The Approximate Optimality of Simple Schedules for Half-Duplex Multi-Relay Networks

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Abstract—In ISIT’12 Brahma, Özgür and Fragiou conjectured that in a half-duplex diamond relay network (a Gaussian noise network without a direct source-destination link and with $N$ non-interfering relays) an approximately optimal relay scheduling (achieving the cut-set upper bound to within a constant gap uniformly over all channel gains) exists with at most $N + 1$ active states (only $N + 1$ out of the $2^N$ possible relay listen-transmit configurations have a strictly positive probability). Such relay scheduling policies are said to be simple. In ITW’13 we conjectured that simple relay policies are optimal for any half-duplex Gaussian multi-relay network, that is, simple schedules are not a consequence of the diamond network’s sparse topology. In this paper we formally prove the conjecture beyond Gaussian networks. In particular, for any memoryless half-duplex $N$-relay network with independent noises and for which independent inputs are approximately optimal in the cut-set upper bound, an optimal schedule exists with at most $N + 1$ active states. The key step of our proof is to write the minimum of a submodular function by means of its Lovász extension and use the greedy algorithm for submodular polyhedra to highlight structural properties of the optimal solution. This, together with the saddle-point property of min-max problems and the existence of optimal basic feasible solutions in linear programs, proves the claim.

I. INTRODUCTION

Adding relaying stations to today’s cellular infrastructure promises to boost both coverage and network throughput. Although higher performances could be attained with Full-Duplex (FD) relays, due to practical restrictions, such as the inability to perfectly cancel the self-interference, currently employed relays operate in Half-Duplex (HD).

This paper studies a general memoryless multi-relay network, where the communication between a source and a destination is assisted by $N$ relays operating in HD mode. The capacity of this network is not known in general. In [1] we showed that Noisy Network Coding (NNC) [2] achieves the cut-set upper bound [3] to within $1.96(N + 2)$ bits per channel use for a general Gaussian noise multi-relay network, universally over all channel gains, thus improving on previously known constant gap results. In general, finding the capacity of a HD multi-relay network is a combinatorial problem since the cut-set upper bound is the minimum between $2^N$ bounds (one for each possible cut in the network), each of which is a linear combination of $2^N$ relay states (since each relay can either transmit or receive). Thus, as the number of relays increases, optimizing the cut-set bound becomes prohibitively complex. Identifying structural properties of the cut-set upper bound, or of a constant gap approximation of the cut-set upper bound, is therefore critical for efficient numerical evaluations and can have important practical consequences for the design of reduced complexity / simple relaying policies.

In [4], the authors analyzed the Gaussian HD diamond relay network, a multi-relay network without a direct source-destination link, with $N = 2$ non-interfering relays and proved that at most $N + 1 = 3$ states, out of the $2^N = 4$ possible ones, suffice to characterize the capacity to within a constant gap. We say that these $N + 1$ states are active and form an (approximately optimal) simple schedule. In [5], Brahma et al verified through extensive numerical evaluations that in Gaussian HD diamond networks with $N \leq 7$ relays an optimal (to within a constant gap) schedule has at most $N + 1$ active states and conjectured this to be true for any $N$. In [6], Brahma et al’s conjecture was proved for Gaussian HD diamond networks with $N \leq 6$ relays; the proof is based on certain properties of submodularity and on linear programming duality; the proof technique does not appear to easily generalize to an arbitrary $N$. Our numerical experiments in [7] showed that Brahma et al’s conjecture on the existence of optimal simple schedules for diamond HD relay networks extends to any Gaussian HD multi-relay network (i.e., not necessarily with a diamond topology) with $N \leq 8$; we conjectured that the same holds for any $N$. Should our more general version of Brahma et al’s conjecture be true, then Gaussian HD multi-relay networks have optimal simple schedules irrespectively of their topology. In [8] we discussed polynomial time algorithms to determine the optimal simple schedule and extensions beyond relay networks.

