GHOST: Efficient Goodness-of-fit HOS Testing Signal Detector for Cognitive Radio Networks

Daniel Denkovski, Vladimir Atanasovski and Liljana Gavrilovska
Ss. Cyril and Methodius University in Skopje
Faculty of Electrical Engineering and Information Technologies
Karpos 2 bb, 1000 Skopje, Macedonia
{danield, vladimir, liljana}@feit.ukim.edu.mk

Abstract—Flexible, reliable and robust signal detectors are vital components of the emerging cognitive radio networks. This paper presents a novel Goodness-of-fit Higher-Order-Statistics Testing (GHOST) signal detector, which eliminates the drawbacks of the previous related detectors and offers dominant signal detection performances. The detection is made based on goodness-of-fit testing comparing the skewness and kurtosis of the received power spectrum with empirically estimated system noise statistics. This allows reliable detection of different types of signals with SNR even lower than -32dB. The detector can be easily applied to the existing radio hardware since it adapts to the specific noise characteristics.

Keywords- GHOST signal detection, Higher-Order-Statistics (HOS), Goodness-of-Fit (GoF) tests, noise adaptation.

I. INTRODUCTION

Reliable and robust spectrum sensing is essential for proper operation of upcoming cognitive networks. The signal detectors used in the spectrum sensing process must provide high sensitivity in order to accurately detect primary users. However, the fading and the other time/frequency/space variations of the radio environment, as well as the noise variations/uncertainties, may significantly affect the primary signal SNR. Thus, signal detectors that can reliably detect weak primary signals (of about -20dB SNR as a target [1]) are necessary at the secondary system radios.

The signal detection methods that exist in the literature can be broadly classified in two main classes: blind and feature detection methods [2]. Blind detectors do not require a priori knowledge on the detected signal [3], whereas the feature detectors [4][5] use some signal characteristics in order to perform the detection.

The most widely used blind detection method is the energy detection that has low complexity, but also low detection performances. Possible improvements are the eigenvalue based blind detection [3], where the detection is based on the autocorrelation or covariance matrices of the received samples, and the Higher-Order-Statistics (HOS) based detection [6][7]. Additionally, the HOS can be also used in Goodness-of-Fit (GoF) testing, i.e. a test that evaluates whether a given sample set follows a specific distribution or not. The combination of both aspects, the HOS and the GoFs, results in powerful detection performances.

This paper presents a novel signal detector that uses HOS and GoF tests on the FFT power spectrum to assess the presence of a signal. The proposed GoF HOS Testing (GHOST) signal detector uses a test statistic formed from empirical estimations (from the noise distribution) of the mean noise skewness and kurtosis and their variance/covariance values. The GHOST detector can be easily applied on present radio hardware since it adapts to its specific noise. A tested evaluation confirms the powerful detection performances of the GHOST detector, i.e. a reliable detection of weak signals with SNR lower than -32dB.

The paper is organized as follows. Section II briefly explains the existing HOS detectors. Section III elaborates the GHOST detector in details. Section IV gives a tested performance evaluation of the GHOST detector and comparisons with the existing detectors. Finally, section V concludes the paper and gives guidelines for future work.

II. RELATED WORK

Two HOS based detectors exist in the literature up to the best of the authors’ knowledge. The former [6] is in the process of standardization as a part of the IEEE 802.22. It performs Gaussianity tests on the estimates of the third-to-sixth order cumulants on both the real and imaginary parts of a 2048 point FFT spectrum and is capable of detecting signals with SNR lower than -20dB. The latter HOS detector [7] is an enhancement of the previous one using the Jarque–Bera (JB) statistic for the detection. The JB statistic is a combination of the third and the fourth order statistics, i.e. the skewness and the kurtosis [8]. In general, JB is a normality test statistics, i.e. a quantization measure of how close the skewness and kurtosis values of the inspected data set to the normal distribution are. Therefore, it is a GoF test for the normal distribution. However, the authors in [7] use this statistic incorrectly (in terms of GoF tests). They calculate the JB statistic on the skewness and kurtosis on the amplitude of the FFT spectrum, which is a Rayleigh distributed random variable when a signal is absent. Although used in suboptimal manner, the JB
detector offers better detection performances than the previous HOS method, detecting signals with SNR lower than -25dB.

