Efficiency Evaluation for Supply Chains Using Maximin Decision Support
Desheng Dash Wu, Cuicui Luo, and David L. Olson

Abstract—The outputs of upstream individual processes (members) become the inputs of downstream members in supply chains. When multiple inputs and outputs are present, data envelopment analysis has been widely applied to assess efficiency. In cooperative groups, such as supply chains, a maximin decision approach can reflect not only overall system efficiency, but also efficiency of system elements. This paper discusses a maximin efficiency multistage supply chain model capable of measuring supply chain members performance as well as overall supply chain performance.

Index Terms—Data envelopment analysis (DEA), efficiency, maximin criteria, supply chain.

I. INTRODUCTION

As the manufacturing environment becomes more and more competitive and interrelated, supply chain management (SCM) has become a topic of interest to many researchers. Data envelopment analysis (DEA) has been applied to many aspects of supply chains. Much of this research has focused on components of the supply chain, such as evaluation of vendors [1], purchasing [2], both [3], or production [4], [5]. The impact of enterprise resource planning systems in support of supply chains has also been evaluated through DEA [6], [7]. However, a supply chain can be identified as efficient by DEA models while components may be inefficient [8]. Although some efforts have been made to integrate these individual components into a single scenario, very limited progress has been accomplished since most of the tradeoffs and the relationships among different supply chain components are often unknown. It is rare that one single performance measure can suffice for the purpose of performance evaluation, and thus, DEA is a useful tool for measuring supply chain performance. Thus, multiple criteria DEA models have been proposed for supply chain evaluation [9], [10], to include analytic hierarchy process [11]–[13], and even case-based reasoning models to assess supplier sourcing [14]. Liu and Hipel [15] used network optimization models to select quality control strategies for a product supply chain. Chow [16] presented a probabilistic model for a supply chain’s inventory policy. Schoenherr and Talhari [17] applied DEA for analysis of environmental sustainability across transcontinental supply chains. This research uses both stochastic and deterministic methods for an integrated supply chain evaluation model.

Efficient and effective SCM strategy involves tradeoffs among concepts, such as value maximization, process integration, responsiveness improvement, and cycle time reduction. Effective SCM calls for the comprehensive collaboration among individual members to coordinate manufacturing, logistics, and materials management functions within a supply chain [18], and thus, to enhance competitive advantage. When all individual members (strategic organizations) in the value chain integrate and act as a single unified entity, performance is enhanced throughout the supply chain system.

Several conceptual frameworks have been proposed for evaluating complete supply chain performance measurement (SCPM) in the literature [19]. Supply chain performance measurement differs from classical performance measurements, such as supplier assessment in several aspects. First, there are two levels of supply chain performance. The first level is the performance of individual supply chain members. The second level is the performance of an entire supply chain system, which are determined and characterized by individual supply chain members. Performance at both levels may change as a result of changes in the supply chain structure. Systematic performance should be taken into account as well as subsystematic performance in the evaluation process. Second, management process at the top level should be emphasized, instead of simply analyzing organization components. Third, linkage between different supply chain individuals is very important [20].

An integrated supply chain can be viewed as a P-stage serial system composed of P processes plotted in Fig. 1. The supply chain is considered to be a set of linked processes connecting downstream processes (e.g., customers) to upstream processes (e.g., suppliers, factories, distribution centers, and retailers). DEA is an effective approach in estimating the empirical efficient frontiers and measuring the relative efficiency of peer units [21] as well as estimation of overall supply chain performance [22].

Examples of the use of DEA to estimate performance of supply chain components, include Wu and Blackhurst [23], who demonstrated the use of DEA as a tool for measuring the performance of vendors on multiple criteria and for use in vendor negotiations. Narasimhan et al. [24] proposed an approach for effective supplier performance based on the
Theorem 3.3: \[ E_p^j = \sqrt{Y_j^p Y_j^T} + \delta_{j}^{p+1} Y_j \] where \( \delta_{j}^{p+1} \) is defined as in Theorem 3.2.
States of nature (input, output variables)

<table>
<thead>
<tr>
<th>States</th>
<th>V1</th>
<th>V2</th>
<th>V3</th>
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<tr>
<td>S3</td>
<td>c_4</td>
<td>c_5</td>
<td>c_6</td>
</tr>
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</table>

Serial chain

\[
Sj = c_k, c_{k-1}, \ldots, c_1
\]

Fig. 2. Generic serial chain decision table.

where \( \delta = 0, i(f, p) = 1; \delta = 1, i(f, p) = 2, \ldots, P, V^f = \{v_1^f, v_2^f, \ldots, v_n^f\} \) and \( U^p = \{u_1^p, u_2^p, \ldots, u_m^p\} \) are the non-negativity weight vector of inputs and outputs associate with Process \( p \), respectively. For example, in Fig. 1, the efficiency given to the first subprocess of the \( j \)th supply chain is

\[
E_j^1 = \frac{U^{1T} Y_j^1}{V^{1T} X_j^1}
\]

and the efficiency given to the second subprocess efficiency of the \( j \)th supply chain is

\[
E_j^2 = \frac{U^{2T} Y_j^2}{V^{2T} X_j^2 + U^{1T} Y_j^1}
\]

(refer to Fig. 1).

