Abstract—In IEEE 802.16 networks, bandwidth allocation is centralized at the base station (BS). For the uplink traffic, data transmission is scheduled after the BS processes bandwidth requests (BW-REQs) sent by mobile stations (MSs). In this paper we focus on the multicast/broadcast polling method, where MSs send their BW-REQs during contention periods. Contention-based polling is deemed suitable for Best-Effort TCP flows, since the latter have less stringent needs, in terms of delay and throughput, than traffic carried by other classes of service. We propose a novel analytical model for Best-Effort TCP performance over 802.16 networks. This model explicitly considers the impact of the uplink request/grant mechanism, including the influence of bandwidth-perception mismatch at the BS. The effect of MAC-layer parameters, such as the MAC queue length, is also studied. The accuracy of the model is verified by means of ns-2 simulations.

I. INTRODUCTION

The IEEE 802.16 wireless metropolitan network standard [1], also known as WiMAX, defines high-performance mechanisms that provide last-mile, high-speed Internet services. In an IEEE 802.16 system, a base station (BS) allocates network resources to mobile stations (MSs). The IEEE 802.16 standards allow for dynamic on-demand reservation of uplink bandwidth [2]. To support various types of traffic, the IEEE 802.16e standard defines five Classes of Service (CoS), which allow setting various priorities to user traffic, and condition the rate, the delay and the jitter experienced by users’ data flows. The prioritization is implemented at the MAC layer via a classifier, a scheduler and an admission control sub-system. The bandwidth scheduling is centralized at the BS. For the downlink, the BS schedules data bursts according to the data waiting in BS queues for downloading traffic. For the uplink traffic, data is scheduled after the BS processes bandwidth requests (BW-REQs) sent by MSs. In this paper, we will consider only the multicast/broadcast polling method, which induces MSs to send their BW-REQs during contention periods.

A. IEEE 802.16 Frame Structure

A frame is divided into a downlink subframe and an uplink subframe, in the time dimension for TDD or the frequency dimension for FDD. In the Point-to-Multipoint architecture, all transmissions between BS and MSs are scheduled by the BS. The scheduling information is broadcast in downlink and uplink MAP (DL-MAP and UL-MAP) messages. MSs receive the downlink packets by analyzing the DL-MAP, and transmit uplink packets in slots defined in the UL-MAP.

B. Bandwidth Request and Grant Scheme

In 802.16 networks, bandwidth request-grant mechanisms are responsible of managing and satisfying the uplink bandwidth needs of MSs. These mechanisms enable the following functions: allowing MSs to dynamically indicate their bandwidth requirements; managing the perception that the BS has of MSs’ bandwidth needs; fulfilling those needs by granting uplink bandwidth allocations.

1) Requesting bandwidth: An MS informs the BS of its bandwidth needs by sending a BW-REQ message, which indicates an amount of bytes waiting for transmission at the MS queue. The standard supports different methods by which an MS can send the BW-REQ to the BS. In the polling-based method, the BS polls the MSs for knowing their bandwidth needs. Multicast and broadcast polling are used for enabling contention-based requests. In this method, the MSs contend to send BW-REQ messages in the bandwidth-request contention slots. The MS considers that a request is lost if no data grant has been given within a certain time interval, called T16 timer in the 802.16-2004 standard (In the 802.16e amendment, T16 has been replaced by a “contention-based reservation timeout”). Request losses are handled by a truncated binary exponential backoff (BEB) algorithm which allows retransmitting lost BW-REQs. Finally, the piggybacking method uses the grant management subheader to attach a bandwidth request to an uplink data packet.

2) Managing bandwidth perception: To satisfy the bandwidth needs of the MSs, the BS has to have a perception of the uplink bandwidth needs and allocation of MSs. This information may be stored in a bandwidth allocation table. Perception-management policies (i.e., the algorithms used for maintaining the allocation table) are implementation-dependent, as they are not specified in the IEEE 802.16 standard [1].