Related works on determining the optimal relay scheduling, but not focused on characterizing the minimum number of active states, are available in the literature. For example [8] studied an iterative algorithm to determine the optimal schedule when the relays use decode-and-forward. In [9] the authors proposed a ‘grouping’ technique to compute the relay schedule that maximizes the approximate capacity of certain Gaussian HD relay networks; because finding a good node grouping is computationally complex, the authors proposed a heuristic approach based on tree decomposition which results in polynomial time algorithms; as for diamond networks in [5], the low-complexity algorithm of [9] relies on the ‘simplified’ topology of certain networks. As opposed to these works, we prove that a linear number of states is sufficient to determine an optimal schedule regardless of the network topology. We also note that in [10], FD relay networks were studied and
that “under the assumption of independent inputs and noises, the cut-set bound is submodular” [10] Theorem 1], a result that we shall use in the derivation of our main result.

The main result of this paper is a formal proof of Brahma et al’s conjecture beyond the Gaussian noise case. In particular, we prove that for any HD network with \( N \) relays, with independent noises and for which independent inputs in the cut-set bound are approximately optimal, the optimal relay policy is simple. The key idea is to use the Lovász extension and the greedy algorithm for submodular polyhedra to highlight structural properties of the minimum of a submodular function. Then, by using the saddle-point property of min-max problems and the existence of optimal basic feasible solutions for Linear Programs (LPs), an (approximately) optimal relay policy with the claimed number of active states can be shown. A polynomial time algorithm to find the optimal simple relay schedule is also discussed.

The rest of the paper is organized as follows. Section II describes the general memoryless HD multi-relay network. Section III summarizes some known results for submodular functions and LPs and then proves the main result. Finally, Section IV concludes the paper.

II. SYSTEM MODEL

A memoryless relay network has one source (node 0), one destination (node \( N+1 \)), and \( N \) relays (indexed from 1 to \( N \)). It consists of \( N+1 \) input alphabets \( \{X_0, X_1, \cdots, X_N\} \) where \( X_0 \) is the input alphabet of source 0 and \( X_1, \cdots, X_N \) are the input alphabets of source 1 to node \( N \). A destination (node \( N+1 \)) receives the output of node \( N \). The state transition probability is defined in the usual way (see for example [1]).

We first summarize some properties of submodular functions and LPs in Section III-A, we then prove Theorem 1 (see the term \( H(S_{[1:N]}, Y_{[1:N]}) \) in (3)) is the mutual information across the network cut \( A \subseteq \{1; N\} \) when a random schedule is employed, i.e., information is conveyed from the relays to the destination by switching between listen and transmit modes of operation at random times [11] (see the term \( H(S_{[1:N]}, Y_{[1:N]}) \) in (3)). \( I_{[A]}^{(\text{fix})} \) in (4) is the mutual information with a fixed schedule, i.e., the time instants at which a relay transitions between listen and transmit modes of operation are fixed and known to all nodes in the network [11] (see the term \( S_{[1:N]} \) in the conditioning in (4)). Note that fixed schedules are optimal to within \( N \) bits.

III. MAIN RESULT

We next consider networks for which the following holds: there exists a product input distribution

\[
P_{X_{[1; N+1]}|S_{[1; N]}} = \prod_{i \in [1; N]} P_{X_i|S_{[1; N]}} \tag{7a}
\]

for which we can evaluate the set function \( I_{[A]}^{(\text{fix})} \) in (4) for all \( A \subseteq \{1; N\} \) and bound the capacity as

\[
C' - G_1 \leq C \leq C' + G_2; \quad C' := \max_{P_{S_{[1; N]}}} \min_{A \subseteq [1; N]} I_{[A]}^{(\text{fix})}, \tag{7b}
\]

and where \( G_1 \) and \( G_2 \) are non-negative constants that may depend on \( N \) but not on the channel transition probability. In other words, we concentrate on networks for which using independent inputs and a fixed relay schedule in the cut-set bound provides both an upper bound, to within \( G_2 \) bits, and a lower bound, to within \( G_1 \) bits, on the capacity.

For example, for a general Gaussian multi-relay network with independent noises, independent Gaussian inputs are optimal to within \( G_1 + G_2 \leq 1.96(N+2) \) bits universally across all channel gains [11]. In [11] we conjectured that the optimal schedule in this case would be a simple one, i.e., the optimal probability mass function \( P_{S_{[1; N]}} \) in (7b) is such that at most \( N+1 \) entries have a strictly positive probability. This paper proves that not only the conjecture is true for the Gaussian noise case, but it also holds in more generality.

