Predictive Control in Power Electronics and Drives

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Abstract—Predictive control is a very wide class of controllers that have found rather recent application in the control of power converters. Research on this topic has been increased in the last years due to the possibilities of today’s microprocessors used for the control. This paper presents the application of different predictive control methods to power electronics and drives. A simple classification of the most important types of predictive control is introduced, and each one of them is explained including some application examples. Predictive control presents several advantages that make it suitable for the control of power converters and drives. The different control schemes and applications presented in this paper illustrate the effectiveness and flexibility of predictive control.

Index Terms—Drives, power electronics, predictive control.

I. INTRODUCTION

The use of power converters has become very popular in the recent decades for a wide range of applications, including drives, energy conversion, traction, and distributed generation. The control of power converters has been extensively studied, and new control schemes are presented every year.

Several control schemes have been proposed for the control of power converters and drives. Some of them are shown in Fig. 1. From these, hysteresis and linear controls with pulse-width modulation (PWM) are the most established in the literature [1]–[3]. However, with the development of faster and more powerful microprocessors, the implementation of new and more complex control schemes is possible. Some of these new control schemes for power converters include fuzzy logic, sliding mode control, and predictive control. Fuzzy logic is suitable for applications where the controlled system or some of its parameters are unknown. Sliding mode presents robustness and takes into account the switching nature of the power converters. Other control schemes found in the literature include neural networks, neuro–fuzzy, and other advanced control techniques.

II. PREDICTIVE CONTROL METHODS

Predictive control is a very wide class of controllers that have found rather recent application in power converters. The classification proposed in this paper for different predictive control methods is shown in Fig. 2.

The main characteristic of predictive control is the use of the model of the system for the prediction of the future behavior of the controlled variables. This information is used by the controller in order to obtain the optimal actuation, according to a predefined optimization criterion.

The optimization criterion in the hysteresis-based predictive control is to keep the controlled variable within the boundaries of a hysteresis area, while in the trajectory-based, the variables are forced to follow a predefined trajectory. In deadbeat control, the optimal actuation is the one that makes the error equal to zero in the next sampling instant. A more flexible criterion is used in MPC, expressed as a cost function to be minimized.

The difference between these groups of controllers is that deadbeat control and MPC with continuous control set need a modulator, in order to generate the required voltage. This will result in having a fixed switching frequency. The other
controllers directly generate the switching signals for the converter, do not need a modulator, and present a variable switching frequency.

One advantage of predictive control is that concepts are very simple and intuitive. Depending on the type of predictive control, implementation can also be simple, as with deadbeat control and finite control set MPC (FS-MPC) (particularly for a two-level converter with horizon \( N = 1 \)). However, in general, some implementations of MPC can be more complex. Variations of the basic deadbeat control, in order to make it more robust, can also become very complex and difficult to understand.

Using predictive control, it is possible to avoid the cascaded structure, which is typically used in a linear control scheme, obtaining very fast transient responses. An example of this is the speed control using trajectory-based predictive control.

Nonlinearities of the system can be included in the model, avoiding the need of linearizing the model for a given operating point and improving the operation of the system for all conditions. It is also possible to include restrictions to some variables when designing the controller. These advantages can be very easily implemented in some control schemes as MPC, but it is very difficult in schemes as deadbeat control.

A more detailed description of each type of predictive control is shown in the next sections.

III. HYSTERESIS-BASED PREDICTIVE CONTROL

Hysteresis-based predictive control strategies try to keep the controlled system variables between the boundaries of a hysteresis area or space. The most simple form of this principle is the so-called “bang–bang controller.” Although bang–bang controllers usually are not considered as predictive controllers in literature, they clearly show the characteristics of a typical predictive controller. An improved form of a bang–bang controller is the predictive current controller proposed by Holtz and Stadtfeld [4]. The block diagram of the hysteresis-based predictive control is shown in Fig. 3.

Using predictive current control, the switching instants are determined by suitable error boundaries. As an example, Fig. 4 shows a circular boundary, the location of which is controlled by the current reference vector \( i_s^* \). When the current vector \( i_s \) touches the boundary line, the next switching state vector is determined by prediction and optimization.

The trajectories of the current vector for each possible switching state are computed, and predictions are made of the respective time intervals required to reach the error boundary again. These events also depend on the location of the error boundary, which is considered moving in the complex plane as commanded by the predicted current reference. The movement is indicated by the dotted circle in Fig. 4. The predictions of the switching instants are based on mathematical equations of the machine. The switching state vector that produces the maximum on-time is finally selected. This corresponds to minimizing the switching frequency.

