<tr>
<td>$1/\mu_{h,1} = 1/\mu_{h,2}$</td>
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<tr>
<td>$1/\mu_{c,1}$</td>
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<tr>
<td>$1/\mu_{c,2}$</td>
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</table>

Fig. 1 shows the blocking probability of new and handoff connections of classes 1 and 2 for radio systems 1 and 2, while Fig. 2 shows the system utilization of classes 1 and 2 for radio systems 1 and 2. From Fig. 1, the proposed scheme gives priority to handoff connections of radio system 1 over new connections of radio system 1, which in turn has priority over handoff connections of radio system 2, which in turn has priority over new connections of radio system 2 in terms of both classes 1 and 2’s blocking probabilities. The lower the blocking probability, the better the performance. Thus prioritization between classes and radio systems are achieved in this proposed scheme.

Fig. 2 shows that class 1 in radio system 1 has higher utilization than that of class 1 in radio system 2. Similar, class 2 in radio system 1 has higher utilization than that of class 2 in radio system 2. Class 2 has higher utilization than that of class 1 as class 2 uses two basic channels as compared to one basic channel for class 1.

VI. CONCLUDING REMARKS

A new analytical formulation of the dynamic spectrum allocation problem for handling multiclass services with two cellular systems sharing a spectrum band is presented in Section 4. A complete sharing scheme with multiple guard channels is used for the resource sharing policy. The analytical model is solved using a K-dimensional Markov Chain for the CS scheme. The GoS performance metrics are connection blocking probability and system utilization. Numerical results are provided to illustrate the performance of the proposed scheme. These results show that prioritization between traffic classes and radio systems is achieved using the proposed scheme.

REFERENCES