Now, we introduce a maximin criterion as a means to reach a maximin efficiency model. Assume that the serial (supply) chain involves perfect knowledge of the entire set of the states (input, output variables), but that evaluation of the probability associated with the realization of each of each state is not possible. Then, a decision problem under complete ignorance can be easily represented through a decision table, as shown in Fig. 2.

\( S_{ij} (j = 1, 2, \ldots, J) \) represents all the units of interests, \( V_3 \) (for \( k = 1, 2, \ldots, K) \) represents the generic state of nature, and \( c_{ij} \) is the consequence or payoff associated with unit \( j \) and variable/state \( k \). Specifically, for the \( p \)th subprocess, we have \( C_{ik} = \{v_i^f, y_j^p\} \), where

\[
i \in \{1, 2, \ldots, m\}, \quad r \in \{1, 2, \ldots, s\}, \quad j \in \{1, 2, \ldots, J\},
\]

\[
k \in \{1, 2, \ldots, K\} \subset \{1, 2, \ldots, \} \cup \{1, 2, \ldots, s\}.
\]

The maximin criterion (MMC) works as follows. For each unit \( S_j \), the decision maker identifies the minimum payoff over the set of states of nature

\[
S_j = \min_{i \in \{1, \ldots, K\}} \{C_{ik}\}
\]

(2)

where \( S_j \) is usually called the security level of the decision [36]. Then, she chooses the act for which this security level is the highest: Choose \( S_p \) such that

\[
S_p = \max_{j \in \{1, \ldots, J\}} \{S_j(\min_{i \in \{1, \ldots, K\}} \{C_{ik}\})\}
\]

(3)

If the payoff in (2) is defined as the efficiency given to subunit \( p \) in (1), then, we yield the maximin efficiency model for subunit \( p \)

\[
S_p^* = \max_{j \in \{1, \ldots, J\}} \{E_j(\min_{i \in \{1, \ldots, K\}} \{C_{ik}\})\}
\]

DMU\( j \)'s maximin strategy is a strategy that maximizes \( j \)'s worst-case payoff, in the situation where all DMUs happen to play the strategies, which cause the greatest harm to \( j \). The maximin value or safety level of the game for DMU\( j \) is that minimum amount of payoff governed by a maximin strategy.

Note that in Problem (3) we define the minimum efficiency level from the efficiency set attached to all units as the security level, and yield the optimized efficiency by choosing an optimal weight strategy from the weight set subject to some constraints.

Let

\[
\beta^p = \min_{j \in \{1, \ldots, J\}} \{E_j^p\} = \min_{j \in \{1, \ldots, J\}} \left\{ \frac{U^{jT} Y_j^p}{V^{jT} X_j^p + \delta(U^{jT} Y_j^p)} \right\}
\]

then, (3) is converted to the following programming model:

\[
\begin{align*}
& \max_{\beta^p} \quad \beta^p \\
\text{s.t.} \quad & \omega^p X_j + \mu^p Y_j^p \geq \omega^p X_j + \mu^p Y_j^p \geq 0, \quad j = 1, 2, \ldots, n \quad (5.1) \\
& \beta^p (U^{jT} Y_j^p + \mu^p Y_j^p) - \mu^p Y_j^p \leq 0 \quad (5.2) \\
& \omega^p X_j + \omega^p Y_j^p = 1 \quad (5.3) \\
& \omega^p \geq 0, \omega^p \geq 0, \mu^p \geq 0 \\
& \delta = 0, \quad \text{if} \quad p = 1, \delta = 1, \quad \text{if} \quad p = 2, \ldots, P \quad (5)
\end{align*}
\]

In (5.1), \( \omega^p \) and \( \omega^p \) are input weight vectors associated with the direct input \( X_j^p \) and intermediate input \( Y_j^{p-1} \) (also outputs produced at the \( p-1 \)th Process), respectively. \( \mu^p \) is the output weight vector associated with the output \( Y_j^p \). Note that the DEA efficiency is upwardly bounded by the unity value by use of constraint (5.1). Constraint (5.2) impose a lower bound of \( \beta^p \), i.e., the minimal DEA efficiency, to each DEA efficiency. Constraint (5.3) means a certain normalization yielded by performing the Charnes–Cooper transformation [37].

In model (5), when we change the evaluated DMU\( j \) (i.e., \( X_j^p \) and \( Y_j^{p-1} \) are changed in the constraints of above problem), different optimal solutions of \( \omega^p \), \( \omega^p \) and \( \mu^p \) are obtained.