The maximum possible switching frequency is limited by the computing time of the algorithms which determine the optimal switching state vector. Higher frequencies can be handled by employing the double prediction method: Well before the boundary is reached, the actual current trajectory is predicted in order to identify the time instant at which the boundary transition is likely to occur. The back electromotive
force (EMF) vector at this time instant is predicted then. It is used for the optimal selection of the future switching state vector using the earlier described procedure.

A further reduction of the switching frequency, which may be needed in very high-power applications, can be achieved by defining a current error boundary of rectangular shape, having the rectangle aligned with the rotor flux vector of the machine. Using field-oriented predictive current control, the switching frequency can be reduced more than with a circular boundary area in stator coordinates [5]. Holtz and Stadtfeld optimized their predictive controller for minimum switching frequency. Today, different optimizing criteria are considered, e.g., low current distortion or low electromagnetic interferences (EMIs). Modifications of the predictive current control are consequently under consideration.

IV. TRAJECTORY-BASED PREDICTIVE CONTROL

The principle of trajectory-based predictive control strategies is to force the system’s variables onto precalculated trajectories. Control algorithms according to this strategy are direct self control by Depenbrock [6] or direct mean torque control by Flach et al. [7]. Some additional methods like sliding mode control [8] or direct torque control [9] are a combination of hysteresis- and trajectory-based strategies, whereas direct speed control (DSPC) by Mutschler [10] can be identified as a trajectory-based control system, although it also has a few hysteresis-based aspects. DSPC will be further explained as an example of trajectory-based predictive controllers.

Unlike cascade controllers, predictive control algorithms offer the possibility to directly control the desired system values. Most predictive control methods published so far only deal with stator currents, torque, or flux (linear) directly; the drive speed is controlled by a superimposed control loop. DSPC, shown in Fig. 5, in contrast, has no control loop of this type; the switching events in the inverter are calculated in a way where speed is directly controlled in a time-optimal manner.

Similar to the methods of Depenbrock [6] and Takahashi and Nogushi [9], the switching states of the inverter are classified as “torque increasing,” “slowly torque decreasing,” or “rapidly torque decreasing.” For small time intervals, the inertia of the system and the derivatives of machine and load torques are assumed as constant values. The behavior of the system leads to a set of parabolas in the speed error versus acceleration area as shown in Fig. 6.

The initial state of the system is assumed to be $e_k/a_k$. In this state, a torque increasing voltage vector has to be produced by the inverter, and therefore, the switching state $S_k$ is chosen. The state now travels along the dotted parabola until the point $e_{k+1}/a_{k+1}$ is reached. This is the intersection with another parabola for a “torque decreasing” switching state $S_{k+1}$, which will pass through the point “+Hy.” The intersection $e_{k+1}/a_{k+1}$ has been precalculated as the optimal switching instant to reach the desired state point “+Hy” as fast as possible. Therefore, in $e_{k+1}/a_{k+1}$, the inverter is commutated into the switching state $S_{k+1}$. Then, the state of the system travels along the new parabola until the point $e_{k+2}/a_{k+2}$ is reached. At this point, the inverter is switched again into a torque increasing state $S_{k+2}$. The corresponding trajectory passes the point “−Hy.” In steady state, the state moves along the path $+Hy - e_{k+2}/a_{k+2} - Hy - e_{k+3}/a_{k+3} - +Hy$. Hence, the speed error $e$ is kept in the hysteresis band between $−Hy$ and $+Hy$. This is the hysteresis aspect of this strategy aforementioned. Of course, the optimal steady-state point would be the point of origin. Since the switching frequency of the inverter is limited, the drive state cannot be fixed to that point. Therefore, the hysteresis band is defined to keep the switching frequency in an acceptable range.

The algorithm of DSPC clearly shows the main principle of predictive control that foreknowledge of the drive system is used to precalculate the optimal switching states instead of trying to linearize the nonlinear parts of the system and then control them by PI controllers. The speed can be controlled directly without a cascade structure.

V. DEADBEAT-BASED PREDICTIVE CONTROL

A well-known type of predictive controller is the deadbeat controller. This approach uses the model of the system to calculate, once every sampling period, the required reference voltage in order to reach the reference value in the next sampling instant. Then, this voltage is applied using a modulator. It has been applied for current control in three-phase inverters [11]–[21], rectifiers [22], [23], active filters [24], [25], power factor correctors [26], power factor preregulators [27], [28], uninterruptible power supplies [29]–[31], dc–dc converters [32], and torque control of induction machines [33]. While this method has been used when a fast dynamic response is required, being deadbeat-based, it is often fragile. Indeed, errors in the parameter values of the model, unmodeled delays and other
errors in the model often deteriorate system performance and may even give rise to instability. Another disadvantage of these deadbeat control schemes is that non-linearities and constraints of the system variables are difficult to incorporate.