The main result of the paper is:

**Theorem 1.** Under the assumptions in (7) and of

\[
P_{Y_{[1; N+1]}|X_{[1; N+1]}, S_{[1; N]}} = \prod_{i \in [1; N+1]} P_{Y_i|X_{[1; N+1]}, S_{[1; N]}} \tag{7c}
\]

i.e., “independent noises”, simple relay policies are optimal in (7), i.e., the optimal probability mass function \( P_{S_{[1; N]}} \) has at most \( N+1 \) non-zero entries / active states.

We first summarize some properties of submodular functions and LPs in Section III-A, we then prove Theorem 1.
in Section [II-B] we discuss the computational complexity of finding optimal simple schedules in Section [II-C] and conclude with an example of a network with $N = 2$ relays in order to illustrate some of the steps in the proof in Section [II-D].

A. Submodular Functions, LPs and Saddle-point Property

The following are standard results in submodular function optimization [12] and LPs [13].

**Definition 1** (Submodular function, Lovász extension and greedy solution for submodular polyhedra). A set-function $f : 2^N \to \mathbb{R}$ is submodular if and only if, for all subsets $A_1, A_2 \subseteq [1 : N]$, we have $f(A_1) + f(A_2) \geq f(A_1 \cup A_2) + f(A_1 \cap A_2)$. Note that submodular functions are closed under non-negative linear combinations.

For a submodular function $f$ such that $f(\emptyset) = 0$, the Lovász extension is a function defined as

$$
\hat{f}(w) := \max_{x \in P(f)} \mathbf{w}^T \mathbf{x}, \quad \forall \mathbf{w} \in \mathbb{R}^N,
$$

where $P(f)$ is the submodular polyhedron defined as

$$
P(f) := \left\{ \mathbf{x} \in \mathbb{R}^N : \sum_{i \in A} x_i \leq f(A), \quad \forall A \subseteq [1 : N] \right\}.
$$

The optimal $\mathbf{x}$ in (8) can be found by the greedy algorithm for submodular polyhedra and has components

$$x_{\pi^{-}} = f(\{\pi_1, \ldots, \pi_i\}) - f(\{\pi_1, \ldots, \pi_{i-1}\}), \quad \forall i \in [1 : N],$$

where $\pi$ is a permutation of $[1 : N]$ such that the weights $\mathbf{w}$ are ordered as $w_{\pi_1} \geq w_{\pi_2} \geq \ldots \geq w_{\pi_N}$. Note that the Lovász extension is a piecewise linear convex function.

**Proposition 2** (Minimum of submodular functions). Let $f$ be a submodular function such that $f(\emptyset) = 0$ and $\hat{f}$ its Lovász extension. The minimum of the submodular function satisfies

$$
\min_{A \subseteq [1 : N]} f(A) = \min_{\mathbf{w} \in [0:1]^N} \hat{f}(\mathbf{w}) = \min_{\mathbf{w} \in [0:1]^N} \hat{f}(\mathbf{w}),
$$

i.e., $\hat{f}(\mathbf{w})$ attains its minimum at a vertex of $[0, 1]^N$.

**Definition 2** (Basic feasible solution). Consider the LP

$$
\begin{align*}
\max & \quad \mathbf{c}^T \mathbf{x} \\
\text{subject to} & \quad \mathbf{A} \mathbf{x} \leq \mathbf{b}, \quad \mathbf{x} \geq 0,
\end{align*}
$$

where $\mathbf{x} \in \mathbb{R}^n$ is the vector of unknowns, $\mathbf{b} \in \mathbb{R}^m$ and $\mathbf{c} \in \mathbb{R}^n$ are vectors of known coefficients, and $\mathbf{A} \in \mathbb{R}^{m \times n}$ is a known matrix of coefficients. If $m < n$, a solution for the LP with at most $m$ non-zero values is called a basic feasible solution.

**Proposition 3** (Optimality of basic feasible solutions). If a LP is feasible, then an optimal solution is at a vertex of the (non-empty and convex) feasible set $S = \{ \mathbf{x} \in \mathbb{R}^n : \mathbf{A} \mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq 0 \}$.

Moreover, if there is an optimal solution, then an optimal basic feasible solution exists as well.

