d-infinite Criteria for MEG Characterization

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Abstract—Magneto Encephalographic (MEG) brain signals are studied using a method for characterizing nonlinear dynamics. This approach uses the value of $d_\infty$ (d-infinite) to characterize the system's asymptotic chaotic behavior. A novel procedure was developed to extract this parameter from time series when the system's structure and laws are unknown. The implementation of the algorithm has proven to be general and computationally efficient. The information characterized by this parameter is furthermore independent and complementary to the signal power since it considers signals normalized with respect to their amplitude. The algorithm implemented here is applied to whole-head 148 channel MEG data during two highly structured yogic breathing meditation techniques. Results, which are relative to spatio-temporal distributions of the calculated $d_\infty$ on the MEG channels, are analyzed and compared during different phases of the yogic protocol.

I. INTRODUCTION

While the activity of single neurons has been well characterized through physiological studies [1], the dynamics of whole brain activity are best studied using electroencephalography (EEG) and magnetoencephalography (MEG). Both methods measure the product of large groups of neurons assembled in a hierarchical manner and thus provide an opportunity to study the relationship of the micro and macro levels of the brain as a physical system. Various linear and nonlinear analysis techniques are described in the literature for studying MEG and other brain signals in an attempt to characterize normal resting activity, effects of sensory stimulation, and pathological states [2]. The work presented here is based on the evaluation of the asymptotic distance $d_\infty$ (d-infinite) and this approach has been previously used for characterizing nonlinear dynamics [3].

A novel implementation to evaluate the $d_\infty$ is introduced here, and this method is computationally less onerous than conventional methods. In general, in the theoretical determination of the $\lambda$ and $d_\infty$, the knowledge of the finite difference equations in the discrete domain as well as the differential equations in the continuous domain are fundamental [4]. When the laws of the systems under study are unknown and we are provided only with experimental data, the need arises for a calculation of the asymptotic distance $d_\infty$ for generic time series. Therefore, the novel procedure developed here evaluates the $d_\infty$ as the asymptotic value of the average distance between trajectories that are extracted directly from the time series. As described in Section II, this method has been applied to MEG data collected during the practice of two highly structured yogic breathing techniques, one of which showed efficacy for the treatment of patients with obsessive compulsive disorder (OCD) [5].

A preliminary study of the theoretical and methodological framework has been tested and verified in Section III. The extraction of the $d_\infty$ parameter has been performed on numeric series coming from a well-known nonlinear system showing chaotic behavior. Furthermore, in order to verify the consistency of the methodology, a comparison was performed between the computation of the $d_\infty$ on trajectories of a discrete map starting from nearby points, and the computation of it on short time sequences starting from nearby values in a long run numeric series generated by the same discrete map.

In Section IV the approach for the $d_\infty$ extraction from time series is presented using MEG signals. Specifically, we exploit the potential of the $d_\infty$ parameter for data analysis comparing these results to the power distribution analysis. This analysis compares brain activity in the different phases of the protocol and correlates it with the potential effect of the breathing exercise. Moreover, the same parameter has been evaluated after filtering the MEG signals in the different frequency bands common to brain activity.

II. CASE STUDY

Recordings were made using a whole-head 148 channel MEG instrument (4-D Neuroimaging, San Diego, CA, USA) located at The Scripps Research Institute (La Jolla, CA). Each of the 148 pick up coils in this instrument is a 2 cm diameter magnetometer, with a 2.2 cm distance between coils center to center. Each coil is connected to a SQUID that produces a voltage proportional to the magnetic field radial to the head, resulting in preferential sensitivity to
neural electrical sources tangential to the surface of the scalp emanating from cortical sulci. This MEG system is contained in a magnetically shielded room that helps reduce the contribution of magnetic fields from more distant sources, and this significantly increases the signal to noise ratio and improves the ability to detect deeper signal sources in the brain. Trained MEG technicians positioned the subject, applied electro oculogram leads, and performed head shape digitization. A subject was employed who is both highly trained with yogic breathing techniques and as a subject for MEG recordings. Head-shape was digitized, based on known locations on the subject's head (tragus of left and right ears and nasion). Head shape data is for later co-registration between measurement coil locations, electrode locations, and scalp landmarks (Figure 1). Eye movements were recorded with electrodes placed above and below the right eye. Electrode impedances were set below 5 kΩ. MEG data was recorded with a sampling rate of 251 Hz, with an analog filter band pass of 1 to 100 Hz.