Suppose \( \{\beta^1, \omega^1, \omega^1, \mu^1, \mu^1\} \) is an optimal solution to model (5), where \( d \) denotes the DMU under evaluation. The set of all DMUs is then divided into two groups DMU\( 1 \) and DMU\( 2 \) as follows:

\[
\begin{align*}
\text{DMU}_1^p &= \left\{ \text{DMU}_j^p | \beta^p (U^{jT} Y_j^p + \delta U^{jT} Y_j^p) - \mu^p Y_j^p \geq 0 \right\} \quad (6) \\
\text{DMU}_2^p &= \left\{ \text{DMU}_j^p | \beta^p (U^{jT} Y_j^p + \delta U^{jT} Y_j^p) - \mu^p Y_j^p > 0 \right\} \quad (7)
\end{align*}
\]

Then, model (5) can be reduced to the following model (8) if we replace the inequality constraint in model (5) by equality.
constraints associated with the DMUs in DMU1:

Max $\beta_j^p$
\[ \text{s.t. } \alpha_1^pT Y_{1} + \mu_1^pT Y_{1} - \mu_1T Y_{1} \geq 0, \text{ for } \text{DMU}_1^p \in \text{DMU}_1^p \]
$\beta_j^p (\alpha_1^pT X_{1} + \beta_1^pT Y_{1} - \beta_1T Y_{1}) - \mu_1^pT Y_{1} \leq 0 \text{ for } \text{DMU}_1^p \notin \text{DMU}_1^p$
$\beta_j^p (\alpha_1^pT X_{1} + \beta_1^pT Y_{1} - \beta_1T Y_{1}) - \mu_1^pT Y_{1} = 0 \text{ for } \text{DMU}_1^p \notin \text{DMU}_1^p$

(8)

After solving model (8), the evaluated DMUs can be further divided into three classes according to the different efficiency given to each of them. Then another model similar to model (8) is yielded and a new solution leads to further partition of DMUs under evaluation. The procedure is repeated until all the inequality constraints (5.2) in model (5) become the following equality constraints:

$\beta_j^p (\alpha_1^pT X_{1} + \beta_1^pT Y_{1} - \beta_1T Y_{1}) - \mu_1^pT Y_{1} = 0, \text{ for } j = 1, 2, \ldots, n \tag{9}$

where $\delta = 0$, if $p = 1; \delta = 1$, if $p = 2, \ldots, P$.

Thus, efficiency equilibrium is achieved based on (9). This equilibrium is obtained for individual chain members (stages), but not for the overall supply chain system. The cooperation among adjacent processes is not taken into account, thus, we name this kind of equilibrium efficiency as independent equilibrium efficiency (IEE).

Let

$W_{dp}^a = (\alpha_d^pT X_{d} + \beta_d^pT Y_{d} - \beta_dT Y_{d}), d = 1, 2, \ldots, n \tag{10}$

be the solution to the problem using the maximin principle with constraint (9) optimized for DMU_d

IEE_d^p (W_d) = \frac{\mu_1^pT Y_{1}}{\alpha_d^pT X_{d} + \beta_d^pT Y_{d} - \beta_dT Y_{d}}.

\[ \delta = 0, \text{ if } p = 1; \delta = 1, \text{ if } p = 2, \ldots, P. \tag{11} \]

If $d \neq j$, then IEE_j^p (W_j) denotes the DEA-efficiency. Then, we have:

Theorem 1: Repeatedly solving model (5) until (9) is yielded, or, solving the following problem (12) using the maximin principle with constraints (9), we yield n optimal weight strategies $W_{dp}^{\alpha_d} = (\alpha_{d1}^p, \alpha_{d2}^p, \ldots, \alpha_{dn}^p), d = 1, 2, \ldots, n$, such that each DMU, has a maximin equilibrium efficiency of $\text{IEE}_{dp}^\alpha (W_{dp}^{\alpha_d}) = \beta_j^p$.

Max $\sum_{d=1}^{n} \beta_j^p$
\[ \text{s.t. } \beta_j^p (\alpha_1^pT X_{1} + \beta_1^pT Y_{1} - \beta_1T Y_{1}) - \mu_1^pT Y_{1} = 0, \text{ for all } j \]
\[ \alpha_1^pT Y_{1} + \mu_1^pT Y_{1} - \mu_1T Y_{1} = 0, \text{ for } \text{DMU}_1^p \in \text{DMU}_1^p \]
\[ \delta = 0, \text{ if } p = 1; \delta = 1, \text{ if } p = 2, \ldots, P. \tag{12} \]

Theorem 1 provides a way of generating Nash equilibrium solution for a performance evaluation problem under maximin criterion. In other words, DMUs under evaluation do not have incentive to deviate from a strategy yielded from Theorem 1 unless extra payoffs are offered to DMUs. We now apply this model to gauge the performance of a supply chain in next section.

III. MAXIMIN EFFICIENCY SUPPLY CHAIN MODEL

The supply chain efficiency in this paper is defined as the weighted sum efficiency of its individual members (process). Formally, the supply chain efficiency is defined by

$SE_j = \sum_{p=1}^{P} w_j^p E_j^p = \sum_{p=1}^{P} w_j^p \frac{U_j^pT Y_j}{\sqrt{V_j^pT X_j + V_j^pT Y_j}} \tag{13}$

where $w_j^p$ is the user-specified weight for the $p$th subprocess, reflecting the preference over supply chain member’s performance, and $E_j^p$ is defined in (1).

