Abstract—The numerical model of CW-DFB optical source for fiber-optic transmission systems employing coherent detection technique is presented. The developed model is based on numerical solution of quantum-mechanical coupled rate equations. The numerical model is suitable for optical transmission systems for purpose of generation of optical carrier wave on the transmitter side as well as it can be used as local laser for coherent detection on the receiver side. The main attention of proposed optical source model is focused on signal, time-dependent characteristics to determine spectral properties of presented laser, which fully satisfies current requirements on coherent optical transmission systems. The efficiency of numerical model is also discussed. We have showed that by employing the re-sampling technique, both computational effort and accuracy of presented laser model are markedly enhanced.

Keywords—Coherent transmission systems, Distributed feedback lasers, Numerical modeling, Optical sources, Runge-Kutta.

I. INTRODUCTION

The investigation of optical sources and semiconductor technology has been rapidly growing in the last decades [1] and significant enhancement has been achieved. For long-haul single- and multi-channel transmission systems, the coherent lasers are preferred such as Fabry-Perot lasers (FPL) [2], single-mode lasers with distributed feedback (DFBL), external cavity lasers (ECL) and tunable lasers. The mentioned types of lasers sufficiently fulfill the requirements on modern coherent systems with sub-MHz linewidths for required wavelength range.

In this paper, we present a numerical model of DFB laser based on solving coupled rate equations by numerical integration using fourth-order Runge-Kutta numerical technique. Several various numerical approaches to model lasers are known and have been reported in [3] or [4], both on emphases on noise properties. Other approaches for modeling ultra-fast and ultra-short fiber optical sources have been studied and described in [5] using full-time Finite Difference Method (FDM) or in [6] via pseudospectral propagation technique, called Split-Step Fourier Method (SSFM).

The solution of coupled rate equations is necessary to determine the fundamental characteristics and signal properties including time-dependent number of photons and electrons during transients and after and of transients (steady state) in our optical source [3], [4]. Mentioned fundamental characteristics of presented model of optical source are illustrated in Figs. 1 and 2, respectively.

The noise properties of laser are mainly dominant in steady state, because the fluctuating nature of corresponding parameters (\(N(t)\) and \(S(t)\)) caused by noise is so much smaller than their average values [3]. From a transmission point of view, the steady state operation of laser is important to ensure that laser radiates the continuous wave.

II. EMPLOYED MODELING APPROACH

A. Numerical Modeling of Coupled Rate Equations

As has been mentioned, the dynamic properties of lasers can be modeled via coupled rate equations, described by Eq. (1)-(3) [3]. These equation characterize the relation between carriers number \(N(t)\), photons number \(S(t)\) and optical phase \(\theta(t)\). This approach takes into account intrinsic fluctuations of described parameters and through the rate equations, we
are able to create efficient numerical model of continuous wave (CW) DFB laser source. Fluctuations of power and phase are caused by non-uniform electron excitation on the excitation level and by non-identical properties of photons in the resonant circuit. For purpose of optical source modeling, it is highly desirable and necessary to take into account the noise characteristics described by Langevin’s noise sources $F(t)$ [3] to correctly described the random behavior of real optical source. Other kinds of noise sources for efficient modeling the effect of random noise in lasers usually utilized Gaussian’s distribution [7], [8], uniform distribution [9] and Poisson’s distribution [10] of probability.

$$
\frac{dN(t)}{dt} = \frac{I(t)}{q} - \frac{N(t)}{\tau_n} - \frac{g}{1 + \varepsilon S(t)} N(t) - N_0 + F_N(t),
$$

(1)

$$
\frac{dS(t)}{dt} = g \left( \frac{N(t) - N_0}{1 + \varepsilon S(t)} S(t) - \frac{S(t)}{\tau_p} + \frac{\beta N(t)}{\tau_p} + F_S(t) \right),
$$

(2)

$$
\frac{d\theta(t)}{dt} = \frac{\alpha}{2} g \left( N(t) - N \right),
$$

(3)

where $I(t)$ denotes the injected current, $q$ is the electron charge, $\tau_n$ is the carrier lifetime, $g$ denotes the gain slope, $\varepsilon$ is the nonlinear gain compression factor, $N_0$ is the carrier number at transparency, $\beta$ is the fraction of spontaneous emission coupled into the lasing mode, $\alpha$ is the linewidth enhancement factor, and $\tau_p$ is time-average carrier number. By using these equations, diffusion and nonlinear effects are lumped together as effective field-dependent optical gain compression. According to [11], the term $1/(1 + \varepsilon S)$ is the best option to employ numerical solution for lasers with high-density number of photons. The optical feedback is not considered in presented numerical model. This simplification can be made, because we do not deal with internal kind of modulation (direct modulated lasers), where the optical feedback represents the light reflection on connector, which makes connection between laser and optical fiber [12].

