A Compact Representation for Topological Decompositions of Non-Manifold Shapes

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Introduction

Manifold shapes (Topological Manifold)

Each neighborhood of every point $p$ is homeomorphic to one connected component of a *ball*, centered at $p$. 
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- many tools based on manifold shapes

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- non-manifold singularities, i.e., points at which the manifold condition is not satisfied
- parts of different dimensions
- assembly of components (FEM analysis)
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Representing Non-Manifold Shapes

Classical approach

- Discretized by **simplicial $d$-complexes** of any dimension, embedded in the Euclidean space

Drawbacks (wrt non-manifolds)

- Only local connectivity for every simplex, meaningful components are not exposed explicitly
- Non-manifold singularities are not exposed directly ($d > 5$, Nabutovski, 1996)

Structural Model

Connections among meaningful components (global structure)
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*connections* among meaningful components (*global structure*)
Our Proposal: a Decomposition Approach

Main Property of Non-Manifold Shapes

*Complex topology* of a non-manifold shape offers *valuable information* for:

- Decomposing a shape into almost manifold components (simpler topology)
- These components are connected by non-manifold singularities

Related Work (see paper)

- Rossignac et al., 1989/1999
- Desaulniers and Stewart, 1992
- De Floriani et al., 2003
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Key Idea of our Approach

Expose explicitly and combine combinatorial and structural information

Compact Manifold-Connected (MC-) graph

Two-level graph-based representation of the MC-decomposition, Hui and De Floriani, 2007
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Topological data structure (Local Connectivity)

Structural Model (Global Structure)
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Manifold-Connected (MC) Complexes

Hui and De Floriani, 2007

Given a simplicial $d$-complex $\Sigma$ and $k \leq d$:

- **Top $k$-simplex**: Does not bound any other simplex.
- **Manifold** $(k-1)$-path (MC-Adjacency): Sequence of top $k$-simplices in $\Sigma$, where each simplex is adjacent through a $(k-1)$-simplex, bounding at most two top $k$-simplices.
- **Always decidable and dimension-independent**
- **MC-complex of dimension $k$**:
  - **Maximal manifold** $(k-1)$-path, starting from a top $k$-simplex $\sigma$.
  - **Representative top simplex** $\sigma$ (arbitrary).
  - **Equivalence class** $[\sigma] \text{ wrt to MC-adjacency}$

Superclass of manifolds, they may contain non-manifold singularities.
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- Decomposition of a simplicial complex $\Sigma$ into its $\textit{MC-complexes (MC-components)}$
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**Examples:**
- 1 MC-component
  - 1 MC-component (for manifolds)
- 6 MC-components
- 8 MC-components
- 3 MC-components
  - Common subcomplex of some non-manifold singularities
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**MC-components’ Intersection**

Common subcomplex of some **non-manifold singularities**
The Compact Manifold-Connected (MC-) graph

A two-level representation of the MC-decomposition, which integrates \textit{combinatorial} and \textit{structural} aspects.
The Compact Manifold-Connected (MC-) graph

A two-level representation of the MC-decomposition, which integrates *combinatorial* and *structural* aspects.

**Lower Level (Combinatorial Aspects)**

Describes a non-manifold shape by any topological data structure $M_\Sigma$ *(unique)*:

- the *Incidence Simplicial (IS) data structure*, De Floriani et al., 2010
- the *Generalized Indexed data structure with Adjacencies (IA*)*, Canino et al., 2011
- *any* topological data structure for *non-manifolds* can be exploited

The Mangrove TDS Framework (Canino, 2012 - PhD. Thesis)

Tool for the fast prototyping of topological data structures

Extensible through dynamic plugins (*mangroves*)

Any type of complexes is supported

The Mangrove TDS Library is released as GPL v3 software for the scientific community at [http://mangrovetds.sourceforge.net](http://mangrovetds.sourceforge.net)
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Describes the *connectivity* of MC-components by a hypergraph $G^C_\Sigma = (N_\Sigma, A^C_\Sigma)$.
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A hypernode in $\mathcal{N}_\Sigma$
- Corresponds to one *MC-component* $C$
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A hyperarc $a$ in $\mathcal{A}^C_{\Sigma}$
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- References to $s_a$ **non-manifold singularities** in $S$
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References are directed toward simplices in $\mathcal{M}_\Sigma$

Similar to a *spatial index* on any non-manifold shape

Storage Cost

$$S_C = n_C + \sum_{a \in \mathcal{A}_\Sigma^C} (k_a + s_a)$$
Other Representations of the MC-decomposition

Properties of our Compact MC-graph

- Few hyperarcs
- Minimizes duplications of intersections
- Maximal list of MC-components in hyperarcs
Other Representations of the MC-decomposition

Our Compact MC-graph resolves all the drawbacks of:

- Pairwise MC-Graph (Boltcheva, Canino, et al., 2011)
  - Arcs \( \equiv \) intersections of only two MC-components, formed by a subcomplex of non-manifold singularities
  -Verbose due to cliques
  -Less robust wrt to simmetry