Now consider the following model:

Max $\sum_{d=1}^{n} \sum_{p=1}^{P} \frac{w_j^p}{\alpha_d^pT X_{d} + \beta_d^pT Y_{d} - \beta_dT Y_{d}}$
\[ \text{s.t. } \frac{\alpha_j^pT Y_{j}}{\alpha_1^pT X_{1} + \beta_1^pT Y_{1} - \beta_1T Y_{1}} \leq 1, \text{ for } j = 1, 2, \ldots, n \]
\[ \frac{\alpha_j^pT Y_{j}}{\alpha_1^pT X_{1} + \beta_1^pT Y_{1} - \beta_1T Y_{1}} \leq 1, \text{ for all } p = 2, 3, \ldots, P \]
\[ U_j^pT \geq 0, V_j^pT \geq 0, V_j^pT \geq 0. \tag{14} \]

In model (4), the objective function seeks to maximize the weighted average sum of each process’s minimal efficiency, i.e., a principle of maximizing the weighted average sum of each process’s minimal efficiency (maximin principle) is utilized here.

Let

$\alpha_j^1 = \min_{j \in [1, n]} \frac{U_j^1T Y_j}{V_j^1T X_j}, \alpha_j^p = \min_{j \in [1, n]} \frac{U_j^pT Y_j}{V_j^pT X_j + U_j^pT Y_j} \tag{15}$

then programming (14) can be stated as

Max $\sum_{d=1}^{n} \sum_{p=1}^{P} \frac{w_j^p}{\alpha_d^pT X_{d} + \beta_d^pT Y_{d} - \beta_dT Y_{d}}$
\[ \text{s.t. } 0 \leq \alpha_j^p \leq \frac{\alpha_j^pT Y_j}{\alpha_1^pT X_{1} + \beta_1^pT Y_{1} - \beta_1T Y_{1}} \leq 1, \text{ for } j = 1, 2, \ldots, n \]
\[ \frac{\alpha_j^pT Y_{j}}{\alpha_1^pT X_{1} + \beta_1^pT Y_{1} - \beta_1T Y_{1}} \leq 1, \text{ for all } p = 2, 3, \ldots, P \]
\[ U_j^T \geq 0, V_j^pT \geq 0, V_j^pT \geq 0. \]
Note that DMU_d represents the DMU under evaluation, we perform the Charnes–Cooper transformation, i.e., we let

\[ t^p = \frac{1}{\delta^p U^p} \]

\[ t^d = \frac{1}{\delta^d U^d} \]

\[ a^p = t^p V^p \]

\[ a^d = t^d V^d \]

\[ \mu^d = t^d U^d \]

and \( \mu^p = t^p U^p \). We note that \( t^p \) is proportional to \( t^d \) that is, for \( \exists k_{p}^{(p-1)} \geq 0 \) such that \( t^p = k_{p}^{(p-1)} t^d \) is satisfied. Therefore, model (15) can be transformed into the following problem:

\[
\begin{align*}
\text{Max} & \quad \frac{1}{n} \sum_{p=1}^{n} w_p \delta^p a^p \\
\text{s.t.} & \quad \omega^p X^p - \mu^p Y^p \geq 0, \quad j = 1, 2, \ldots, n \\
& \quad \omega^d X^d + k_{p}^{(p-1)} (\mu^d Y^d - \mu^p Y^p) \geq 0 \\
& \quad \omega^d (\omega^d X^d + k_{p}^{(p-1)} (\mu^d Y^d - \mu^p Y^p)) - \mu^p Y^p \geq 0 \\
& \quad \omega^d X^d + k_{p}^{(p-1)} (\mu^d Y^d - \mu^p Y^p) \geq 0 \\
& \quad \omega^d X^d + k_{p}^{(p-1)} (\mu^d Y^d - \mu^p Y^p) = 1, \quad p = 2, 3, \ldots, P \\
& \quad \omega^d X^d + k_{p}^{(p-1)} (\mu^d Y^d - \mu^p Y^p) \geq 0.
\end{align*}
\]

Employing the same procedure in Section II, the following equilibrium (17) are obtained:

\[
\begin{align*}
& \omega^d (\omega^d X^d + k_{p}^{(p-1)} (\mu^d Y^d - \mu^p Y^p)) - \mu^p Y^p = 0, \quad j = 1, 2, \ldots, n \\
& \delta^d = 0, \quad \delta^p = 1, \quad \delta^d = 1, \quad \delta^p = p, \ldots, P.
\end{align*}
\]

Let

\[ W^p = (\omega^p t^p A^p, \mu^p t^p A^p, \mu^p t^p A^p, k_{p}^{(p-1)} t^p A^p) \]

\[ W^d = (\omega^d t^d A^d, \mu^d t^d A^d, \mu^d t^d A^d, k_{p}^{(p-1)} t^d A^d) \]

be the solution to the problem using the maximin principle with constraints (17) solved for DMU_d and

\[ CEE^d (W^d) = \sum_{j=1}^{n} \omega^d t^d X^j + k_{p}^{(p-1)} t^d Y^j A^j \]

\( j = 1, 2, \ldots, n, p = 1, 2, 3, \ldots, P \).