3) Satisfying MSs’ bandwidth needs: The BS fulfills the MSs’ bandwidth needs by allocating physical slots in the uplink subframe, which is defined in the UL-MAP. The standard states that the BW-REQs should be done on a per-connection basis, allowing the base station to properly implement a
fair and QoS-oriented uplink scheduler [3]. However, grants are made per MS, and not per Connection Identifier (CID). Depending on the scenario and the used policy, this system may lead to a mismatch between the perception of the BS and the actual needs of MSs.

C. Objective and structure of the paper

In [4], [5], we analyzed several perception-management policies and studied their behavior by means of simulation. Essentially, such policies differ in the way the BS bandwidth-allocation table is updated with incoming requests—in particular, whether the update is based on the granted bandwidth or on the actual uplink bandwidth usage. Our simulation results in [4], [5] highlight how, in different circumstances, different policies may indeed result in the BS getting out of sync with respect to the MSs, in terms of bandwidth demands and allocations. In such a case, the BS may not respond as it should to BW-REQ packets, which results often in uplink bandwidth allocation failure. However, from simulation results alone it is difficult to predict to what extent TCP performance is affected by a given “amount of desynchronization” between the BS and the MSs.

In this paper we propose a novel analytical model of TCP performance over IEEE 802.16 networks, so as to address the question of how the out-of-sync state of the BS results in poorer TCP performance. In this model, we explicitly consider the impact of the uplink bandwidth request-grant mechanism and the bandwidth perception scheme at the BS. Other MAC layer factors are also considered in this model, including queue length limit and the T16 timer. Based on the analysis of such MAC layer factors, we have developed expressions for TCP performance (i.e., for delay and throughput).

The paper is structured as follows. In Section II we develop our analytical model for the TCP throughput focusing on 802.16 MAC layer characteristics. Then, based on this model, in Section III we evaluate the aggregate throughput of TCP flows as a function of parameters such as: number of MSs, BS queue sizes and length of the T16 timer. Results given by the model are validated by means of ns-2 simulations. In Section IV, we briefly survey different works related to the bandwidth-request contention mechanism. Finally, in Section V we conclude the paper.

II. MATHEMATICAL MODEL

In this section we propose a simple model for the performance of best-effort TCP flows over WiMAX networks. With this model, the effect of the BS desynchronization problem on TCP performance is isolated. To this end, we take into account and model several MAC layer’s critical subsystems to finally offer a simple model for the TCP throughput. Later, in Section III, we will see how this model predicts the TCP throughput while varying characteristics such as the queue size and the number of stations. Such results may offer hints for correctly dimensioning 802.16 system parameters, like BS queue sizes.

A. Uplink delay

We will begin by computing the expectation of the uplink delay as a function of the bandwidth perception scheme. The behavior of the latter (in terms of mismatch between bandwidth requests and grants) is captured in a simple way by a single parameter \( q \), defined below. Assume there are \( N \) MSs in one IEEE 802.16e cell. The T16 timer value is denoted as \( T_{16} \), and there are \( n \) bandwidth request transmission opportunities in each frame, the duration of which is \( T_f \). For the sake of simplicity, we assume that the probability of collision in each slot is constant and that a transmission in each slot happens with equal probability, which is reasonable in the presence of long-lived traffic flows.

We denote the collision probability as \( p \), and the probability of no bandwidth being allocated to this MS by the BS as \( q \). Then, the failure probability of one BW-REQ is \( p_f = p + (1 - p)q \). The probability that a bandwidth request is both successfully received and managed\(^1\) by the BS in the \( i \)-th transmission attempt, \( i \geq 1 \), is \( p_f^{-1}(1 - p_f) \). Therefore, the average number of transmissions of a BW-REQ is:

\[
N_{tx} = \sum_{i=1}^{M} ip_f^{i-1}(1 - p_f) + Mp_f^M \tag{1}
\]

\[
= \sum_{i=0}^{M-1} (i+1)p_f^i - \sum_{i=1}^{M} ip_f^i + Mp_f^M = \sum_{i=1}^{M} p_f^{i-1}, \tag{2}
\]

where \( M \) is the maximum number of retransmissions.