Figure 1: Channels Map.

A. Yogic Protocol

The subject was recorded while reclining and supported at 45 degrees. The subject followed a well-practiced protocol that involves 10 min of resting baseline recording (rest phase I), followed by a 31 min exercise recording phase, and followed by 10 min of resting recording (rest phase II). The three phases are separated by a one-minute recording pause. The exercise phase consists of selectively breathing through only one nostril (using a plug for the other side, with both arms resting in the lap) at a respiratory rate of one breath per minute (15 s slow inspiration, 15 s breath retention, 15 s slow expiration, and 15 s breath hold out). On day one (data set 5) the technique employing the left nostril was used (the pattern that has shown efficacy in treating OCD [5], and on the following day (data set 6), the same pattern was employed using the right nostril. This approach is used to study the potential differential brain effects that may result from these two unique meditation techniques, and to help insure that the effects of one technique do not carry over into the effects of the other if both techniques are practiced on the same day.

III. FROM TIME SERIES

A. Theoretical Background

Two key aspects of chaos are the stretching of infinitesimal displacements and the existence of complex orbit-like structures, in the form of a vast variety of possible unstable orbits, confined in a region of the phase space called the attractor [3]. The stretching property is strictly related to sensitive dependence on initial conditions. A quantitative characterisation of stretching properties is provided by the Lyapunov exponents. Let us assume that x, denotes a k-dimensional vector, and consider the dynamical system specified by the discrete map:

\[ x_{n+1} = G(x_n, r) \]  

where \( r \) is a parameter (not mentioned in the following).

Let us consider \( N \) couples of trajectories starting from two nearby points separated by a small distance \( d_o \)

\[ x_j^{(i)} = G_e(x_0^{(i)}) \quad x_j^{(o)} = G_e(x_0^{(i)} + d_o) \]  

averaging on the \( N \) couples of trajectories, choosing randomly the \( N \) initial starting points, the mean distance between two trajectories after \( j \) iteration can be defined as:

\[ d_j = \frac{1}{N} \sum_{i=1}^{N} \left| x_j^{(i)} - x_j^{(o)} \right| \]  

where the \( |\cdot| \) operator denotes the usual absolute value. The \( d_j \) asymptotic value is defined as:

\[ d_\infty = \lim_{n \to \infty} \frac{1}{n} \sum_{j=1}^{n} d_j \]  

It is well known [3] that, after \( n \) iterations, the stretching phenomenon stretches the distance \( d_o \) as:

\[ d_{n+1} = e^{\lambda_1} d_o = e^{\lambda} d_o = \lambda d_o \]  

where \( \lambda \) is the Lyapunov exponent of the map. After a sufficiently large number of iterations, the folding process takes place to keep the trajectories bound in the phase space. It is important to notice that, while \( \lambda \) is only sensitive to the stretching mechanism, \( d_\infty \) is sensitive to both the stretching and the folding mechanisms, being an important measure for characterisation of chaos when the \( \lambda \) is not easily computed, or to distinguish between series which have the same Lyapunov exponent. For a continuous system, the above assumptions still apply and the results are obtained in [4].

B. Numerical Algorithm for time series

A numerical algorithm implements the theoretical calculation of the \( d_j \). Given a time series and given a range for the initial conditions \( d_0 \), particular trajectories \( (x_j) \) starting from a point in the that range is extracted from the original signal. It selects couples \( (x_j, x'_j) \) among the found trajectories, and if they have the same initial slope in sign,
calculates the trajectory distance between them. This operation is performed iteratively for all the possible couples \((i, j)\). The average of the trajectory distances \(d_{ij}\) is calculated. The \(d_{\infty}\) represents the average asymptotic value of \(d_i\).