Drawbacks of the HOS detectors are the high computational complexity and the requirement of new hardware blocks for their proper operation. Hence, they cannot be implemented onto existing radios.

Similarly as the JB detector in [7], the GHOST detector proposed in this paper uses the skewness and kurtosis, but in proper GoF tests on the FFT power spectrum. The next section explains the GHOST detector and its specific operations.

III. GHOST DETECTOR

This section presents the novel GHOST detector that uses HOS and GoF testing to determine signal presence. It covers the essential aspects of HOS and GoF, the GHOST detector block scheme and the analytical background on GHOST.

A. HOS and GoF testing

The HOS reflect some specific characteristics of the distribution of a random variable. The third and fourth order statistics, the skewness and kurtosis [9], represent the most important HOS. Skewness is a measure of the symmetry of the distribution of the random variable x quantifying the level of asymmetry in the referred distribution. The sample skewness ($\gamma_3$) of the sample set $\{x_i, i=1..N\}$ can be calculated as:

$$\gamma_3 = \frac{\bar{x}^3}{\hat{\sigma}_3^3} = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^3$$

(1)

where $\bar{x}$ represents the sample mean, $\hat{\mu}_3$ and $\hat{\sigma}_3$ correspond to the estimates of the second and the third central moments of the target sample set with size N. A zero skewness calculated with this equation implies a symmetric distribution.

The kurtosis reflects the “peakness” of the distribution of the random variable x measuring whether the distribution is peaked or flat relative to the normal one. Eq. (2) gives the estimation of the sample kurtosis ($\gamma_4$) of a sample set:

$$\gamma_4 = \frac{\bar{x}^4}{\hat{\sigma}_4^4} = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^4$$

(2)

According to this definition, a normal distribution has a kurtosis of 3. The combination of skewness and kurtosis is distribution specific and, thus, can be used in GoF testing.

The GoF tests are particular type of tests that evaluate whether a given random process (sample set) follows a specific distribution based on the comparisons with some known statistics of a target distribution. Besides the skewness and the kurtosis, a subject to GoF testing can be various distribution specific metrics such as: mean values, variances, fifth and higher-order moments and cumulants, autocorrelations, distribution function parameters etc.

The GHOST detector optimally combines the HOS with the GoF testing to yield powerful signal detection. It uses GoF tests on the skewness and kurtosis of the received power spectrum bins to determine whether they are drawn from the system noise power spectrum distribution or not using the following hypothesis model:

$$H_0: \text{the samples are drawn from the noise distribution}$$

$$H_1: \text{the samples are not drawn from the noise distribution}$$

The GoF testing requires knowledge on the system noise power spectrum skewness and kurtosis values and their covariances. They can be calculated empirically in the case when the system is under the hypothesis $H_0$.

The following part presents the GHOST detector scheme.

B. GHOST detector operation

The GHOST detector performs GoF tests on the skewness and kurtosis calculated from the received power spectrum, checking if they are compliant with the empirically estimated respective system noise power spectrum statistics. Therefore, the GHOST detector has two operational modes: an initialization mode and a detection mode. The initialization mode (the case of hypothesis $H_0$) of the detector is required for setting up the detection parameters evaluated from the noise. These parameters are used in the detection mode to test if a signal exists in the received samples performing GoF tests.

Fig. 1 presents the GHOST detector block scheme. The receiver chain starts with a switch between the receiver antenna and a 50$\Omega$ terminator needed to perform the system noise evaluation. Following three blocks (i.e. RF front end, ADC and FPGA) are common receiver components. The output of the FPGA block is a flow of complex time-domain samples fed into a serial-to-parallel (S/P) converter. A vector of N time domain complex samples is the output of the S/P block that is afterwards subject to FFT analysis. The next block converts the resulting N complex FFT bins to FFT power spectrum bins, afterwards used for HOS analyses.

The subsequent switch controls the mode of operation of the GHOST detector. In the initialization mode, the received samples correspond to the system noise samples. The calculated noise power spectrum statistics are passed to an S/P converter accumulating M consequent noise skewness and kurtosis values. These M skewness and kurtosis values are input to the subsequent block performing the estimation of the mean vector $\omega_n$ and the inverse covariance matrix $\Sigma_w^{-1}$ on the noise skewness and kurtosis vector $w_n$ as:
The detection threshold \( \lambda \) is tightly related to the distribution of the estimators of \( \gamma \) and \( \eta \). It can be evaluated analytically if the distribution of the GHOST test statistic \( (\gamma, \eta) \) is known or empirically estimated in the initialization mode.