If \( d \neq j \), then CEE^j (W^j) \((p = 1, 2, 3, \ldots, P)\) is the cross efficiency given to DMU_j when DMU_d is under evaluation.

Theorem 2: Repeatedly solving model (16) until (17) is yielded, or, solving the problem using the maximin principle with constraints of (17), we get n optimal weight strategies

\[ W^d = (\omega^p t^p A^p, \mu^p t^p A^p, \mu^p t^p A^p, k_{p}^{(p-1)} t^p A^p) \] such that each DMU_j has an equilibrium efficiency of

\[ CEE^j (W^j) = \omega^j t^j. \]

Similarly, Theorem 2 provides a way of generating Nash equilibrium solution for a supply chain performance evaluation problem under maximin criterion, which means supply chains under evaluation do not have incentive to deviate from a strategy yielded from Theorem 2.

Definition 1: (Coordinated maximin equilibrium efficiency for members) Suppose

\[ W^d = (\omega^p t^p A^p, \mu^p t^p A^p, \mu^p t^p A^p, k_{p}^{(p-1)} t^p A^p) \]

\[ \delta^d = 0, \quad \delta^p = 1, \quad \delta^d = 1, \quad \delta^p = p, \ldots, P. \]

\[ CEE^d (W^d) = \frac{\mu^p t^p A^p}{\omega^d t^d A^d + \delta^p t^p A^p - \mu^d t^d A^d} \]

\[ \forall d, j = 1, 2, \ldots, n, d \neq j \]

we call CEE^j (W^j) coordinated maximin equilibrium efficiency given to the jth process stage of the jth supply chain if CEE^j (W^j) = CEE^j (W^j).

Definition 2: Coordinated equilibrium efficiency (for the supply chain) we call CEE_j coordinated equilibrium efficiency given to the jth supply chain (DMU_j) if

\[ CEE = \frac{1}{\sum_{p=1}^{n} w_p} CEE^d (W^d^p). \]

Note that \( \omega^d (\omega^d t^d X^d + k_{p}^{(p-1)} t^d Y^d - \mu^p Y^p) \geq 0 \), where \( \delta^d = 0, \quad \delta^p = 1, \quad \delta^d = 1, \quad \delta^p = p, \ldots, P \). Therefore, we have

Corollary 1:

(I) \( a^p t^p = a^d t^d A^p, \mu^p t^p = a^d t^d A^p \)

\[ A^p = \left\{ \begin{array}{ll}
\frac{1}{\omega^d t^d A^d}, & p = 1 \\
\frac{1}{\omega^d t^d A^d}, & p = 2, \ldots, P.
\end{array} \right. \]

(II) \( k_{p}^{(p-1)} t^p = k_{p}^{(p-1)} t^d A^p \)

\[ A^p = \frac{1}{\omega^d t^d A^d}. \]

This corollary reveals that when maximin equilibrium is achieved, the optimal solution to variables (including input and output weights and \( k_{p}^{(p-1)} \)) solved for different DMUs are proportional to each other.
Similar to model (12), the maximin equilibrium DEA efficiency for supply chains can be easily yielded by solving model (18) as follows:

\[
\begin{align*}
\text{Max} & \quad \omega_1^T, \omega_p^T, \mu_1^T, \mu_p^T, \kappa_p^T, \\
\text{s.t.} & \quad \alpha_j \sum_{p=2}^{P} \omega_p^T X^p_j - \mu_1^T Y^{j_1} = 0, \quad j = 1, 2, \ldots, n, \\
& \quad \omega_1^T X_j^1 + k^{p-1} \mu_1^T Y_p^{j_1} = 1, \quad p = 2, 3, \ldots, P, \\
& \quad \omega_1^T \geq 0, \mu_1^T \geq 0, \mu_p^T \geq 0, \quad k^{p-1} \geq 0.
\end{align*}
\]

Model (18)

**IV. Relations Between Supply Chain Maximin Efficiency and Substage Maximin Efficiency**

As an option to derive maximin equilibrium efficiency models (12) and (18) exhibit the advantage of simplicity in implementation. Naturally, our following discussion about the supply chain maximin equilibrium efficiency and substage maximin efficiency is based on these two models.

**Corollary 2:** \( CEE_j \geq \frac{1}{\sum_{i=1}^{P} w_i} (w_i \times IEE_j^p (W_j^p)) \) where \( CEE_j \) is defined in Definition 2 and \( IEE_j^p (W_j^p) = \beta_j^p \) is the optimal solution to model (12).

**Theorem 3:** Supply chain is coordinated maximin equilibrium DEA efficient, if and only if each process (member) in the supply chain is coordinated maximin equilibrium DEA efficient.