If a BW-REQ packet is successfully received and managed by the BS in the \( i \)-th transmission, then the average number of waiting slots for this BW-REQ is:

\[
N_{s,i} = \frac{nT_{16}}{T_f} (i - 1) + \frac{W_0}{2} + \ldots + \frac{2^{i-1}W_0}{2} \tag{3}
\]

\[
= \frac{nT_{16}}{T_f} (i - 1) + \frac{W_0}{2}(2^i - 1), \tag{4}
\]

where \( W_0 \) is the initial contention window size. Therefore, the average number of waiting slots until the end of current contention period is:

\[
N_s = \sum_{i=1}^{M} (p_f^{i-1}(1 - p_f)N_{s,i}) + p_f^M (\frac{nT_{16}}{T_f}M + \frac{W_0}{2}(2^M - 1))
\]

\[
= \frac{W_0}{2} - p_f^M W_0 2^M - 1 + \sum_{i=1}^{M} p_f^i(\frac{nT_{16}}{T_f} + W_0 2^{i-1}) \tag{5}
\]

Therefore, the probability of transmission in a slot is given by:

\[
p_{tr} = \frac{N_{tx}}{N_s + T_{idle}} \tag{6}
\]

where \( T_{idle} \) is the average number of slots before next request. We assume all nodes are identical in terms of application

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\(^{1}\)By “successfully managed”, we mean that the perception management scheme allocates the requested bandwidth to the MS.
type and backoff scheme, so the transmission probability is the same for all nodes. Thus, the probability of collision can be expressed as

$$p = 1 - (1 - p_{tr})^{N-1}$$

(5)

The collision probability $p$ of every transmitted BW-REQ packet can be obtained by solving the above fixed-point equation, which then can be used to compute $N_{CW}$ according to (3). The average uplink delay can thus be expressed by:

$$E[D_u] = \frac{N_{BS}T_f}{n}$$

(6)

B. Congestion window, BS queue and packet losses

In this section, we will develop an expression for the expectation of TCP's congestion window size of last round in a TCP cycle, which is denoted by $N_{CW}$. Assume that the BS is the bottleneck for TCP traffic. For the downlink queue in the BS, the arrival rate is $N_{CW}P_{tr}T_{RTT}$, where $T_{RTT}$ is the round-trip time, $P_{tr}$ is the TCP packet size. $T_{RTT}$ includes the delay $D_{wired}$ on the wired network, the downlink delay $D_d$ and the uplink delay $D_u$ on the wireless link. Since we assume the bottleneck for TCP traffic is the wireless link, $D_{wired}$ is considered to be constant. In a TCP cycle, when the congestion window size of last round in a TCP cycle, which is denoted by $N_{CW}$. Assume that the BS is the bottleneck for TCP traffic. For the downlink queue in the BS, the arrival rate is $N_{CW}P_{tr}T_{RTT}$, where $T_{RTT}$ is the round-trip time, $P_{tr}$ is the TCP packet size. $T_{RTT}$ includes the delay $D_{wired}$ on the wired network, the downlink delay $D_d$ and the uplink delay $D_u$ on the wireless link. Since we assume the bottleneck for TCP traffic is the wireless link, $D_{wired}$ is considered to be constant. In a TCP cycle, when the congestion window size is smaller than the service rate (the download transmitting rate), the queue length reaches the queue capacity ($Q_{BS}$). Therefore, the average queue length can be considered as $Q_{BS}/2$. The downlink delay for user $k$ consists mainly of the queueing delay at the BS, which can be expressed as:

$$E[D_d] = \frac{P_{tr}Q_{BS}}{r_k/N}$$

(7)

where $r_k$ means the transmitting rate for user $k$, and $N$ means the number of MSs. The service rate for the BS queue is $r_{BS}/N$, in case of slot fair scheduling.