The detection mode of the GHOST energy detector is entered when both switches are in alternate positions (Fig. 1). The first block after the second switch in the detection mode performs the test statistic calculation based on the input skewness and kurtosis values of the received FFT power spectrum bins. Setting up the mean vector and the inverse covariance matrix calculated in the initialization mode, the GHOST test statistic \( T_{\text{ghost}} \) is calculated as follows [11]:

\[
T_{\text{ghost}} = (w - \omega_n)^T \Sigma^{-1} N M \sum_{i=1}^{M} \sum_{j=1}^{N} (\gamma_i - \bar{\gamma})(\eta_j - \bar{\eta})
\]

where \( w \) is the vector of the inspected skewness and kurtosis. Comparing the GHOST test statistic \( T_{\text{ghost}} \) with the threshold \( \lambda \) the last block in the detector makes the signal detection:

\[
\begin{align*}
T_{\text{ghost}} &< \lambda, \text{there is no signal detected} \\
T_{\text{ghost}} &\geq \lambda, \text{a signal is detected}
\end{align*}
\]

The GHOST statistic is a measure of how close the FFT power spectrum bins are to the underlying noise distribution. In terms of signal detection, if the GHOST statistic is lower than the detection threshold, it implies that an external signal does not exist or cannot be detected. Otherwise, if it is higher, it means that a signal causes the FFT power spectrum bins distribution to vary from the respective noise distribution. Although the block scheme implies that the decision is based on a single test statistic output, multiple consecutive measurements can make the detection even more robust to primary signal fluctuations.

The GHOST detector is easily applicable to nowadays present radio hardware. It requires processing demanding calculations depending on the FFT sample size. However, the initialization can be performed only during calibration, since the detector estimates of the mean vector and covariance matrix can be stored for each specific configuration (frequency, bandwidth, FFT sample size etc.) and restored when required. The initialization mode provides the ability to cope with hardware non-idealities such as different I/O characteristics on different frequencies, limited precision and data resolution etc. This makes it possible for the GHOST detector to adapt to the noise characteristics of low-cost and less precise hardware devices and be able to detect various types of signals (cellular, video broadcast etc.). Furthermore, the GHOST detector is resistant to noise uncertainties since the skewness and kurtosis statistics, calculated by (1) and (2) are normalized and therefore noise power independent. These are the main advantages over the previous HOS detectors.

The next part focuses on the analytical calculation of the noise power spectrum mean vector \( \omega_n \) and covariance matrix \( \Sigma_{\text{cov}} \) in the case of hypothesis \( H_0 \).

C. Analytical background

Theoretically, the FFT power spectrum bins follow a Chi-Square distribution \( \chi^2 \) with two degrees of freedom (with PDF \( f(x) = \frac{1}{2\sigma_{\text{iq}}^2} e^{-x/2\sigma_{\text{iq}}^2} \)), representing the distribution of a sum of the squared magnitudes of two zero mean Gaussian variables [6] with variances \( \sigma_{\text{iq}}^2 \). The asymptotic mean \( \mu \) and variance \( \sigma^2 \) of the \( \chi^2 \) random variable \( x \) can be calculated as:

\[
\mu = \mathbb{E}(x) = \int_{-\infty}^{\infty} x f(x) dx = \int_{0}^{\infty} \frac{x}{2\sigma_{\text{iq}}^2} dx = 2\sigma_{\text{iq}}^2
\]

\[
\sigma^2 = \mathbb{E}(x - \mu)^2 = \int_{-\infty}^{\infty} (x - 2\sigma_{\text{iq}}^2)^2 f(x) dx = 4\sigma_{\text{iq}}^4
\]

Using these values, the theoretical mean skewness and kurtosis of the \( \chi^2 \) distribution can be computed as well:

\[
\begin{align*}
\gamma &= \mathbb{E}(x - \mu)^3 / \sigma^3 = \int_{-\infty}^{\infty} (x - 2\sigma_{\text{iq}}^2)^3 f(x) dx / (4\sigma_{\text{iq}}^4)^{3/2} = 2 \\
\eta &= \mathbb{E}(x - \mu)^4 / \sigma^4 = \int_{-\infty}^{\infty} (x - 2\sigma_{\text{iq}}^2)^4 f(x) dx / (4\sigma_{\text{iq}}^4)^2 = 9
\end{align*}
\]

It is clear that the skewness and kurtosis are statistics independent on the variance of the \( \chi^2 \) random variable \( x \), or the variance of the two composing Gaussian variables \( \sigma_{\text{iq}}^2 \). Therefore, these statistics can make the decision process invariant to noise uncertainties and variations.

