Power Systems as Dynamic Networks

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Abstract—The theory of power systems dynamics has been developed largely from detailed studies of the dynamics of simple system structures with emphasis on the effects of modeling details of the dynamics. In contrast, the recent study in the science of complex networks, motivated by numerous real-world examples in various areas including power systems, employs rather simple dynamics and yet places much more emphasis on network structures. It appears that the two approaches can usefully inform each other for further progress into an integrated study of very large (or massive) systems as well as very complex dynamics. This paper reviews recent progress in these areas and suggests fruitful lines of future research.

I. INTRODUCTION

Power systems are naturally modeled as large-scale and highly nonlinear structured dynamical systems. These nonlinear models can be shown to exhibit a wide variety of complex behaviour described mathematically in terms of Lyapunov stability, structural stability, bifurcations, and chaos. Previous planning and operating practice has been able to proceed largely without deep concern for these aspects of complex system behaviour. Reference [1] in 1995 gave a presentation of the state-of-art in nonlinear phenomena and predicted that in the future they will be more important to power systems. Economic and environmental restrictions, along with the trend to open access, have caused operating margins to be reduced as systems become more heavily loaded. Consequently, system behaviour will be more dependent on nonlinear characteristics. Recent experience, especially a number of major blackouts worldwide in 2003, suggests that we have entered an era of less predictable behaviour in more complex power networks. We now talk of power systems as complex networks with a tendency to catastrophic failures [2]. This in turn requires much more sophisticated analysis. Generally, we can be confident that the scientific basis for operating large systems like power systems needs to be strengthened. This paper explores the possible synergy between the nonlinear theory of power systems, with emphasis on stability analysis, and recent results in network science.

The theory of dynamics for power systems has been developed to an advanced state following contributions dating back to the 1930’s. There are theories for the major phenomena using various mathematical techniques. Of particular importance are synchronization, oscillations, bifurcations, collapse and chaos [1]. The theories have emerged largely from detailed studies of dynamics of simple system structures, often just with one or two generators, with emphasis on the effects of modeling details of the dynamics. In recent years, there has been an explosion of scientific effort aimed at study of complex networks with applications in numerous areas across physics, computer and life sciences, and engineering systems such as power systems. A major topic is synchronization phenomena. This work employs rather simplistic dynamics and places much more emphasis on the network structure including those becoming popular recently in the social and physical sciences, i.e. small-world and/or scale-free models. It appears that the two approaches, which will be referred to respectively as dynamics oriented and structure oriented, have many connections.

II. POWER SYSTEMS STABILITY THEORY

From a practical viewpoint, there are four major analytical problems: a) using power flows to compute equilibria; b) applying angle stability (commonly called transient stability or large-disturbance stability) to check preservation of generator synchronism; c) studying oscillations (commonly treated as a linear issue); and d) employing voltage instability to check the possibility of voltage collapse. Of course, theoretically they are all aspects of the one overall stability question [3], but it is possible to focus on each via appropriate modelling simplifications.

A power system consists of generators (power sources) and loads (power sinks) connected to buses which are interconnected by transmission lines in a network structure. For transient stability, i.e. the electro-dynamical behaviour, the loads are usually taken as static. Most papers present this model with the loads taken as impedances and the network
modelled in reduced forms, i.e. the load buses absorbed into
the network. Following the work by Bergen and Hill [4, 5]
and subsequently (see for instance [6-9]) we use a network-
preserving model (NPM) here.