When the arrival rate at the BS queue is lower than the service rate, the queue can be emptied in downloading transmission. When TCP increases its congestion window size, the arrival rate increases. When the arrival rate gets larger than the service rate, the queue begins to grow until packets are dropped. Henceforth, in a TCP cycle, TCP congestion window size increases from 1 (or half of previous congestion window size) to $N_{CW}$, and packet drop occurs, which ends current TCP cycle. As a result, $N_{CW}$ is around the value that arrival rate equals serving rate,

$$\frac{N_{CW}P_{tr}}{2D_{wired} + D_d + D_u} = \frac{r}{N}$$

(8)

By solving the above equation, we can get the expectation of maximum TCP congestion window size, which is expressed as:

$$N_{CW} = \frac{Q_{BS}}{2} + \frac{r_k}{NP_{tr}}(D_u + D_{wired})$$

(9)

2A cycle starts when the TCP sender enters the congestion avoidance phase and ends when the sender receives three duplicate ACKs.

When TCP congestion window size reaches $N_{CW}$, BS queue overflow occurs and packets are dropped. Therefore, the probability of packet loss event can be expressed as:

$$p_{\text{lost}} = \frac{1}{b(N_{CW} + \cdots + N_{CW})} = \frac{3}{4b} \frac{N_{CW}(N_{CW} + 1)}{2}$$

(10)

where $b$ is the number of packets that are acknowledged by a received ACK.

C. TCP Throughput

Based on the results in previous sections, we can now obtain an expression for the throughput of TCP flows in the system. TCP throughput can be modeled in a simple way by the following expression, directly derived from the well-known equation by Padhye et al. [6]:

$$B = \frac{P_{tr}(E[Y] + Q * E[Y_{TO}])}{E[A] + Q * E[A_{TO}]} = \frac{P_{tr}\left(1 - p_{\text{lost}}\right) + N_{CW} + Q \frac{1}{T_{RTT}(N_{CW} + 1) + QT_{PTO}f(p_{\text{lost}})}}{T_{RTT}(N_{CW} + 1) + QT_{RTO}\frac{f(p_{\text{lost}})}{1 - p_{\text{lost}}}}$$

(11)

where $Y$ and $A$ are the number of packets sent and the duration of a TCP cycle, respectively. $Q$ is the probability that a loss is signaled by a timeout. $Y_{TO}$ and $A_{TO}$ are the number of packets sent and the period between the timeout and the end of the slow start phase. $T_{RTT}$ is the TCP retransmission timeout, and $f(p_{\text{lost}})$ is given by:

$$f(p_{\text{lost}}) = \sum_{i=1}^{M_{TO}} (2p_{\text{loss}})^{i-1}$$

(12)

where $M_{TO}$ is the maximum number of TCP retransmissions of timeout.

1) Asymptotic Throughput: As the queue size increases, the maximum TCP congestion window size increases and thus the number of data packets sent increases. However, the queue delay at the BS also increases, so there should exist an upper bound for TCP throughput. From (11), we can get the asymptotic throughput for user $k$:

$$\lim_{Q_{BS} \to \infty} \frac{B_k}{\sum_{i=1}^{M_{TO}} (2p_{\text{loss}})^{i-1}} = \frac{P_{tr}\left(1 - p_{\text{lost}}\right) + N_{CW} + \frac{3}{4}N_{CW}\frac{1}{2}}{T_{RTT}(N_{CW} + 1) + \frac{3}{4}N_{CW}T_{RTO}\frac{f(p_{\text{lost}})}{1 - p_{\text{lost}}}}$$

(13)

Then the numerator and the denominator can be expressed as follows:

$$\lim_{Q_{BS} \to \infty} \frac{P_{tr}\left(1 - p_{\text{lost}}\right) + \frac{3}{4}N_{CW}\frac{1}{2}}{T_{RTT}(N_{CW} + 1) + \frac{3}{4}N_{CW}T_{RTO}\frac{f(p_{\text{lost}})}{1 - p_{\text{lost}}}} = \frac{3}{8}P_{tr}b$$

$$\lim_{Q_{BS} \to \infty} \frac{1}{T_{RTT}(N_{CW} + 1) + \frac{3}{4}N_{CW}T_{RTO}\frac{f(p_{\text{lost}})}{1 - p_{\text{lost}}}} = \frac{N_{BS}P_{tr}}{r_k}$$