A. Deadbeat Current Control

A typical deadbeat current control scheme is shown in Fig. 7. It can be noted that, compared to a classic current control scheme, the PI controller has been replaced by the deadbeat controller. The reference voltage is applied using a modulator.

The load model for a generic RLE load is described by the following space vector equation

$$v = Ri + L \frac{di}{dt} + e \tag{1}$$

where $v$ is the voltage space vector, $i$ is the current space vector, and $e$ is the EMF voltage space vector.

The following discrete-time equation can be obtained from (1) for a sampling time $T_s$:

$$\frac{1}{\delta} i(k + 1) - \chi i(k) = v(k) - e(k) \tag{2}$$

where $\delta = e^{-T_s R/L}$ and $\chi = 1/R(1 - e^{-T_s R/L})$.

Based on the discrete-time model (2), the reference voltage vector is obtained as

$$v^*(k) = \frac{1}{\delta} [i^*(k + 1) - \chi i(k)] + e(k). \tag{3}$$

Reference voltage $v^*$ is applied in the converter using a modulator.

The basic operating principle of deadbeat current control is shown in Fig. 8. Here, the load current $i$ at time $k$ is different to the reference current $i^*$. This error is used for calculation of the reference voltage $v^*$, which is applied to the load at time $k$. Ideally, at time $k + 1$, the load current will be equal to the reference current.

B. Modifications to Basic Algorithm

When implemented in a real system, several problems may appear and deteriorate the performance of a deadbeat controller. One of them is the delay introduced by calculation time and modulation. This problem has been solved in [17], [18], and [20] by considering this delay in the model.

Another important issue is the sensitivity to plant uncertainties and errors in the model parameter values. This problem has been studied, and several solutions have been proposed, including the use of an adaptive self-tuning scheme [34], a predictive internal model [35], and neural networks [36]. Some results comparing the implementation of the conventional deadbeat current controller, i.e., using (3), implemented in a full digital system without any compensation of the calculation delay, and the modified deadbeat controller proposed in [34] are shown in Fig. 9.

In some applications, information about the disturbances is needed by the controller, and these include variables which are not measured. In these cases, the use of disturbance observers has been proposed [28], [30]. Other specific applications can require a modified algorithm for reduced switching frequency, as proposed in [37].

VI. MPC

MPC, also referred to as receding horizon control, is the only one among the so-called advanced control techniques (usually understood as techniques more advanced than a standard PID control) which has been extremely successful in practical applications in recent decades, exerting a great influence on research.
and development directions of industrial control systems. Applications and theoretical results abound; see, e.g., the books [38]–[40] and survey papers [41]–[44]. An attractive feature of MPC is that it can handle general constrained nonlinear systems with multiple inputs and outputs in a unified and clear manner.

In this section, we will focus on MPC formulations where a continuous control set is considered and controller outputs are first passed through modulators, which then provide the switch positions. A survey on FS-MPC formulations which are first passed through modulators, which then provide the switch positions. Consequently, we will study configurations where the controller output feeds into a modulator stage which then provides the switch positions. Subsequently, we will restrict the system inputs \{u(k)\} in (4) according to

\[
\{ u(k) \} \subseteq \mathbb{R}^p, \quad k \in \{0, 1, 2, \ldots\}
\]

where \(\mathbb{R}\) is a polytope and \(p\) denotes the number of switches. For example, the components of \(u(k)\) could correspond to duty cycles or PWM reference signals, in which case, \(\mathbb{U}\) is formed of intervals, namely, \(\mathbb{U} = [0, 1]^p\). Clearly, the aforementioned model can only approximate switching effects, see also [45]. Nevertheless, as we will see, several interesting proposals for power electronics and drives have been developed by using this simple setting.

In addition to the constraints on the system inputs, MPC also allows one to incorporate state constraints, for example

\[
x(k) \in \mathbb{X} \subseteq \mathbb{R}^n, \quad k \in \{0, 1, 2, \ldots\}.
\]

State constraints can, for example, correspond to constraints on capacitor voltages in flying capacitor converters or neutral point clamped converters. Constraints on inductive load currents can also be modeled as state constraints.