Besides the asymptotic mean vector \( \omega_n = [2 \ 9]^T \), the covariance matrix \( \Sigma_{\text{cov}} \) can be also theoretically evaluated for the \( \chi^2 \) distribution. Using the respective influence functions [10] of skewness and kurtosis for the \( \chi^2 \) distribution, the asymptotic covariance matrix is computed to be [11]:

\[
\Sigma_{\text{cov}} = \begin{bmatrix} 72 & 720 \\ 720 & 8060 \end{bmatrix}
\]

However, the practical results in the next section suggest that this theoretical \( \omega_n \) and \( \Sigma_{\text{cov}} \) do not comply with the respective ones, empirically calculated from the noise. This stems from the hardware non-idealities. Therefore, for optimal detection operation of GHOST, \( \omega_n \) and \( \Sigma_{\text{cov}} \) should be empirically calculated under the hypothesis \( H_0 \).

IV. PERFORMANCE EVALUATION

This section aims to prove the optimality of the GHOST detection and its dominance over the existing detectors. All evaluations are performed using an Anritsu MS2690A signal analyzer [12] with a built-in signal generation option. A coaxial cable was used to connect the signal generator output.
and the signal analyzer input with measured losses 1.4dB. The signal analyzer was connected to a host PC extracting the IQ samples and performing the detector’s operations in Matlab.

A. Detection parameters evaluation

Before the detection mode operation of the GHOST detector, the mean vector ($\omega_n$) and covariance matrix ($\Sigma_{\omega}$) should be calculated from the system noise according to (3) and (4). Ten million IQ samples (N*M) were used to evaluate these terms, as well as the probability of false alarm ($P_{fa}$) thresholds, for each FFT size ranging from 1024 up to 65536. For instance, the estimated mean vector and covariance matrix for the case of FFT size of 2048 are:

$$\omega_n = \begin{bmatrix} 1983 \\ 8826 \end{bmatrix}^T, \quad \Sigma_{\omega} = \begin{bmatrix} 65045 & 619629 \\ 619629 & 6545347 \end{bmatrix}$$  \hspace{1cm} (12)

These mean skewness and kurtosis slightly vary from the theoretical values for the $\chi^2$ distribution, which are 2 and 9, respectively (section III C). The covariance matrix also varies from the theoretical covariance matrix for the $\chi^2$ distribution. It proves that these values are hardware dependent, since the hardware non-idealities contribute to the final power distribution, even in the case of high-precision spectrum analyzers. The conclusion is that the FFT power spectrum bins have $\chi^2$-like distribution. Therefore, the empirical calculations of the mean vector and covariance matrix are essential for proper operation of the GHOST detection.

Fig. 2a presents the Empirical Probability Distribution Function (EPDF) of the GHOST test statistic value in the case of hypothesis $H_0$. The distribution has a similar shape to Chi-Square distribution and most of the GHOST test statistic values are lower than 17. However due to estimation errors, values significantly higher than 17 are also possible (not visible in Fig. 2a). The probability of occurrence of these high values is low and decreasing with the FFT size increase (Table I). Namely, the probability that the GHOST test statistic is higher than 17 is 0.96% for the FFT size of 1024, decreasing to 0.59% when the FFT size is 65536. Therefore, a fixed decision threshold of 17 can guarantee $P_{fa}$ lower than 1% for any FFT size higher than 1024. Figure 2b presents the $P_{fa}$ threshold dependence on the FFT size for $P_{fa}$, ranging from 1% to 50%. The same conclusion applies for the thresholds, i.e. the FFT size increase does negligibly affect the $P_{fa}$ thresholds.