The logistic map, as a discrete series (Eq. 6), was used as an example of the developed \(d_{\infty}\) algorithm \(a=4\).

\[
x_{n+1} = a \times x_n \times (1-x_n) \quad (6)
\]

Furthermore, Figure 2 shows the consistency between the \(d_j\) resulting from the application of the developed algorithm on the logistic time series (red line) and the same curve computed using 30 pairs of trajectories obtained by the iterations of the known Logistic Map starting from different initial conditions with an initial distance \(d_{ij}=0.02\) (blue line). The consistency between the two curves shows the reliability of the developed algorithm for nonlinear iterative maps. While taking in account the mentioned constraints, this method can be used for the characterization of time series coming from measurements performed on real systems when the laws and structures are unknown and chaotic dynamics are suspected. Since it is computationally efficient, it can be easily applied to large data sets.

III. D-INFINITE IN MEG DATA

Power analysis is performed in order to characterize the distribution on the scalp using the maximum of the autocorrelation functions for all channels. The autocorrelation function is calculated on one-minute time series (N samples) for each channel \(c_h\) and the value in zero \(C_i(0)\) represents the power of the signal in that minute. The two spatio-temporal maps, shown in Figure 3, respectively for the left nostril \(\text{data set 5}\) and \(\text{right nostril data set 6}\) breathing protocols, are a complete representation of the whole-head MEG power time evolution. The color of the pixel \((j-th, i-th)\) represents the value of the power intensity related to the \(j\)-th minute for the \(i\)-th channel, using a conversion that is described by the color-bar on the right of the image. It is worth noting the emergence of patterns in the maps for the single channel columns. As also shown in a previous work [6], some differences can be observed in the three different phases of the yogic protocol and in the different channels during the different phases. The two spatio-temporal maps, shown in Figure 4, respectively for the left (a) and the right nostril (b) breathing protocols, give a representation of the whole-head MEG data \(d_{\infty}\) time evolution. The image's \(i\)-th column represents in a color code the time evolution by one-minute time-window steps of the logarithmic value of \(d_{\infty}\) for the \(i\)-th MEG channel. A further step in the visualization of the MEG signal dynamics is the spatial representation of the average logarithmic value of the \(d_{\infty}\) through the three phases. The \(d_{\infty}\) head maps were obtained averaging the spatio-temporal maps for the three phases as it is described in Section II.

While the power spatial distributions show differences between the protocols [6], at a first analysis from the \(d_{\infty}\) spatial patterns (Figure 5) striking differences between them are not visible since the \(d_{\infty}\) emphasizes nonlinear dynamics in MEG signals, different from high power brain phenomena. In order to further investigate a potential difference between the effects of the exercise phase for the two protocols, MEG signals were filtered in the alpha (8-12 Hz), beta (12-20 Hz), theta (4-8 Hz), and gamma (34-60 Hz) frequency bands and the \(d_{\infty}\) analysis was performed. As shown in Figure 6, beside the results on the beta band showing an expected asymmetric dynamics due to the breathing protocols, several other interesting patterns arise. The gamma band maps show features that are common to both the left and the right breathing protocol while the alpha band maps show dramatic differences between the two protocols. This proofs that the
frequency bands analysis allows a more detailed detection of conscious and unconscious brain states for the two protocols.

Figure 3: Power Spatio-temporal Maps: (a) left nostril and (6) right nostril breathing.

Figure 4: Spatio-temporal Maps: (a) left nostril and (6) right nostril breathing.

Figure 5: The $d_\infty$ head maps obtained averaging the spatio-temporal maps, data set 5 (left column) and data set 6 (right column).

IV. CONCLUSIONS

A novel implementation to evaluate the $d_\infty$ has been introduced; this method is computationally less onerous than the conventional methods and also fulfills the requirement of being general and applicable to experimental data coming from several instrumentation for diagnostics, including EEG, ECG, etc. Furthermore, the potentiality of the application of the $d_\infty$ parameter to MEG data analysis is evaluated comparing the results to the power distribution analysis. The $d_\infty$ spatial distribution highlights low power activities emphasizing spatially distributed dynamics that are not necessarily related to high power neuronal activity and that are likely to be very important for decoding brain activity. Finally the $d_\infty$ parameter was extracted from the filtered MEG signals in the alpha, beta, theta and gamma bands suggesting the value of conducting further analyses in this direction.

Figure 6: The $d_\infty$ head maps obtained filtering the data of the exercise phase, left nostril (left column) and right nostril (right column) breathing.

REFERENCES