**Proof:** Sufficient condition: Suppose each process (member) of DMU \( j \) is maximin equilibrium DEA efficient, i.e., in model 12, for sub(p)-process, \( IEE_j^p (W_j^p) = 1 \), where \( W_j^p = (\omega_1^p, \omega_2^p, \mu_1^p, \mu_2^p, \kappa_p^p)^T \) is the corresponding optimal weights. Note that \( \omega_2^p \geq 0, \mu_1^p \geq 0, \mu_2^p \geq 0, \quad k^{p-1} \geq 0 \)

\[
\begin{align*}
\omega_1^p & = h^{p-1} \omega_1^{p-1}, \\
\mu_2^p & = h^{p-1} \mu_2^{p-1}, \quad \kappa_p^p = h^{p-1} \kappa_p^{p-1}.
\end{align*}
\]

(18)

\( (\omega_1^p, \omega_2^p, \mu_1^p, \mu_2^p, \kappa_p^p) \) is a feasible solution to model (18)

\( CEE_j = \frac{1}{\sum_{i=1}^{P} w_i} (w_i \times IEE_j^p (W_j^p)) = 1. \)

Therefore, the \( j \)-th supply chain (DMU \( j \)) is coordinated maximin equilibrium DEA efficient.

**Necessary condition:** Note that \( CEE_j = 1 \) if and only if \( CEE_j^p (W_j^p) = 1 \). Note also that \( CEE_j^p (W_j^p) = 1 \) and

**TABLE II**

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<th>Observation</th>
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<td>155</td>
<td>180</td>
<td>145</td>
<td>155</td>
<td>180</td>
</tr>
<tr>
<td>Operating cost ( X_j^2 )</td>
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<td>80</td>
<td>110</td>
<td>100</td>
<td>100</td>
<td>85</td>
<td>95</td>
<td>125</td>
</tr>
<tr>
<td>Revenue ( Y_j^1 )</td>
<td>35</td>
<td>25</td>
<td>23</td>
<td>24</td>
<td>27</td>
<td>25</td>
<td>28</td>
<td>35</td>
</tr>
<tr>
<td>Manufacturing cost ( X_j^3 )</td>
<td>218</td>
<td>190</td>
<td>180</td>
<td>205</td>
<td>185</td>
<td>180</td>
<td>190</td>
<td>180</td>
</tr>
<tr>
<td>Manufacturing lead time ( X_j^4 )</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>6</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Fill rate ( Y_j^2 )</td>
<td>0.7</td>
<td>0.9</td>
<td>0.78</td>
<td>0.88</td>
<td>0.73</td>
<td>0.95</td>
<td>0.89</td>
<td>0.87</td>
</tr>
<tr>
<td>Products shipped from manufacturer to retailer ( Y_j^3 )</td>
<td>800</td>
<td>700</td>
<td>750</td>
<td>500</td>
<td>650</td>
<td>550</td>
<td>650</td>
<td>600</td>
</tr>
<tr>
<td>Retailer inventory ( X_j^5 )</td>
<td>90</td>
<td>100</td>
<td>80</td>
<td>70</td>
<td>85</td>
<td>77</td>
<td>78</td>
<td>90</td>
</tr>
<tr>
<td>Average number of backorders ( X_j^6 )</td>
<td>10</td>
<td>11</td>
<td>13</td>
<td>12.5</td>
<td>14</td>
<td>13.5</td>
<td>12.5</td>
<td>15.5</td>
</tr>
<tr>
<td>Retailer profit ( Y_j^4 )</td>
<td>3000</td>
<td>2200</td>
<td>3200</td>
<td>2300</td>
<td>3300</td>
<td>2400</td>
<td>3500</td>
<td>3800</td>
</tr>
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</table>
TABLE III
Maximin Equilibrium Efficiency Using Model (12)

<table>
<thead>
<tr>
<th>DMU1</th>
<th>DMU2</th>
<th>DMU3</th>
<th>DMU4</th>
<th>DMU5</th>
<th>DMU6</th>
<th>DMU7</th>
<th>DMU8</th>
</tr>
</thead>
<tbody>
<tr>
<td>IFF given to subsystem 1</td>
<td>1.000000</td>
<td>0.711500</td>
<td>0.959100</td>
<td>0.959100</td>
<td>0.626000</td>
<td>0.697450</td>
<td>0.711900</td>
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<tr>
<td>IFF given to subsystem 2</td>
<td>0.756920</td>
<td>0.949977</td>
<td>1.000000</td>
<td>0.756920</td>
<td>0.823830</td>
<td>0.881250</td>
<td>0.865970</td>
</tr>
<tr>
<td>IFF given to subsystem 3</td>
<td>0.936550</td>
<td>0.673950</td>
<td>0.854110</td>
<td>0.700370</td>
<td>0.866670</td>
<td>0.673950</td>
<td>1.000000</td>
</tr>
<tr>
<td>Average</td>
<td>0.897820</td>
<td>0.778410</td>
<td>0.815300</td>
<td>0.683000</td>
<td>0.727120</td>
<td>0.750880</td>
<td>0.859290</td>
</tr>
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TABLE IV
Maximin Equilibrium Efficiency Using Model (18) or Repeatedly Optimized Model (16)