A set-function $f$ is supermodular if and only if $-f$ is submodular, and it is modular if it is both submodular and supermodular.

**Proposition 4** (Saddle-point property). Let $\phi(x, y)$ be a function of two vector variables $x \in \mathcal{X}$ and $y \in \mathcal{Y}$. By the minimax inequality we have

$$
d^* := \max_{y \in \mathcal{Y}} \min_{x \in \mathcal{X}} \phi(x, y) \leq \min_{x \in \mathcal{X}} \max_{y \in \mathcal{Y}} \phi(x, y) := p^*.
$$

and equality holds, i.e., $p^* = d^*$, if (i) $\mathcal{X}$ and $\mathcal{Y}$ are both convex and one of them is compact; (ii) $\phi(x, y)$ is convex in $x$ and concave in $y$; (iii) $\phi(x, y)$ is continuous.

B. Proof of Theorem 7

The objective is to show that simple relay policies are optimal in (7b). The proof consists of the following steps:

1) We first show that the function $I_{\mathcal{A}}(\mathcal{I})$ defined in (4) is submodular under the assumptions in (7).

2) By using Proposition 2 we show that the problem in (7b) can be recast into an equivalent max-min problem.

3) With Proposition 3 we show that the max-min problem is equivalent to solve a min-max problem. The min-min problem is then shown to be equivalent to solve $N!$ max-min problems, for each of which we obtain an optimal basic feasible solution by Proposition 5 with the claimed maximum number of non-zero entries.

**STEP 1**: We show that $I_{\mathcal{A}}(\mathcal{I})$ in (4) is submodular. The result in [10] Theorem 1 showed that $f_s(\mathcal{A})$ in (5) is submodular for each relay state $s \in [0 : 1]^N$ under the assumption of independent inputs and independent noises (the same work provided an example of a diamond network with correlated inputs, and showed that in this case the cut-set bound is neither submodular nor supermodular). Since submodular functions are closed under non-negative linear combinations (see Definition 1), this implies that $I_{\mathcal{A}}(\mathcal{I}) := \sum_{s \in [0:1]^N} \lambda_s \mathcal{f}_s(\mathcal{A})$ is submodular under the assumptions of Theorem 1.

**STEP 2**: Given that $I_{\mathcal{A}}(\mathcal{I})$ in (4) is submodular, we would like to use Proposition 2 to ‘replace’ the minimization over the subsets of $[1 : N]$ in (7b) with a minimization over the cube $[0 : 1]^N$. Since $I_{\mathcal{A}}(\mathcal{I}) = I(\mathcal{X}_{[1:N+1]}; \mathcal{Y}_{N+1}; S_{[1:N]}) \geq 0$ in general, we define a new submodular function $g(\mathcal{A}) := I_{\mathcal{A}}(\mathcal{I}) - I_{\emptyset}(\mathcal{I})$ and proceed as in (9) at the top of the next page to show that the problem in (7b) is equivalent to

$$
C' := \max_{\lambda_{\text{vect}}} \min_{w \in [0,1]^N} \left\{ \mathbf{1}_x \mathbf{w}^T \mathbf{H}_{\pi,f}\lambda_{\text{vect}} \right\},
$$

where $\lambda_{\text{vect}}$ is the probability mass function of $S_{[1:N]}$ (in particular, $\lambda_{\text{vect}} := [\lambda_s] \in \mathbb{R}^{2^{N+1} \times 1}$ where $\lambda_s := \mathbf{P}[S_{[1:N]} = s] \in [0, 1]$, for $s \in [0 : 1]^N$ such that $\sum_{s \in [0:1]^N} \lambda_s = 1$, $\mathbf{H}_{\pi,f} \in \mathbb{R}^{N(N+1) \times 2^{N+1}}$ and $\mathbf{P}_{\pi} \in \mathbb{R}^{N(N+1) \times 2^{N+1}}$ are defined in (10) at the top of the next page, $\mathbf{P}_{\pi} \in \mathbb{R}^{N(N+1) \times 2^{N+1}}$ is the permutation matrix that maps $[1, w_1, \ldots, w_N]$ into $[1, w_2, \ldots, w_N, w_1]$, and $f_s(\mathcal{A})$ was defined in (6). We thus express our original optimization problem as the max-min problem in (14).