(14)

Finally, the asymptotic throughput with infinite queue length limit is
\[ \lim_{Q_{BS} \to \infty} B_k = \frac{3 r_k}{4 N} \] (15)

This means that as queue length limit increases to infinity, no matter what kind of bandwidth perception scheme is applied, TCP throughput reaches an upper bound, which is just the transmitting rate achieved by the MS. It also means that when queue length limit is small, TCP throughput increases as queue length limit increases. Finally remark that, when the queue length limit is large, TCP throughput is constrained by the BS transmitting rate.

III. PERFORMANCE EVALUATION AND VALIDATION

In this section we present \( ns - 2 \) simulation results that validate the model developed in the previous section. Our objective is twofold. First, we corroborate what the model predicts, i.e., the sensitivity of the TCP throughput to different values of system parameters such as \( T16 \), queue length and number of MSs. Second, we observe how the failure ratio \( q \) synthesizes the misperceived bandwidth needs of the BS. We remark that specific values of \( q \) have been found through simulation for this specific long-lived flows scenario. Thus, different TCP traffic configuration may give a different \( q \) set.

These simulation results, compared with our analytical model, give a fairly good matching. We have chosen \( q = 0.3 \) for DDA-i, meaning that there is a non-negligible probability of getting out of sync. For the other three policies we have set \( q = 0 \) since every BW-REQ was served with at least the minimum number of bandwidth slots.

Fig. 1 shows the BW-REQ failure ratio which corresponds to a desynchronization metric between the BS and the MSs in a steady state of a long transfer. Note that for DDA-i there is a sustained failure ratio as the BW-REQ are not correctly served. On the contrary, observe that the failure rate is 0 for RPG, DPG and DDA-d. Although there is also a possibility that the allocated uplink bandwidth is smaller than MSs’ demands (another form of desynchronization), the failure ratio only accounts for scenarios in which BS does not allocate any uplink bandwidth.

Fig. 2 shows aggregated TCP throughput of downlink-only traffic for different number of MSs. From the observations, our analytical model predicts the throughput fairly well. We remark however, that the analytical throughput is a little larger than that of simulation. This is because we have not take into consideration the signaling overhead in downlink subframe (e.g., the overhead of UL-MAP and DL-MAP). In addition, not all downlink bandwidth is allocated in simulation, which contributes to a lower throughput for simulations. Remark that for RPG, DPG and DDA-i, throughput with 2 MSs is less than that of 1 MS. This is because from 2 MSs on, the collision frequency of BW-REQ packets increases considerably, therefore increasing the probability of request transmission failure. This in turn, decreases the downloading TCP throughput. For DDA-i, throughput is much lower than those of other bandwidth perception schemes. This is because, when compared with
other policies, DDA-i’s desynchronization problem increases the probability that the BS does not allocate any bandwidth to a demanding MS.

Fig. 3 gives the analysis results and simulation results of aggregated TCP throughput with different MAC queue length limit. From Fig. 3b, as the queue length limit increases, throughput increases. When the queue length limit is larger than 10, throughputs of RPG, DPG, DDA-d reaches the maximum throughput and keeps stable. This means that when queue length limit is less than 10, then maximum congestion window size is constrained by queue length limit and TCP throughput is guided by queue length limit. When queue length limit is larger than 10, TCP throughput is constrained by BS transmitting rate. The analytical results in 3a also give similar results.

Finally, Fig. 4 gives analysis and simulation results of aggregated TCP throughput with different T16 timer timeout. From Fig. 4b, when T16 timer is small, TCP performance is better than larger T16 timeout. This is because an MS retransmits its BW-REQ quickly, and thus BS bandwidth perception can get synchronized quickly with the MS’s bandwidth demand. From Fig. 4a, our analytical results give similar results.

