Performance Analysis of A Correlation-Based Optical Flow Algorithm under Noisy Environments

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Abstract—This paper presents the performance test and analysis of a correlation-based optical flow estimation (CBOFE) algorithm, the Camus method, under various noisy environments. The noise sensitivity of the original algorithm is tested under different noise sources such as additive Gaussian, bit-quantization, multiplicative noises and combinations of multiplicative and additive noises. We show the efficiency of pre-filtering under different noisy environments. The SNR equations are derived for various additive and multiplicative noise cases for analyzing the performance degradation. The simulation results show that the multiplicative noise is comparable to additive noise if the standard deviations of noises are scaled to the same percentage of the full range of signal.

I. INTRODUCTION

An optical flow field, which is crucial information for computer vision tasks, is a vector field which describes the instantaneous motion of all pixels in a sequence. Generally, optical flow estimation methods can be classified into gradient-based and correlation-based methods. The latter use the block-wise similarity of intensity values between current and previous frames, and the former solves the linear equations of spatiotemporal derivatives [1].

Although volumes of research on the topic of optical flow have been performed, most researchers have focused on accuracy and real-time issues [1],[7]. Because of this trend, most performance tests and evaluations have been performed under noise-free environments. However, real world environments are noisy since it is hard to avoid sensor and circuit noise. For example, a CMOS image sensor, which is an important image sensor due to its low power consumption, has relatively high gain variations and offset compared to a CCD image sensor. Therefore, it causes multiplicative and additive noise problems. Previous works regarding noise sensitivity of optical flow estimation (OFE) algorithms are reported in [6] and [5]. Liu et al. [6] conducted tests of various OFE algorithms for additive Gaussian noise case. Lee and Anderson [5] performed in-depth tests of a gradient-based OFE algorithm under various noises.

In this paper, we present the performance test and analysis of a correlation-based optical flow estimation (CBOFE) algorithm, the Camus method, in various noise environments for real system design. This builds on the work reported in [5] in which the behavioral simulations of a gradient-based optical flow estimation algorithm were reported. More refined analysis for multiplicative noise cases and the efficiency of pre-filtering under different noise sources are investigated for a different type of OFE algorithm compared to [5]. The rest of the paper is organized as follows. A brief introduction of CBOFE method is presented in section 2. The signal model is described in section 3. The SNR’s for different noises are derived in section 4. In section 5, we analyze and discuss of the performance of the Camus algorithm in noise-free and various noisy conditions. Conclusions are presented in section 6.

II. CORRELATION-BASED OPTICAL FLOW ESTIMATION ALGORITHM

In correlation-based optical flow methods, the motion of a pixel at \((x, y)\) in one frame to a successive frame is determined by a matching function, such as the sum of squared difference (SSD), sum of absolute difference (SAD), and correlation, over a searching area (SA) of \((2\mu+1) \times (2\mu+1)\) using the squared window patch of \(p \times p\) pixels, which is centered at \((x, y)\). A typical matching function of SSD is

\[
\arg\min_{dx,dy} \sum_{i,j} (I_t(x, y) - I_{t+dt}(x + dx, y + dy))^2
\]

Considering the relationship of the velocity equation,

\[
velocity = \frac{\Delta distance}{\Delta time}
\]

correlation-based optical flow methods can be categorized into spatial and temporal search methods based on the searching axis. Spatial searching methods select the best candidate pixel of the current frame within the SA of one previous frame.
This is the case that $\Delta t = 1$ is fixed and the searching axis is distance in (2) for different velocities. The location differences between the current frame pixel $(x, y)$ and best candidate pixel in the previous frame generate an optical flow field. This requires enough memory for two frames, current and previous frames. However, the complexity is quadratic with respect to the size of search range of $\mu$. When $\Delta distance = 1$ is fixed and time is searching axis in (2) for different velocities, this is temporal searching case. Temporal searching method determines the best candidate pixel of the current frame over several previous frames with fixed search range in spatial domain. An optical flow field derived using a temporal searching method is calculated as $1/dt(dx, dy)$, where $dx/dt$ and $dy/dt$ are motion vectors. An advantage of this algorithm is the linear complexity with respect to the number of frames. However, the size of memory linearly increases with the number of previous frames. The Camus algorithm, which we will investigate in this paper, is a typical temporal searching method [4].

### III. A Signal Model for Noisy Environment

Correlation-based motion sensors, such as the Reichardt motion sensor, are widely implemented in analog VLSI (AVLSI) due to low power and simple structure. Even though the correlation-based Camus method, the simplified 2D version of Reichardt OFE in discrete time, achieves satisfactory performance in noise-free environments, the performance can deteriorate when noise exists in OFE system. Therefore, the accuracy of OFE is not satisfactory as the noise level of OFE system increases.