**STEP 3**: In order to solve (14) we would like to reverse the order of min and max. We note that the function $\phi(\lambda_{\text{vect}}, \mathbf{w}) := [\mathbf{1}_x \mathbf{w}^T \mathbf{H}_{\pi,f}\lambda_{\text{vect}}]$ satisfies the properties in Proposition 2 (it is continuous, convex in $\mathbf{w}$ by the convexity of the Lovász extension and linear, thus concave, in $\lambda_{\text{vect}}$;
\[
\min_{\mathcal{A} \subseteq [1:N]} I_A^{(\text{fix})} = I_0^{(\text{fix})} + \min_{\mathcal{A} \subseteq [1:N]} g(\mathcal{A})
\]
\[
= I_0^{(\text{fix})} + \min_{w \in [0,1]^N} \left[ w_{\pi_1}, w_{\pi_2}, \ldots, w_{\pi_N} \right]
\]
\[
\begin{bmatrix}
g(\{\pi_1\}) - g(\emptyset) \\
g(\{\pi_1, \pi_2\}) - g(\{\pi_1, \ldots, \pi_{N-1}\}) \\
\vdots
\end{bmatrix}
\]
\[
= I_0^{(\text{fix})} + \min_{w \in [0,1]^N} \left[ w_{\pi_1}, w_{\pi_2}, \ldots, w_{\pi_N} \right]
\]
\[
\begin{bmatrix}
I^{(\text{fix})}_{\{\pi_1\}} - I_0^{(\text{fix})} \\
I^{(\text{fix})}_{\{\pi_1, \pi_2\}} - I^{(\text{fix})}_{\{\pi_1, \ldots, \pi_{N-1}\}} \\
\vdots
\end{bmatrix} := \min_{w \in [0,1]^N} \left\{ [1, w^T] \mathbf{H}_{\pi, f} \right\};
\]
\[
\mathbf{H}_{\pi, f} := \mathbf{P}_\pi
\]
\[
\begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ -1 & 1 & 0 & \ldots & 0 \\ 0 & -1 & 1 & \ldots & 0 \\ \vdots \\ 0 & 0 & \ldots & -1 & 1 \end{bmatrix}
\]
\[
(N+1) \times (N+1)
\]
\[
g(\mathcal{A}) = I_A^{(\text{fix})} - I_0^{(\text{fix})}, \mathcal{A} \subseteq [1:2]: \quad \tilde{g}(w_1, w_2) = \begin{bmatrix} w_1 g(\{1\}) + w_2 g(\{1,2\}) - g(\{1\}) \\ w_2 g(\{2\}) + w_1 g(\{1,2\}) - g(\{2\}) \end{bmatrix}
\]
\[
\text{if } w_1 \geq w_2 \quad \text{if } w_2 \geq w_1
\]
\[
P_1 : \max_{\lambda_{\text{vect}}} \min_{0 \leq w_2 \leq w_1 \leq 1} \left[ 1 - w_1 \right]
\]
\[
\begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \end{bmatrix}
\]
\[
\begin{bmatrix} f_0(\emptyset) & f_1(\emptyset) & f_2(\emptyset) & f_3(\emptyset) \\ f_0(\{1\}) & f_1(\{1\}) & f_2(\{1\}) & f_3(\{1\}) \\ f_0(\{1,2\}) & f_1(\{1,2\}) & f_2(\{1,2\}) & f_3(\{1,2\}) \end{bmatrix}
\]
\[
\begin{bmatrix} \lambda_0 \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}
\]
\[
P_2 : \maximize_{\tau} \text{subject to}
\]
\[
\tau \leq f_0(\emptyset)\lambda_0 + f_1(\emptyset)\lambda_1 + f_2(\emptyset)\lambda_2 + f_3(\emptyset)\lambda_3,
\]
\[
\tau \leq f_0(\{1\})\lambda_0 + f_1(\{1\})\lambda_1 + f_2(\{1\})\lambda_2 + f_3(\{1\})\lambda_3,
\]
\[
\tau \leq f_0(\{1,2\})\lambda_0 + f_1(\{1,2\})\lambda_1 + f_2(\{1,2\})\lambda_2 + f_3(\{1,2\})\lambda_3,
\]
\[
\lambda_0 + \lambda_1 + \lambda_2 + \lambda_3 = 1, 
\quad \lambda_i \geq 0 \quad \forall i \in [0:3]
\]