The generators in the system are modelled as constant
voltages behind a reactance. The phase of this voltage is
determined by the rotor dynamics with respect to a
synchronous reference and is modelled by the swing
equation
\[ M \dot{\theta}_i + D \dot{\theta}_i + \sum_{j \in C_i} V_j V_i b_{ij} \sin(\theta_i - \theta_j) = P_i \]  
(1)
This expresses a dynamic real power balance. The subscript
\( i \) denotes machine \( i \) and \( j \in C_i \) refers to the machines
connected to machine \( i \). At each network bus, the voltage
magnitude and phase are variables. These buses generally
have loads attached. Adding equations for the associated
power balance gives, for each load \( k \),
\[ \sum_{j} V_k V_j b_{kj} \sin(\theta_k - \theta_j) = P_k (V_k) \]  
(2)
The key symbols in (1)-(2) are as follows: \( \theta \) is the bus
current phase (including internal voltages of generators); \( V \) is a
bus or nodal voltage (rms value); \( b_{ij} \) is the admittance
of the line joining nodes \( i \) and \( j \); \( M \) is the rotor’s inertia
constant; \( D \) is the damping coefficient; \( P \) and \( Q \) are the real
and reactive power input to a bus (which for the generator is
the mechanical power applied to the rotor shaft). For the
loads, the powers are shown as voltage dependent to reflect
the effect of load characteristics). There are two state
variables associated with each generator, i.e. angle \( \theta_i \) and
frequency \( \omega_i = \dot{\theta_i} \). If loads are modeled as dynamic, there
will be other dynamic state variables corresponding to
internal behavior of the loads, e.g. motor slips.

These equations are typically assembled into a higher-
dimensional set of differential algebraic equations (DAE) of
the form
\[ \dot{x} = f(x, y) \]  
\[ 0 = g(x, y) \]  
(3)
The (dynamic) state \( x \) includes generator angles, frequencies
and maybe higher-order effects in the generator dynamics
and load dynamics. The algebraic variables \( y \) include load
bus voltages. The above-mentioned stability questions can
be summarised as follows (see [3] for more detail): a) a
solution of (3) in steady state, modulo some subtleties [10],
provides the equilibria which of course depend on the power
flow imposed on the system; b) the generator angle
differences \( \theta_i - \theta_R \), where \( \theta_R \) is a reference defined by one
machine or some specially defined angle, e.g. centre of
inertia, must remain small to maintain synchronous operation
in the presence of large disturbances; this is
commonly embedded in a stability question for the post-
fault equilibrium point, i.e. the post-fault angle differences
and frequency must lie in a region of attraction of that
equilibrium; c) the tendency to get undamped oscillations in
frequencies \( a_i \), which are precisely analysed as Hopf
bifurcation, must be avoided; d) the voltages \( V_i \) are subject
to different problems on different time-scales including
short-term [8] and long-term voltage collapse [3], which are
analyzed in terms of local and global bifurcations (see [1]
and references therein).

The earliest contributions to power systems theory
recognized the highly nonlinear nature of (3). We now list
some points of the dynamical systems theory on power
systems which relate to the theme of this paper:

- There are rigorous Lyapunov functions for NPMs
  where the real-power loads are independent of voltages
  [4-6, 11]; this had proved problematic for decades with
  reduced models;
- There are extensive theory and algorithms for
  characterizing and computing the regions of stability
  around stable equilibria;
- The Lyapunov (or energy) functions can be expressed
  in a topological form [4], i.e. a sum of contributions
  from all generators and lines; denoting the line angle
differences by \( \sigma_i = \theta_i - \theta_j \), the Lyapunov
  function can be written
\[ E(a_i V) = \sum_{\sigma_i} \frac{1}{2} M_i \omega_i^2 + \sum_{\sigma_i} W_i (\sigma_i - \sigma_i^0) \]  
(4)
- The vulnerability of the network to large disturbances
  in certain loading conditions can be related to network
cutsets [4, 12] (although a complete theory has not been
developed);
- Allowing for multiple equilibrium points and the local
  solvability of (3), and depending on the load
  characteristics, the energy function can be seen as
  defined over multiple hypersurfaces corresponding to
different stable equilibria and voltage solutions [8];
- In some cases, this application inspired extensions to
  the basic stability theory, e.g. [11].