### A. System Model

Most MPC strategies are formulated in a discrete-time setting with a fixed sampling interval, for example, \(h > 0\). Here, system inputs are restricted to change their values only at the discrete sampling instants, i.e., at times \(t = kh\), where \(k \in \{0, 1, 2, \ldots\\}\) denotes the sampling instants.

Since power electronics applications are often governed by nonlinear dynamic relations, it is convenient to represent the system to be controlled in discrete-time state space form via

\[
x(k+1) = f(x(k), u(k)), \quad k \in \{0, 1, 2, \ldots\}
\]

where \(x(k)\) denotes the state value at time \(k\), whereas \(u(k)\) is the plant input.

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### B. Cost Function

In MPC, at each time instant \(k\) and for a given (measured or estimated) plant state \(x(k)\), a cost function over a finite horizon of length \(N\) is minimized. The following choice encompasses many alternatives documented in the literature:

\[
V(x(k), \bar{u}(k)) \triangleq \sum_{\ell=k}^{k+N-1} L(x(\ell), u(\ell)) + F(x'(k + N))
\]

Here, \(L(\cdot, \cdot)\) and \(F(\cdot)\) are weighting functions which serve to penalize predicted system behavior, e.g., differences between voltage references and predicted values; see Section VI-D. Predicted plant state values are formed according to

\[
x'(\ell+1) = f(x'(\ell), u'(\ell)), \quad \ell \in \{k, k+1, \ldots, k+N-1\}
\]

where

\[
u'(\ell) \in \mathbb{U}, \quad \ell \in \{k, k+1, \ldots, k+N-1\}
\]

refers to tentative plant inputs. The recursion (8) is initialized with the current plant state measurement, i.e.,

\[
x'(k) \leftarrow x(k).
\]

Thus, (8) refers to the predictions of the plant states which would result if the plant input at the time instances \(\{k, k+1, \ldots, k+N-1\}\) was set equal to the corresponding values in\(^1\)

\[
\bar{u}'(k) \triangleq \{u'(k), u'(k+1), \ldots, u'(k+N-1)\}.
\]

Both the predicted plant state trajectory and the plant inputs are constrained in accordance with (5), i.e., we have

\[
u'(\ell) \in \mathbb{U} \quad x'(\ell) \in \mathbb{X} \quad \forall \ell \in \{k, k+1, \ldots, k+N-1\}.
\]

In addition, \(x'(k+N)\) is typically required to satisfy a given terminal state constraint, for example

\[
x'(k+N) \in \mathbb{X}_f \subseteq \mathbb{X}.
\]

Note that, generally, the selection of the terminal state constraint is related to the stability issues [38], [39].

The constrained minimization of \(V(\cdot, \cdot)\) as in (7) gives the optimizing control sequence at time \(k\) and for state \(x(k)\)

\[
\bar{u}(k) \triangleq \{u(k; k), u(k+1; k), \ldots, u(k+N-1; k)\}.
\]

### C. Moving Horizon Optimization

Despite the fact that \(\bar{u}(k)\) contains feasible plant inputs over the entire horizon, in standard MPC, only the first element is used, i.e., the system input is set to

\[
u(k) \leftarrow u(k; k).
\]

At the next sampling step, i.e., at time \(k+1\), the system state \(x(k+1)\) is measured (or estimated), the horizon is shifted by

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that is large enough, the effect of \(u(k)\) on \(x'(\ell)\) for \(\ell > k + N\) will be negligible, and consequently, MPC will approximate the performance of an infinite horizon optimal controller. On the other hand, the constrained optimization problem which needs to be solved online to find the controller output, has computational complexity which, in general, increases with the horizon length. As a consequence, the optimization horizon parameter \(N\) allows the designer to tradeoff performance versus online computational effort. Fortunately, excellent performance can often be achieved with relatively small horizons.

Interestingly, in some situations, the stability of the closed loop model \(x(k + 1) = f(x(k), u(k; k))\) can be ensured through the choice of the MPC design parameters, see, e.g., [40], [42], and [48]. For power electronics applications controlled via MPC, establishing stability results remains, to date, an open problem. However, there is a significant repository of tools in the control community that merit an investigation of their applicability in power electronics applications (see [49] and [50]).

### E. Linear Quadratic Case

A particularly simple case of (4)–(7) arises when the system model is linear and the cost function is quadratic, i.e.,

\[
x(x(k + 1) = Ax(k) + Bu(k)
\]

\[
V(x(k), \bar{u}(k)) = x'(k + N)TPx'(k + N)
\]

\[
+ \sum_{\ell = k}^{k + N - 1} \{x'(\ell)TQx'(\ell) + u'(\ell)TRu'(\ell)\}
\]

where \(A, B, P, Q,\) and \(R\) are given matrices of appropriate dimensions.