IV. RELATED WORK

In order to improve BE traffic performance, researchers have rather focused on other critical subsystems of the bandwidth request-grant mechanism, namely: the length or the repartition of the contention period, and the sending frequency of BW-REQs.

Cho et al. [7] show that maximum throughput for a number $N$ of uploading BE connections is reached when the contention window size (in number of slots) is equal to $N$. Similarly, Ni et al. [2] focus their study on the mean number of frames required to successfully send BW-REQs using contention or unicast polling. For a fixed contention window size, they show that periodical contention is more efficient than unicast polling for a small BW-REQ arrival rate. On the other hand, for a larger number of MSs, multicast/broadcast polling is more efficient when the arrival rate of BW-REQ increases. Moreover, when the truncated-binary exponential back-off (BEB) parameters are well chosen, the delay it takes to successfully send a BW-REQ in contention mode is bounded.

Vinel et al. [8] show the benefits of grouping all Connection Identifier (CID) bandwidth needs by MSs, to reduce the total number of BW-REQs and to improve the mean delay to pass them to the BS. Recently, Delicado et al. [9] have also found that this optimization reduces the number of BW-REQs, improving the uplink throughput at the expense of detailed information on the individual connections. Another way of treating BW-REQs is proposed in [10]. The authors propose to divide the contention period in two non-overlapping subsets. The first period is used to send new BW-REQs, and the second period to solve collisions. This method allows decreasing the
mean delay time to successfully transmit BW-REQs, compared to the legacy BEB.

Delicado et al. [11] propose to independently adapt the contention windows (CW) of the MSs to reduce the number of collisions. As an extension to the standard, each connection sends its BW-REQs in an adapted number of frames once the MS finds the adequate size of the CW (i.e., by exponentially increasing the CW). As a result, the different uplink BW-REQs experience less collisions. Moreover, after a silence period of a flow, a connection uses an adapted (reduced) back-off period based on the last size of the CW in which a BW-REQ was transmitted with success.

On the other hand, there have been few works on WiMAX performance models that explicitly include an analysis of the uplink request-grant scheme. In [12] and [13], the authors propose analytical models for studying the contention based BW-REQ and grant scheme in IEEE 802.16. [12] use a two-dimension Markov to model the back-off procedure when transmitting BW-REQ packets. [13] give an analytical model when subchannelization is enabled. In [14], the authors give a WiMAX analytical model based on a Markovian model, whose states represent the number of MSs with an active traffic. However, the uplink scheme and its impact to downlink traffic are not considered in this model.

V. CONCLUSION AND FUTURE WORK

In this paper we have proposed an analytical model of the performance of best-effort TCP flows over WiMAX networks. Such model aims at capturing the impact of uplink bandwidth request-grant schemes, and especially the influence of bandwidth perception policies at the BS. By analyzing the evolution of BS queue length, we also give the probability of TCP packet loss. Finally, we have developed expressions for TCP performance in terms of throughput. These results may help in obtaining guidelines for the dimensioning of 802.16 system parameters, like the size of BS queues, which may have a significant effect on the performance of users’ application flows.

As a future work we plan to extend the model so that it captures better the different bandwidth perception policies, not only by having more than a single parameter $q$ for capturing the out-of-sync phenomenon but also by considering more sophisticated TCP throughput models.

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APPENDIX

BANDWIDTH-PERCEPTION MANAGEMENT SCHEMES

The following four bandwidth perception schemes are considered in this paper (see [5] for details):

- **Reset Per Grant (RPG):** after granting bandwidth to an MS, the BS resets to zero the bandwidth perception for the served MS in the allocation table.

- **Decrease at Data Arrival with immediate BW-REQ handling (DDA-i):** this policy decreases the perception upon the arrival of successfully transferred data to the BS. The allocation table is updated as soon as BW-REQs are received at the BS.

- **Decrease at Data Arrival with delayed BW-REQ handling (DDA-d):** this policy behaves mostly like DDA-i but the BS scheduler delays the BW-REQs handling.

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