- Exploded MC-Graph (Canino and De Floriani, 2011)
  - A hyper-arc \( \equiv \) one non-manifold singularity \( \sigma \), and connects all the MC-components, bounded by \( \sigma \)
  -Too much hyperarcs
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Experimental Results (with our Mangrove TDS Library)

Digital shapes are freely available from http://indy.disi.unige.it/nmcollection

### 2D shapes (Storage cost and Properties of MC-graphs)

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<thead>
<tr>
<th>Shape</th>
<th>$n_C$</th>
<th>$a_E$</th>
<th>$a_P$</th>
<th>$a_C$</th>
<th>$S_E$</th>
<th>$S_P$</th>
<th>$S_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carter</td>
<td>45</td>
<td>641</td>
<td>79</td>
<td>48</td>
<td>3.8k</td>
<td>2.6k</td>
<td>1.2k</td>
</tr>
<tr>
<td>Chandelier</td>
<td>130</td>
<td>616</td>
<td>328</td>
<td>96</td>
<td>2.6k</td>
<td>2.6k</td>
<td>1k</td>
</tr>
<tr>
<td>Pinched Pie</td>
<td>120</td>
<td>1.4k</td>
<td>1.4k</td>
<td>192</td>
<td>4.8k</td>
<td>9.6k</td>
<td>1.9k</td>
</tr>
<tr>
<td>Tower</td>
<td>169</td>
<td>1.4k</td>
<td>13k</td>
<td>165</td>
<td>5.9k</td>
<td>43k</td>
<td>2.1k</td>
</tr>
</tbody>
</table>

$n_C$: #MC-components
$a_E, a_P, a_C$: #(hyper)arcs
$S_E, S_P, S_C$: storage costs

For 2D shapes:

\[
a_E \approx 8.9 \times a_C, \ a_P \approx 23 \times a_C
\]

\[
S_E \approx 2.8 \times S_C, \ S_P \approx 7.7 \times S_C
\]

### 3D shapes (Storage cost and Properties of MC-graphs)

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<th>$a_P$</th>
<th>$a_C$</th>
<th>$S_E$</th>
<th>$S_P$</th>
<th>$S_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chime</td>
<td>27</td>
<td>29</td>
<td>47</td>
<td>28</td>
<td>133</td>
<td>210</td>
<td>127</td>
</tr>
<tr>
<td>Flasks</td>
<td>8</td>
<td>76</td>
<td>10</td>
<td>6</td>
<td>300</td>
<td>232</td>
<td>98</td>
</tr>
<tr>
<td>Teapot</td>
<td>2.9k</td>
<td>1.2k</td>
<td>18.1k</td>
<td>1k</td>
<td>10.4k</td>
<td>57.5k</td>
<td>10.1k</td>
</tr>
<tr>
<td>Wheel</td>
<td>115</td>
<td>136</td>
<td>520</td>
<td>88</td>
<td>675</td>
<td>1.7k</td>
<td>563</td>
</tr>
</tbody>
</table>

For 3D shapes:

\[
a_E \approx 4.1 \times a_C, \ a_P \approx 6.8 \times a_C
\]

\[
S_E \approx 1.6 \times S_C, \ S_P \approx 3.2 \times S_C
\]

Our experimental results confirm properties of the Compact MC-graph
## Experimental Results (cont’d)

### Comparisons with the Incidence Graph, Edelsbrunner, 1987

<table>
<thead>
<tr>
<th>Shape</th>
<th>$S_C$</th>
<th>$S_{IA^*}$</th>
<th>$S_{IG}$</th>
<th>$S_C + S_{IA^*}$</th>
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<tr>
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Combined with the IA* data structure, *Canino et al., 2011*

$S_{IA^*}$: storage cost of the IA*

$S_{IG}$: storage cost of the IG

For 2D shapes: $S_{IG} \approx 1.45 \times S_{IA^*}$
For 3D shapes: $S_{IG} \approx 3.2 \times S_{IA^*}$
For 4D shapes: $S_{IG} \approx 8 \times S_{IA^*}$
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Experimental Results (cont’d)

Comparisons with the Incidence Graph, Edelsbrunner, 1987

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<th>Shape</th>
<th>$S_C$</th>
<th>$S_{IA^*}$</th>
<th>$S_{IG}$</th>
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Interesting result (wrt the Incidence Graph)

The Compact MC-graph, combined with the IA* data structure, is more compact than the incidence graph:

- our contribution is a structural model (topological + structural aspects)
- the IG data structure is a topological data structure (local connectivity)
Conclusions and Future Work

Compact Manifold-Connected (MC-) graph

Two-level graph-based representation of the

**MC-decomposition**, Hui and De Floriani, 2007
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Acknowledgements

We thank:

- anonymous reviewers for their useful suggestions
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- the National Science Foundation (contract IIS-1116747)

These slides are available on
http://www.disi.unige.it/person/CaninoD

Thank you much for your attention!
Interesting Papers and References

- M. Botsch, L. Kobbelt, M. Pauly, P. Alliez, B. Lévy, *Polygon Mesh Processing*, CRC Press, 2010
Interesting Papers and References (cont’d)