If the system inputs and states in (16) are unconstrained, then the moving horizon optimization of the aforementioned cost function leads to a linear time invariant controller. This simple approach was investigated in [51] and [52] for use in drive applications.

On the other hand, if the inputs and states are constrained to belong to polytopes, then minimizing the cost function amounts to solving a convex quadratic program. In this case, semieplcit solutions to the MPC optimization problem can be found. In the associated Explicit MPC schemes, some of the computations can be carried out offline, thus alleviating online computational burden; see [53] and [54]. Several works, including [44] and [51], have studied constrained linear quadratic MPC in the context of power electronics and drive applications. We will next revise the approach taken in [44].

### F. Explicit MPC for Drive Control

The work [44] illustrates the advantages of replacing the PI current controller in the field-oriented drive control scheme in Fig. 11 by Explicit MPC.

The dynamics of an induction machine are governed by nonlinear differential equations. To obtain a model suitable for Explicit MPC, see (16), in [44], the time-continuous machine model was discretized, and the nonlinear cross-coupling
Fig. 11. Typical field-oriented control of an induction machine [44].

Fig. 12. Experimental tracking performance of current control: (a) With PI controllers designed according to symmetrical optimum and (b) with model predictive controller [44].

between the stator current components $i_{sd}$ and $i_{sq}$ was neglected. The state variable $x(k)$ in (16) was chosen as

$$x(k) = [i_{sd}(k) \quad i_{sq}(k)]^T.$$ 

Despite the fact that the model used is only approximate, it turns out that the Explicit MPC formulation adopted can outperform standard PI control without requiring significant additional computation times. Fig. 12(a) and (b) shows experimental results. In this figure, $i_{sq}^*$ is the reference of the stator current component, whereas $u_{sq}^*$ refers to the stator voltage component, i.e., the control input, which is fed to the PWM module, see Fig. 11.

G. Comments

The results of [44] are certainly promising. However, we feel that MPC has significantly more to offer than replacing individual modules within a cascaded control structure. Indeed, the main advantage of MPC, with respect to other control approaches, lies in the possibility to control nonlinear and constrained systems. In addition, various objectives, such as reference tracking and disturbance compensation, can be incorporated. Thus, it is worth studying the feasibility of developing MPC architectures which govern the entire drive control architecture without using any additional PI loops or PWM modules. For that purpose, nonlinearities need to be taken into account, and the finite set nature of switching states should be respected.

In the following section, we will revise some MPC formulations where the switches are controlled directly, without using PWM modules.

VII. FS-MPC

Considering the discrete nature of power converters, it is possible to simplify the optimization problem of MPC by avoiding the use of modulators. Taking into account the finite set of possible switching states of the power converter, which depends on the possible combinations of the on/off switching states of the power switches, the optimization problem is reduced to the evaluation of all possible states and the selection of the one which minimizes the given cost function. In addition to this, if the horizon length is set to $N = 1$, the calculation of the optimal actuation is very simple and easy to implement experimentally, as will be shown later in this section.

A. System Model

When modeling a converter, the basic element is the power switch, which can be an insulated-gate bipolar transistor, a thyristor, a gate turn-off thyristor, or others. The simplest model of these power switches considers an ideal switch with only two states: on and off. Therefore, the total number of switching states of a power converter is equal to the number of the different combinations of the two switching states of each switch. However, some combinations are not possible, for example, those combinations that short-circuit the dc link. Let us consider the example of an H-bridge single-phase inverter. It has four switches; therefore, the total number of combinations is $2^4 = 16$. However, two switches in the same leg of the inverter cannot be on at the same time; therefore, it is usual to drive them as complementary switches. This way, the number of possible states is reduced, and each leg has only two states. Therefore, the number of possible states is $2^2 = 4$.