A signal model is required for performance tests and analysis considering the sensor and circuit noises. The noisy signal model for simulations is as follows [5]:

$$y(m, n; t) = (1 + p(m, n))x(m, n; t) + q(m, n; t)$$  \hspace{1cm} (3)

where

$$p(m, n) = r(m)e(n)e(m, n),$$

$$q(m, n; t) = a(m, n; t) + b(m, n; t).$$

In (3), $y(\cdot)$ is a noisy signal, $p(\cdot)$ is a multiplicative noise, $x(\cdot)$ is original input signal, and $q(\cdot)$ is an additive noise. $p(\cdot)$ is mainly due to multiplicative fixed pattern noise (FPN) which is the dominant source of noise in the CMOS image sensor, and $q(\cdot)$ is composed of general additive gaussian noise (AGN) of $a(\cdot)$ and bit resolution uniform noise of $b(\cdot)$. $p(\cdot)$ is the combination of row-, column-, and element-wise multiplicative noises of $r(\cdot), e(\cdot)$, and $e(\cdot)$, which are constants in some period of time.

### IV. The SNR Derivations for Performance Analysis

The signal-to-noise ratio (SNR) is a useful measure for analyzing the performance degradation by various noises. The general SNR formula for additive noise is

$$SNR_{an} = 10 \cdot \log_{10} \frac{\sigma_x^2}{\sigma_n^2}$$  \hspace{1cm} (4)

where $\sigma_x^2$ is the signal power and $\sigma_n^2$ is the additive noise power. For additive noise cases, we can use this formula if we assume that noise and signal are uncorrelated and have zero mean. However, SNR for the multiplicative noise case is not self-evident since this noise is signal dependent. To derive the SNR for the multiplicative noise case, we employ the same assumption as the additive noise case. Based on these assumptions, we can derive the SNR for the multiplicative noise case. At first we have to expand the signal modeling equation as follows:

$$y = (1 + n_2)x$$
$$= x + n_2 \cdot x$$
$$= x + z$$  \hspace{1cm} (5)

where $x$ is the original signal and $n_2$ is the multiplicative noise.

From (5), we can derive the following equation.

$$SNR_{mn} = 10 \cdot \log_{10} \frac{\sigma_x^2}{\sigma_z^2}$$
$$= 10 \cdot \log_{10} \frac{\sigma_x^2}{\sigma_{n_2}^2 \cdot \sigma_z^2}$$  \hspace{1cm} (6)

where $\sigma_x^2$ is the signal power and $\sigma_{n_2}^2$ is the multiplicative noise power. For (6), it is necessary to be statistically independent between $n_2$ and $x$, which is a strong constraint to satisfy in practical condition due to non-linear phenomenon. We derive (6) to show the ideal linear case for comparison with additive noise case. Unlike the SNR of additive noise case, the SNR for multiplicative noise case (6) looks depending on only the variance of multiplicative noise. However, both (4) and (6) have logarithmic linear relationship with the noise power.

### V. Experiments and Discussions

In this section, we show the experimental results of the Camus optical flow algorithm as a function of various noise powers and pre-filtering (PF). Three frames are employed for the previous frames of the search area in Camus algorithm. To investigate the performance effects of each noise component
in (3), we test separately for each noise and then test with several noise component combinations based on the magnitude error (ME) and angle error (AE) of motion fields.

Two test sequences of different complexity were employed. The first sequence is zooming-in office images with a constant velocity. Another sequence which is globally moving right is a simulated busy street corner. The camera follows a car which turns into a side-street. Figure 1 shows typical images of these sample sequences.

The standard deviations of additive and multiplicative noises, $\sigma_n1$ and $\sigma_n2$, are scaled to the same percentage with respect to the full range of each signal. For example, $\sigma_n1$ is scaled to the 256 for the gray image level and $\sigma_n2$ is based on the full swing of gain, which is assumed to be 1.

A. The efficiency of pre-filtering

Compared to gradient-based optical flow algorithms [4], the original Camus algorithm performs optical flow estimation without a pre-filtering block. The lack of pre-filtering is reasonable only if frame rate is fast enough to detect motion fields without temporal aliasing and input image of OFE system is not noisy. However, in reality temporal aliasing occurs depending on the relative speed of object compared to the frame rate. Pre-filtering increases the SNR by reduction of temporal aliasing and noise.

We employ a spatiotemporal Gaussian filter of standard deviation of 2.0 pixels per frame, which is a low-pass filter for pre-filtering. Figures 2 and 3 show that pre-filtering is effective for additive noise when applied to Camus optical flow algorithm. As noise level increases, pre-filtering becomes more effective. Therefore, even simple pre-filtering can improve the performance of optical flow estimation for noisy images.