III. NETWORK SCIENCE

Most systems in nature and engineering can be described
by a network consisting of nodes connected by links, e.g.
the buses and lines respectively in a power system reviewed
above. Engineers tend to describe complexity in terms like
dimension, nonlinearity, uncertainty and structure [13].
Scientists are more likely to mention structural features,
more specifically e.g. diversity, evolution, self-similarity or
behavioural features like emergence even though many of
these concepts remain somewhat difficult to define [14].
Here, we focus on results related to dynamic networks
Network science has identified classes of structures which are commonly observed in the real world. These include small-world and scale-free networks [14-16]. The latter property has been shown to apply to numerous large-size networks including the Internet, WWW, metabolic networks and power systems. It refers to absence of scale in the sense of connectivity of the nodes, i.e. a few nodes have high connectivity (or hubs) and numerous nodes have low connectivity. The current synchronization literature has, and still is producing large numbers of results in the spirit of (6) on synchronization of such networks with structure (5); the generalizations are too numerous to list here, but include time-varying, nonlinear, and delayed interconnections (see e.g. [19, 20]). For example, these results infer the ability to enhance synchronization in a large-size nearest-neighbor coupled network by just adding a tiny fraction of distant links, thereby making the network become a small-world model. The robustness of synchronization in a scale-free dynamical network has also been investigated, against either random or specific removal of a small fraction of nodes in the network. Although the scale-free structure is particularly well-suited to tolerate random errors, it is also particularly vulnerable to deliberate attacks. The error tolerance and attack vulnerability in scale-free networks are rooted in their extremely inhomogeneous connectivity patterns.

There are further topics in network science that relate to power systems issues. In particular, we mention the need to understand cascading collapses [21] and how to design networks to be optimal in desired senses, e.g. for synchronizability [22, 23].

IV. FUTURE RESEARCH OUTLOOK

The two theories that have been described in Sections II and III respectively have considerable commonality in the problem studied. They are both essentially about synchronization in networked systems. Generally speaking, the power systems results are more oriented to node dynamics while the complex networks results are oriented to system structures. However, neither theory can be currently used to make statements about the other. It appears that the theories provide two approaches, which will be referred to as dynamics oriented and structure oriented, respectively, and they can very usefully inform each other for further progress into the study of very large (or massive) systems and very complex dynamics. Some issues which could be further studied are:

- The power systems theory needs to be pushed further in the direction of exploiting structural features of the networks; for instance how to exploit a scale-free structure (or a refined version) in dynamical analysis; there are challenges in this task; realistic power systems models have at least two different types of node dynamics (generators, loads) and the directional power flows between them play a major role;
• The power systems theory needs to follow the ideas of collapse dynamics in networked systems;
• The networks results are typically for the same dynamics at each node; they need to be extended to classes of node dynamics, e.g. same form but different parameters as in power systems theory;
• Further possibilities are in the use of properties like passivity and small-gain on the node dynamics, bringing a connection to results on the stability (and instability) of large-scale systems [24];
• The networks results are typically local, i.e. around a trajectory in the state space; power systems theory has characterized regions of stability (and synchronization) and even global behavior; the tools of absolute stability theory, normal forms, bifurcations and chaos have all been used;
• In the application of Lyapunov methods, there would appear to be value in using different forms such as Lur'e-Postnikov, weighted sums, and so on;
• The networks results are focused on synchronization; as the power systems case illustrates, there are other dynamical phenomena that are related to structures, i.e. collapse phenomena, oscillations, and collapse;
• Many networks such as power systems and the Internet involve some underlying flows which determine equilibria and bifurcations; this needs to be brought into the models used for generic results;
• There are some steps towards robustness analysis in networks, but more refined studies are needed, i.e. the effect of link switching on regions of convergence;
• In both areas, attention needs to be given to the hybrid nature arising from switching events and how this integrates with the dynamics and the network.

V. CONCLUSIONS

This paper has reviewed the theories of power system stability and dynamical network synchronization. While sharing considerable overlap in their concern for synchronous behavior, the results differ in emphasis. There appear to be some very fruitful synergies to pursue for progress in power systems, and networks generally, especially towards large complex systems.

REFERENCES