As a general rule, the number of possible switching states can be calculated as $x^y$, where $x$ is the number of possible states of each leg of the converter and $y$ is the number of phases (or legs) of the converter. This way, a three-phase two-level converter has $2^3 = 8$ possible switching states, a three-phase three-level converter has $3^3 = 27$ switching states, and a five-phase two-level converter has $2^5 = 32$ switching states. Some examples of different converter topologies and their corresponding number of possible switching states are shown in Table I. For some other converter topologies, the way of calculating the possible switching states may be different.
### TABLE I

**POSSIBLE SWITCHING STATES FOR DIFFERENT CONVERTER TOPOLOGIES**

<table>
<thead>
<tr>
<th>Converter</th>
<th>Switching states</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-phase inverter</td>
<td>8</td>
</tr>
<tr>
<td>3-phase NPC inverter</td>
<td>27</td>
</tr>
<tr>
<td>5-phase inverter</td>
<td>32</td>
</tr>
<tr>
<td>Matrix converter</td>
<td>27</td>
</tr>
<tr>
<td>Direct converter</td>
<td>9</td>
</tr>
<tr>
<td>Flying capacitor</td>
<td>8</td>
</tr>
</tbody>
</table>

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**Fig. 13.** FS-MPC.

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**TABLE II**

**POSSIBLE SWITCHING STATES AND VOLTAGE VECTORS FOR A THREE-PHASE INVERTER**

<table>
<thead>
<tr>
<th>$S_a$</th>
<th>$S_b$</th>
<th>$S_c$</th>
<th>$v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$v_0 = 0$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$v_1 = \frac{1}{3} V_{dc}$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>$v_2 = \frac{1}{3} V_{dc} + j \frac{\sqrt{3}}{3} V_{dc}$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$v_3 = -\frac{1}{3} V_{dc} + j \frac{\sqrt{3}}{3} V_{dc}$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>$v_4 = -\frac{1}{3} V_{dc} - j \frac{\sqrt{3}}{3} V_{dc}$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$v_5 = -\frac{1}{3} V_{dc} - j \frac{\sqrt{3}}{3} V_{dc}$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>$v_6 = \frac{1}{3} V_{dc} + j \frac{\sqrt{3}}{3} V_{dc}$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$v_7 = 0$</td>
</tr>
</tbody>
</table>

---

**B. Horizon Length**

When a horizon length $N$ is used, the number of possible input sequences, considering the possible switching states of the converter, can be quite large. Then, the idea of predicting the behavior of the system for all possible switching state sequences becomes difficult to apply in a real system. A simple solution is the use of $N = 1$, reducing the number of calculations to the number of possible switching states of the converter.

The block diagram of an FS-MPC considering a prediction horizon $N = 1$ is shown in Fig. 13. Here, the state variables of the system $x(k)$ are measured (or estimated) and used as initial condition for the predictions. The $n$ predicted values $x(k+1)$, corresponding to the $n$ possible switching states of the converter, are evaluated using the cost function. The switching state $S$ which minimizes the cost function is selected and applied.

**C. Some Application Examples**

1) **Current Control:** Considering a three-phase two-level inverter, the eight possible switching states and voltage vectors are summarized in Table II. Here, variables $S_a$, $S_b$, and $S_c$ represent the switching states of the $a$, $b$, and $c$ legs of the inverter.

A simple example of current control using FS-MPC for an inverter is shown in [55]–[58]. Here, the following cost function is defined as:

$$g = \left| i_\alpha^* (k) - \hat{i}_\alpha (k+1) \right| + \left| i_\beta^* (k) - \hat{i}_\beta (k+1) \right| \quad (17)$$

where $i_\alpha^* (k)$ and $i_\beta^* (k)$ are the real and imaginary parts of the reference current vector, respectively, and $\hat{i}_\alpha (k+1)$ and $\hat{i}_\beta (k+1)$ are the real and imaginary parts of the predicted current vector, respectively.

As shown also in [47], a discrete-time model suitable for the prediction of the load current is

$$\hat{i}(k+1) = \left( 1 - \frac{RT_s}{L} \right) i(k) + \frac{T_s}{L} \left( v(k) - \hat{e}(k) \right) \quad (18)$$

where $R$ and $L$ are the load resistance and inductance, respectively, $T_s$ is the sampling frequency, $\hat{e}$ is the estimated back EMF of the load, $i(k)$ is the measured load current, and the inverter voltage $v(k)$ is the decision variable to be calculated for the control.

Experimental results for the current control of a three-phase inverter, obtained from [58], are shown in Fig. 14.

A similar current control strategy has been presented for three- [59], [60] and four-level [61] inverters. In [60], the possibility of including additional terms to the cost function is proposed, adding capacitor voltage balancing and reduction of the switching frequency.

For the case of the three-phase PWM rectifier, the possible switching states and voltage vectors generated by the inverter are the same as the ones shown in Table II for an inverter. As well as the current can be controlled, it is also possible to control the active and reactive powers [62]–[64].