![Figure 2a](image1.png)

![Figure 2b](image2.png)

**Figure 2.** a) Empirical PDF of the test statistic under hypothesis $H_0$; b) $P_{fa}$ thresholds (1% to 50%) estimation for different FFT sizes

<table>
<thead>
<tr>
<th>FFT size</th>
<th>1024</th>
<th>2048</th>
<th>4096</th>
<th>8192</th>
<th>16384</th>
<th>32768</th>
<th>65536</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{fa}$</td>
<td>0.96</td>
<td>0.88</td>
<td>0.83</td>
<td>0.81</td>
<td>0.73</td>
<td>0.62</td>
<td>0.59</td>
</tr>
</tbody>
</table>

**Table 1.** $P_{fa}$ dependence on fixed threshold for different FFT sizes

B. Detection results

The performances of the GHOST detector were tested with two different signals, an EDGE MCS-9 signal with four active slots and an ISDBT signal (the Japanese digital television terrestrial standard, also based on OFDM) using GMSK modulation and 2/3 convolution coding. In the case of the EDGE signal, the GHOST detector was tested for SNR levels ranging from -30dB up to 0dB (with steps of 2dB) with an analyzed bandwidth of 2MHz. The tested SNR range for the ISDBT signal was -32dB to -2dB (with steps of 2dB) with an analyzed bandwidth of 10MHz around the central frequency. Both signal cases were tested with different FFT sizes ranging from 1024 up to 65536. The GHOST detector performances were compared to two other detectors: the suboptimal JB

![Figure 3a](image3a.png)

![Figure 3b](image3b.png)

**Figure 3.** $P_d$ vs SNR for $P_{fa} = 1\%$, various FFT sizes: a) EDGE signal; b) ISDBT signal
 detector (focusing solely on the detection process in [7]) and the classical energy (power) detector (performing the detection based on the average over the FFT power spectrum bins). The \( P_d \) thresholds were empirically estimated from the noise (case of hypothesis \( H_0 \)) for all three tested detectors.

Figures 3a) and 3b) present the GHOST detector performances in terms of the probability of detection (\( P_d \)) dependence on the input signal SNR and different FFT sizes for the GSM and ISDBT signal type, respectively. It is clear that the detection performances increase for about 3dB for every doubling of the FFT size, for the both signal types. Namely, an FFT size of 32768 yields a \( P_d \) of 100% for the GSM signal and an FFT size of 65536 results in \( P_d \) of 100% for the ISDBT signal for any tested input signal level.

Figures 4a) and 4b) depict comparison evaluations of the GHOST detector performance in terms of the \( P_d \) vs. SNR dependence with the other detectors. It is evident that the GHOST detection has dominant performance. In the case of GSM signal with -30dB SNR and FFT size of 16384, the GHOST detector has \( P_d \) of around 92% and the JB detector and classical power detectors have 56% and 0% \( P_d \). In the ISDBT signal case, with FFT size of 32768 and -32dB SNR, the GHOST detector achieves \( P_d \) of around 93%, while the JB detector and classical power detectors have 45% and 0%.

The results prove that the GHOST detector is a powerful HOS and GoF based detector which offers the ability to detect very weak signals, i.e. signals with SNR even below -32dB when the FFT size is large. These performances are significantly better than the existing HOS and energy detectors. Furthermore, the tested evaluation demonstrates the applicability of the sensing scheme on existing hardware and on the detection of different types of signals.

V. CONCLUSION

This paper presents a novel and efficient signal detector (GHOST) based on Goodness-of-Fit (GoF) Higher-Order-Statistics (HOS) based testing of the FFT power spectrum. The proposed scheme is robust and flexible overcoming the drawbacks and the sub-optimality of the previous HOS detectors. GHOST allows detection of user signals at SNR values lower than -32dB. The increase in the FFT size results in increase of the detection performance not affecting the detection thresholds. Therefore, a fixed threshold can be retained regardless of the sample size. Moreover, the GHOST scheme can be applied to off-the-shelf radio hardware capable of performing IQ sampling, regardless on its sensitivity levels and precision, adapting to the specific noise characteristics, not depending on the noise uncertainties.

Future work will focus on evaluations of the GHOST scheme with different input signals, the impact of cooperation between multiple GHOST detectors etc.

ACKNOWLEDGMENT

This work was funded by the EC project FARAMIR [13] and inspired by the EC project ACROPOLIS [14]. The authors would like to thank everyone involved. The authors also extend their gratitude to the COST IC0902 action [15].

REFERENCES


Figure 4. \( P_d \) vs. SNR for \( P_d = 1\% \), two FFT sizes, different detectors: a) EDGE signal; b) ISDBT signal