<table>
<thead>
<tr>
<th>DMU1</th>
<th>DMU2</th>
<th>DMU3</th>
<th>DMU4</th>
<th>DMU5</th>
<th>DMU6</th>
<th>DMU7</th>
<th>DMU8</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEE given to subsystem 1</td>
<td>1.000000</td>
<td>0.711500</td>
<td>0.959100</td>
<td>0.959100</td>
<td>0.626000</td>
<td>0.697450</td>
<td>0.711900</td>
</tr>
<tr>
<td>CEE given to subsystem 2</td>
<td>0.805000</td>
<td>0.997430</td>
<td>1.000000</td>
<td>0.805000</td>
<td>0.880000</td>
<td>0.899580</td>
<td>0.961920</td>
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<tr>
<td>CEE given to subsystem 3</td>
<td>0.983950</td>
<td>0.661880</td>
<td>0.900300</td>
<td>0.846670</td>
<td>0.926000</td>
<td>0.661880</td>
<td>1.000000</td>
</tr>
<tr>
<td>Average</td>
<td>0.929650</td>
<td>0.790270</td>
<td>0.830700</td>
<td>0.694800</td>
<td>0.811000</td>
<td>0.785100</td>
<td>0.891270</td>
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TABLE V
Optimal Weights

<table>
<thead>
<tr>
<th>Designed by DMU1</th>
<th>Designed by DMU2</th>
<th>Designed by DMU3</th>
<th>Designed by DMU4</th>
<th>Designed by DMU5</th>
<th>Designed by DMU6</th>
<th>Designed by DMU7</th>
<th>Designed by DMU8</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>( k^{1/2} )</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>( \omega_1 )</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>( \mu_1 )</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>( \omega_2 )</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>( \mu_2 )</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>( \omega_3 )</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>( \mu_3 )</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>( \omega_4 )</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>( \mu_4 )</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

Remark 1: Corollary 2 reveals an efficiency improvement capacity by use of collaboration between context supply chain processes (members). Theorem 3 suggests that supply chains with our maximin equilibrium DEA efficiency are also coordinated efficient.

V. CASE ILLUSTRATION

We consider the relevance and adaptation of the above results in evaluating supply chain performance. Supply chain systems can be represented by an integrated input-output system, where each supply chain member uses inputs (related to costs and resources) to produce outputs (associated with benefits and outcomes). In this section, we apply our models to a three-stage supply chain which is composed of three processes (members): the supplier, the manufacturer, and the retailer, i.e., \( P = 3 \) in models (12) and (18). The input-output indexes are shown in Table I. The supplier consumes input resources of labor and operating cost to generate output of revenue, which then become the input cost to the downstream manufacturer. The manufacturer spends manufacturing cost and lead time to produce certain quantity of products and achieve speedy fill rate. Products are then shipped to the retailer, who bear inventory cost and backorders in inventory to earn profits.
Table II presents the input–output data with eight observations, i.e., n = 8. Specially, in model (18), we use an arithmetic mean method reflecting the same preference over supply chain member’s performance. Then, program codes are written in Lingo language and results are reported in Tables III, IV, and V.

Table III shows the IEE scores given to the related supply chain member by running model (12). Table IV provides the CEE scores given to the related supply chain member either by the model (18) or by repeatedly optimized model (16).

Note that the average CEE scores, i.e., coordinated maximin equilibrium DEA efficiency of the related supply chain, in Table IV, are all larger than the average IEE scores in Table III. This validates Theorem 3.

Table V reports optimal weights for each variable. To validate the rationality of the weights, correlation analysis among all the variables using raw data and results is presented in Table VI.

Table V indicates two variables, manufacturing lead time and retailer inventory cost, are assigned zero weight, respectively. Neither of them is intermediate variable. At the third subprocess, an empty weight attached to the variable of retailer inventory cost implies a trivial contribution in gauging the supply chain performance. This variable is highly correlated with another key variable, products shipped from manufacturer, both the output of the second subprocess and the input to the third subprocess. The other one of the two variables is manufacturing lead time, a direct input variable at the second stage. This variable has a high correlation with the key input variable, manufacturing cost, as indicated in Table VI. Note also that in Table V all the intermediate variables were assigned with positive weights, which validates the rationality of the estimated weights.

TABLE VI

<table>
<thead>
<tr>
<th>Variable</th>
<th>Correlation matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor ( X^1 )</td>
<td>1</td>
</tr>
<tr>
<td>Operating cost ( X^2 )</td>
<td>0.306 1</td>
</tr>
<tr>
<td>Revenue ( Y^1 )</td>
<td>0.23 0.327 1</td>
</tr>
<tr>
<td>Manufacturing cost ( X^3 )</td>
<td>-0.306 -0.336 0.335 1</td>
</tr>
<tr>
<td>Manufacturing lead time ( X^4 )</td>
<td>-0.37 -0.459 -0.03 0.663 1</td>
</tr>
<tr>
<td>Fill rate ( Y^2 )</td>
<td>0.216 0.176 0.349 -0.412 0.113 1</td>
</tr>
<tr>
<td>Products shipped from manufacturer ( Y^3 )</td>
<td>-0.558 0.124 0.286 0.254 0.096 -0.679 1</td>
</tr>
<tr>
<td>to retailer ( Y^4 )</td>
<td></td>
</tr>
<tr>
<td>Retailer inventory cost ( X^5 )</td>
<td>0.089 -0.146 0.419 0.005 -0.239 -0.204 0.574 1</td>
</tr>
<tr>
<td>Average number of backorders ( X^6 )</td>
<td>0.629 0.719 0.026 -0.755 -0.701 0.252 -0.543 -0.247</td>
</tr>
<tr>
<td>Retailer profit ( Y^5 )</td>
<td>0.292 0.731 0.566 -0.273 -0.253 -0.371 0.251 0.061 -0.506 1</td>
</tr>
</tbody>
</table>