For more complex converter topologies, the control strategy is the same but with a different set of possible switching states. This is the case of the matrix converter, as presented in [65], the direct converter [66]–[68], and the flying capacitor converter [69].
In terms of the variables to be controlled, depending on the converter and the application, several compositions of the cost function have been proposed.

2) Torque and Flux Control: For the same three-phase inverter, considering an induction motor as a load, the torque and flux of the machine can be directly controlled, as shown in [70]. An appropriate cost function is defined as

\[
g = \left| T_e^*(k) - \hat{T}_e(k+1) + A \left| \psi_s(k) - \hat{\psi}_s(k+1) \right| \right| \tag{19}
\]

where \( T_e^*(k) \) is the reference torque and \( \hat{T}_e(k+1) \) is the predicted electric torque for a given switching state, \( |\psi_s(k)| \) is the reference amplitude for the stator flux, and \( |\hat{\psi}_s(k+1)| \) is the amplitude of the predicted stator flux. The weighting factor \( A \) allows one to adjust the importance of the flux error with respect to the torque error.

3) Power Control: For a three-phase PWM rectifier, the same possible switching states and voltage vectors shown in Table II are valid. Here, the inductive filter model is considered for the prediction of the input current and power of the converter. As well as a predictive current control can be used in the rectifier, another approach for the control of this converter considers the direct control of the active and reactive powers [64]. As proposed in [62] and [63], the cost function evaluates the error in the active and reactive power

\[
g = (P^* - P(k+1))^2 + (Q(k+1))^2 \tag{20}
\]

where the reactive power reference is zero and the active power reference is obtained from the dc link voltage control loop.

4) Control of NPC Converter: In a three-level three-phase inverter, the number of switching states is 27, generating 19 different voltage vectors. Here, it is possible to take advantage of the redundancy of switching states by considering additional terms in the cost function. As proposed in [60], it is possible to control the load currents while balancing the capacitor voltages and reducing the average switching frequency by using the following cost function:

\[
g = \left| i_{\alpha}^* - \hat{i}_{\alpha}(k+1) \right| + \left| i_{\beta}^* - \hat{i}_{\beta}(k+1) \right| + A|V_{dc1} - V_{dc2}| + Bn \tag{21}
\]

where \( V_{dc1} \) and \( V_{dc2} \) are the dc link capacitor voltages and \( n \) is the number of commutations needed to change from the present switching state to the next switching state. The weighting factor \( A \) represents the importance of the capacitor voltage balance, and the weighting factor \( B \) allows the reduction of the average switching frequency.

5) Control of Matrix Converter: The control strategy proposed in [65] for the matrix converter takes into account the 27 possible switching states of the nine bidirectional switches. The goal of the control method is to receive only active power at the input and that the load current follows its reference with good accuracy. These two requirements are expressed as the following cost function:

\[
g = \left| i_{\alpha}^* - \hat{i}_{\alpha}(k+1) \right| + \left| i_{\beta}^* - \hat{i}_{\beta}(k+1) \right| + A|Q(k+1)| \tag{22}
\]

where \( i_{\alpha}^* \) and \( i_{\beta}^* \) are the load current references, \( \hat{i}_{\alpha}(k+1) \) and \( \hat{i}_{\beta}(k+1) \) are the predicted load currents for a given switching state, and \( Q(k+1) \) is the predicted reactive power at the input of the converter.

6) Control of Direct Converter: The direct converter has six bidirectional switches which allow nine possible switching states. The control strategies presented in [66]–[68] consider the control of the capacitor voltages of the LC filter at the input of the converter in a cascaded control structure, in order to obtain unity power factor and controlled amplitude of the output voltage. The cost function used in these works for the control of the capacitor voltages is defined as

\[
g = \|v_{cap}^*(k+1)v_{cap}(k+1)\|^2 \tag{23}
\]

where \( v_{cap}(k+1) \) is the reference capacitor voltage vector, obtained from the outer input current control loop. The capacitor voltages \( v_{cap}(k+1) \) are predicted for each possible switching state.