moreover the optimization domain in both variables is compact. Thus, we now focus on the problem
\[
C' = \min_{w \in [0,1]^N} \max_{\lambda_{\text{vect}}} \left\{ [1, w^T] \mathbf{H}_{\pi, f, \lambda_{\text{vect}}} \right\},
\]
which can be equivalently rewritten as
\[
C' = \min_{\pi \in \mathcal{P}_N} \min_{w_{\pi} \in [0,1]^N} \max_{\lambda_{\text{vect}}} \left\{ [1, w_{\pi}^T] \mathbf{H}_{\pi, f, \lambda_{\text{vect}}} \right\},
\]
\[
= \min_{\pi \in \mathcal{P}_N} \min_{\lambda_{\text{vect}}} \min_{w_{\pi} \in [0,1]^N} \left\{ [1, w_{\pi}^T] \mathbf{H}_{\pi, f, \lambda_{\text{vect}}} \right\},
\]
where \(\mathcal{P}_N\) is the set of all the \(N!\) permutations of \([1:N]\).

In (16), for each permutation \(\pi \in \mathcal{P}_N\), we first find the optimal \(\lambda_{\text{vect}}\), and then the optimal \(w_{\pi} = w_{\pi_1} \geq w_{\pi_2} \geq \ldots \geq w_{\pi_N}\).

This is equivalent to (17), where again by Proposition 4 for each permutation \(\pi \in \mathcal{P}_N\), we first find the optimal \(w_{\pi} = w_{\pi_1} \geq w_{\pi_2} \geq \ldots \geq w_{\pi_N}\), and then find the optimal \(\lambda_{\text{vect}}\).

Let’s now consider the inner optimization in (17), that is, the problem
\[
P_1 : \max_{\lambda_{\text{vect}}} \min_{w_{\pi} \in [0,1]^N} \left\{ [1, w_{\pi}^T] \mathbf{H}_{\pi, f, \lambda_{\text{vect}}} \right\}.
\]

From Proposition 2 we know that, for a given \(\pi \in \mathcal{P}_N\), the optimal \(w_{\pi} = w_{\pi_1} \geq w_{\pi_2} \geq \ldots \geq w_{\pi_N}\) is a vertex of the cube \([0:1]^N\). For a given \(\pi \in \mathcal{P}_N\), there are \(N + 1\) vertices whose coordinates are ordered according to \(\pi\). In (18), for each of the \(N + 1\) feasible vertices of \(w_{\pi}\), it is easy to see that the product \([1, w_{\pi}^T] \mathbf{H}_{\pi, f}\) is equal to a row of the matrix \(\mathbf{F}_\pi\). By considering all possible \(N + 1\) feasible vertices compatible with \(\pi\) we obtain all the \(N + 1\)
we have a polynomial time algorithm that converges to the worst-case dual LP is solvable in polynomial time in $\lambda_{\text{vec}}$ by this is solvable in strongly polynomial time in approach would be prohibitive for large functions, each given by one of the rows of in (13) at the top of the previous page is the minimum of three $w$ $P$ $w$ $P$ schedule requires the solution of of Theorem 1. has an optimal basic feasible solution with at most zero), it means that $\tau > 0$ for (otherwise the channel capacity would be zero), these are optimal in the cut-set upper bound to within a constant gap for all choices of the channel matrices.

IV. CONCLUSIONS

In this work we studied networks with $N$ half-duplex relays. For such networks, the capacity achieving scheme must be optimized over the $2^N$ possible listen-transmit relay configurations. This paper formally proved that, if noises are independent and independent inputs are approximately optimal in the cut-set bound, then the approximately optimal schedule only uses $N + 1$ relay configurations.

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REFERENCES


As mentioned earlier, the result of this paper proves our original conjecture in [2] for Gaussian SISO networks for any number $N$ of relays. Our framework also immediately extends to Gaussian networks with MIMO relays and independent noises since also in this setting independent inputs at all nodes are optimal in the cut-set upper bound to within a constant gap for all choices of the channel matrices.