VI. Conclusion

This paper has demonstrated how a maximin equilibrium efficiency methodology may be applied to improve efficiency of a sequence of processes when the outputs of one become the inputs of the next in the sequence. It is shown that a supply chain is coordinated maximin equilibrium efficient if and only if all supply chain members are coordinated maximin equilibrium efficient. By use of collaboration between adjacent supply chain processes (members), efficiency can be improved.

Illustration of supply chain maximin equilibrium models is demonstrated by a concrete application to supply chain with three processes (members). Numerical data are used to construct a common empirical maximin equilibrium supply chain performance frontier. Based upon a common frontier, supply chains performance is estimated. The relevance of the weight distribution is also validated by use of correlation analysis.

Our generic model has many potential uses. One of the important aspects is the use in supply chain design including identification of efficient supply chain members. Another possible application is the monitoring of a specific supply chain such as product flow [38], [39]. For example, based on past data on observed cycles, model (7) provides a set of optimal output targets. All these targets will be technically feasible based on the observed data and Maximin equilibrium assumptions. The maximin criterion considered in this paper reflect a form of decision maker’s pessimism and general rationality. Another further consideration is to employ
maximin or minimax criterion, where the decision maker is an optimist or opportunist, respectively [41]. However, such researches are beyond the scope of the current paper and left with further exploration.

APPENDIX

Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCC</td>
<td>Banker Charnes Cooper DEA Model</td>
</tr>
<tr>
<td>CEE</td>
<td>Independent Equilibrium Efficiency</td>
</tr>
<tr>
<td>CCR</td>
<td>Coordinated Equilibrium Efficiency</td>
</tr>
<tr>
<td>DEA</td>
<td>Data Envelopment Analysis</td>
</tr>
<tr>
<td>DMU</td>
<td>Decision Making Unit</td>
</tr>
<tr>
<td>IEE</td>
<td>Independent Equilibrium Efficiency</td>
</tr>
<tr>
<td>MMC</td>
<td>Maximin Criterion</td>
</tr>
<tr>
<td>SCM</td>
<td>Supply Chain Management</td>
</tr>
<tr>
<td>SPCM</td>
<td>Supply Chain Performance Measurements</td>
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</table>

REFERENCES

Desheng Dash Wu is currently an Affiliate Professor and the Managing Director with the Risk Laboratory, University of Toronto, Toronto, ON, Canada. He was elected into the 100-Talents Professor Program from the University of Science and Technology of China, Chinese Academy of Sciences, Beijing, China, in 2012, and the 1000-Talents Program for Distinguished Young Scientist in China in 2013. He has published over 80 papers in refereed journals, such as Production and Operations Management, Decision Sciences, Risk Analysis, IEEE Transactions on Systems, Man, and Cybernetics. His current research interests include enterprise risk management in operations, performance evaluation in financial industry, and decision sciences.

He is the editor of Springer book series entitled Computational Risk Management. He has served as an Associate Editor/Guest Editor in journals such as the IEEE Transactions on Systems, Man, and Cybernetics, Annals of Operational Research, Computers and Operations Research, International Journal of Production Economics, and Omega.

Cuicui Luo received the B.Sc. degree in both actuarial science and statistics, and the M.Sc. degree in mathematics finance from the University of Toronto, Toronto, ON, Canada. She is currently pursuing the Ph.D. degree in mathematics finance with Risk Laboratory, University of Toronto. She is currently a Research Associate with Risk Laboratory, University of Toronto. Her current research interests include financial market analysis and decision analysis.

David L. Olson is currently a James and H.K. Stuart Professor in management information systems (MIS) with the University of Nebraska, Kearney, NE, USA, and the Chancellor’s Professor with the University of Nebraska. He has published research in over 100 refereed journal articles, primarily on the topic of multiple objective decision-making and information technology. He teaches in the MIS, management science, and operations management areas. He has authored 17 books, to include Decision Aids for Selection Problems, Introduction to Information Systems Project Management, and Managerial Issues of Enterprise Resource Planning Systems as well as co-authored the books Introduction to Business Data Mining, Enterprise Risk Management, Advanced Data Mining Techniques, New Frontiers in Enterprise Risk Management, Enterprise Information Systems, and Enterprise Risk Management Models.

Mr. Olson is an Associate Editor of the Service Business and a Co-Editor-in-Chief of the International Journal of Services Science. He has made over 100 presentations at international and national conferences on research topics. He is a member of the Association for Information Systems, the Decision Sciences Institute, the Institute for Operations Research and Management Sciences, and the Multiple Criteria Decision Making Society. He was named Best Enterprise Information Systems Educator by IFIP. He is a fellow of the Decision Sciences Institute.