B. The additive noise (AN) case

To reduce power consumption, reduced precision arithmetic is preferred provided performance is not deteriorated. Figure 3 shows the result of bit-level simulation of Camus algorithm. The simulation is divided into two categories. One applies same bit resolutions for all three-dimensional input signals, and the other employs different bit resolutions for input signals. According to Fig. 3, Camus original algorithm is less affected by the bit quantization noise (BQN). Since the gray level is $2^b$, where $b$ is bit-number, the noise power is small for low bit-number quantization. Therefore, the SNR does not decreased much.

Generally, there are noises in the circuitry such as shot noise and thermal noise. The thermal and shot noises can be modeled as additive Gaussian noise (AGN). Therefore, we need to test the performance under the AGN model. Figure 2 shows the results of Camus OFE algorithm with AGN. The percentage increase indicated in the figures is in comparison to the mean of magnitude and angle errors for the noise-free condition. As the variance of AGN increases, the performance of the original Camus algorithm, which does not use pre-filtering, deteriorates. In contrast, AGN has less effects on the performance of the Camus algorithm after pre-filtering.

Figure 4 shows the result of testing for the AGN and BQN case. The performance of the Camus algorithm more depends on the AGN since AGN is the dominant noise factor. The SNR for this case is shown in equation (7).

$$SNR_{bq-agn} = 10 \cdot \log \left( \frac{\sigma_x^2}{\sigma_{bqn}^2 + \sigma_{agn}^2} \right)$$  \hspace{1cm} (7)

where $\sigma_{bqn}^2$ is the bit quantization noise power, $\sigma_{agn}^2$ is the additive Gaussian noise power, and $\sigma_x^2$ is the signal power.

C. The multiplicative noise (MN) case

The gain variation of image sensors in an imager causes multiplicative noise, which manifests as a fixed pattern noise (FPN) [2],[3]. Since each image sensor has a pixel-wise amplifier, in general pixel-wise multiplicative noise exists. Depending on the architecture of imager, there are row and/or


column-wise multiplicative noises. We test the performance for column-wise multiplicative noise under the normal distribution case (see Fig. 5). Even with a very small amount of multiplicative Gaussian noise (MGN), the performance of the original Camus algorithm degrades since the effective noise power increase as in (6). For the office image sequence, the performance with pre-filtering is slightly less than the that of without pre-filtering. However, the result is opposite for the street image sequence. Therefore, pre-filtering without considering image contents and FPN noise characteristics may not be helpful for multiplicative noise since it is signal dependent.

FPNs are anisotropic in contrast with general additive noise which is isotropic noise in 3-D signal space. Due to the anisotropic property, FPNs have directionally different sensitivity to motion and filtering. For example, Fig. 5(b) has more performance degradation compared to Fig. 2(b) after pre-filtering. Since the street image is globally right moving sequence, Camus OFE is affected severely by the column-wise FPN. However, Fig. 5(a) shows less noise effect compared to Fig. 2(a). Since the office image is a zoom-in image sequence. These results support that FPN noise is signal dependent and the statistical independence between signal and noise is hard to achieve in real situation.

D. The multiplicative and additive noise (MAN) case

Figure 6 shows the simulation results of the performance degradation due to the change in multiplicative and additive noises under different combinations of the AGN, MGN, and BQN. The experiment cases are listed below,

Case 1: $\sigma_{n_1} = 8$ for AGN and $\sigma_{n_2} = 8$ for MGN under effective scaling
Case 2: $\sigma_{n_1} = 8$ for AGN and $\sigma_{n_2} = 13$ for MGN under effective scaling
Case 3: $\sigma_{n_1} = 13$ for AGN and $\sigma_{n_2} = 8$ for MGN under effective scaling

In these experiments, we can evaluate whether additive or multiplicative noise is more influential to the performance degradation for the same amount of change in the standard deviation. According to Fig. 6, the dominant noise factor depends on the image contents. Therefore, the impacts of multiplicative noise and additive noise are comparable. In addition, multiplicative noise is hard to remove since it is signal-dependent. Therefore, a multiplicative noise suppression block is necessary for an accurate real optical flow system based on a CMOS image sensor. As we can see in Fig. 6, the degradation for complex image is higher than that for simple image.

VI. CONCLUSIONS

In this paper, we present the performance test and analysis of a correlation-based optical flow estimation (CBOFE) algorithm, the Camus method, under various noise environments. Pre-filtering applied to the Camus optical flow algorithm is effective in reducing additive noise.

Pre-filtering without considering image contents is not much helpful for multiplicative noise since it is signal dependent. Therefore, the effects of multiplicative noise are comparable to that of additive noise if the standard deviations of noises are scaled to the same percentage of the full range of signal.

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