7) Control of Flying Capacitor Converter: The application of FS-MPC to a single-phase four-level flying capacitor converter is presented in [69]. This converter has three pairs of switches allowing eight possible switching states that generate four different voltage levels. The output current and capacitor voltages are controlled considering also the reduction of the switching frequency and control of the spectrum of the output current. This is achieved by using the following cost function:

\[
J[k, u] = ||e[k + N]||^2_P \tag{24}
\]

where \( u' \) is the switching state to be applied and \( e[k] \) is the predicted value of the overall error signal \( e[k] \), defined as

\[
e[k] = \begin{bmatrix}
V_1[k] - \frac{V_{DC}}{2} \\
V_2[k] - \frac{V_{DC}}{2} \tag{25}
\end{bmatrix} W(\rho) \begin{bmatrix}
i_L[k] - \bar{i}_L^e[k] \end{bmatrix}
\]

where \( V_1 \) and \( V_2 \) are the capacitor voltages and \( i_L \) is the output current. The capacitor voltage references are defined as a fraction of the dc link voltage, and the output current reference is \( i_L^e \). The filter \( W(\rho) \) allows one to include a frequency weighting in the cost function in order to control the spectrum of the output current.

The norm \( ||e'[\ell]||_P \) is defined as

\[
||e'[\ell]||_P = e'[\ell]^T Pe'[\ell], \quad P = \text{diag}\{\lambda_1, \lambda_1, 1\}. \tag{26}
\]

The parameters \( \lambda_1 \) and \( \lambda_2 \) and the filter \( W(\rho) \) are the design parameters for the controller.

As well as the aforementioned examples are implemented as an online calculation of the optimal switching state, it is also possible to use the offline calculation of an explicit solution, as proposed in [71] for the torque control of an induction machine.

These examples show the flexibility and wide range of applications on MPC when the finite set of possible states
is considered. This idea has been applied to several other converters, and it is open for new applications.

D. Spectrum and Switching Frequency

In all the examples described in Section VII-C, the switching state of the converter is changed at equidistant instants in time, i.e., in each sampling period, an optimal switching state is selected and applied during a whole period. This way, variable switching frequency is obtained, where the maximum switching frequency is limited to half the sampling frequency.

The resulting spectrum of the voltages and currents is spread over a wide range of frequencies and will change depending on the sampling frequency and the operating conditions. This kind of spectrum is not desirable in some applications. In order to get a concentrated spectrum, similar to the one obtained using PWM, a frequency weighted cost function has been proposed in [47], where a narrowband stop filter is included in the cost function.

Some other applications may require a reduced switching frequency, in order to reduce the switching power losses. In these cases, a weighting factor related to the commutations can be included in the cost function, as proposed in [60] where it is demonstrated that the average switching frequency can be considerably reduced.

A different approach is the use of a constant switching frequency, as will be explained in the next section. A comparison between the variable and constant switching frequency algorithms is presented in [72] for the power control of an active front-end rectifier.

E. Constant Switching Frequency Algorithm

In most commercial applications, the constant switching frequency algorithm is preferred because it allows easy EMI filter design and protection of semiconductor power components. Therefore, a constant switching frequency predictive direct power control (P-DPC) algorithm has been developed in [73] and [74]. This algorithm selects, in every sampling period, an appropriate voltage vector sequence and calculates duty cycles in order to minimize instantaneous active and reactive power errors. The lack of linear controllers and modulator makes the system very fast in transients. However, the P-DPC algorithm is sensitive to measured grid voltage distortion, which causes grid current distortion. Using estimated virtual flux (VF) for instantaneous power calculation, the grid side voltage sensors can be eliminated and sampling frequency reduced [75]. Moreover, in operations under distorted grid voltage, the THD factor of the grid current can also be considerably reduced without deterioration of high dynamic performance (see Fig. 15). Therefore, the VF-based P-DPC (VF-P-DPC) algorithm is a very universal system that can be used for very low switching frequencies (below 2 kHz).

VIII. CONCLUSION

This paper has reviewed the most important types of predictive control used in power electronics and drives. The predictive control methods are divided into following groups: deadbeat control, hysteresis-based control, trajectory-based control, and continuous MPC and FS-MPC. The basic principles and the latest developments of these methods have been systematically described, and application examples have been indicated.

It is demonstrated that predictive control is a very powerful and flexible concept for designing controllers. It presents several advantages that make it suitable for the control of power converters and drives. The use of all available information of the system to decide the optimal actuation allows one to achieve very fast dynamics, by including the nonlinearities and restrictions of the system and avoiding the cascaded structure. It is also possible to take advantage of the discrete nature of the power converters and choose from the possible switching states the optimal solution according to the minimization of a predefined cost function.

Predictive control has been applied to a very wide range of systems, and it is open for new applications and converter
topologies. However, the best suited type of predictive control will depend on the application and requirements of the system.

As a conclusion of survey, it is the belief of the authors that predictive control strategies will continue to play a strategic role in the development of modern high-performance power electronics and drive systems and will offer a new interesting perspective for future research in this area.

REFERENCES


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