This numerical approach [3] based on Eq. (1)-(3) is primary oriented on external type of modulation and the effect of light reflection is excluded from Eq. (1)-(3). On the other hand, it has to be said, if one wants to include light reflections, the Eq. (1)-(3) are no longer suitable and valid. The light reflection effects are governed by so-called single-mode longitudinal laser rate equations. The influence of reflected light can have significant impact on laser time and spectral properties [12].

In case of DFB lasers, the threshold current is expressed as follows [3]:

$$
I_{th} = \frac{d}{\tau_n} \left( N_0 + \frac{1}{g \tau_p} \right).
$$

(4)

Usually the value of $I_{th}$ is approximately from 10 mA to 20 mA [3], [13].

Based on time-dependent variation of number of photons, the lasing power of laser can be expressed as [3]:

$$
p_s(t) = \eta_d \frac{hf}{\tau_p} S(t),
$$

(5)

where $\eta_d$ is deferential quantum efficiency, $h$ is Planck’s constant and $f$ is fundamental laser frequency before modulation. In Fig. 3 are depicted the time-dependent fluctuations of power and phase of the presented optical source.

For purpose of creation the numerical model of optical source with emphasis on signal characteristics and properties, the Runge-Kutta numerical approach is the best option to solve differential equations [3], [4], which have been defined earlier. Generally, in numerical modeling the iteration step size is important parameter to keep the required accuracy. In case of modeling of presented optical source characteristics, the iteration step size has been chosen to be 10 ps [3], [4] as a result between accuracy and computational effort.

The presented numerical model CW DFB laser is consistent in terms of various bit rates. The most important thing is to ensure the criterion of simulation step size. The value 10 ps of step size can also be smaller resulting in higher accuracy, but the computation effort also increases. The value of simulation step size is directly related to the employed value of bit rate depending on duration of used modulating pulse. The time window of corresponding simulation is adaptively changed. The step size of 10 ps is suitable for bit rates higher than 1 Gbit/s with very good numerical stability. We do not consider lower values of transmission bit rates for two main reasons. The first is the fact that they exhibit numerical instabilities in proposed laser model. The second one, from the perspective of current requirements for employed bit rates in optical transmission systems, the lower bit rates are not interesting in terms of their influence on signal transmission and impact of various fiber-channel transmission impairments.

Our attention in this paper is primary focused on representation of signal characteristics of laser. But it is important to note that for the presented model is suggested to use external type of modulation. In our case, the arbitrary kind of modulation can be used with this laser model following the mathematical representation of various modulation schemes described in [14]. However, the deep and full-behavioral model of external modulators could be interesting, it is out of scope of this paper and need further investigation.

B. Representation of Complex Analytical Signals

From a mathematical point of view, optical signals are high-frequency signals. However, the frequency used for purpose of optical transmission is too high to fulfill the sampling theorem [14]. From this reason, it is better to use complex envelope representation of quasi-monomochromatic optical signals with following condition; the carrier frequency of optical signal is so much larger than the spectral bandwidth of
optical signal ($\Delta \omega \ll \omega_0$). Complex envelope of signal $s(t)$ in baseband can be expressed as equivalent signal to the real high-frequency signal $x(t)$ [15]. Following the procedure described in [15], the complex envelope of optical signal is given by [15]:

$$s(t)e^{i\theta(t)} = 2z(t)e^{i\lambda t},$$

(6)

where $X(f) = \mathcal{F}\{x(t)\}$ and $Z(f) = X(f)$ for $f \geq 0$.

From a telecommunication point of view, we are interested in time-dependent power and phase parameters, from which the complex envelope can be estimated from following relation:

$$s(t) = \sqrt{2p_X(f)e^{i\varphi_X(f)}}.$$

(7)

The most important characteristic of lasers is one-side power spectral density (PSD) and it can be expressed through the complex envelope of optical high-frequency signal as follows:

$$\Phi_{ss}(f)_{\text{one-side}} = 10\log \left( \frac{1}{2T} \mathcal{F}\left\{ s(t) \right\} \right) + 30,$$

(8)

where $T$ is the observation time interval. The Eq. (8) is valid for frequencies ranging from 0 to $\infty$. The characteristic one-side PSD of presented optical source is depicted in Fig. 4.

III. RESULTS AND DISCUSSION

In this paper, we turn our attention to signal characteristics and properties of optical source for coherent fiber-optic transmission systems. The created numerical model represents CW DFB laser, which is one of the most used optical source in single- and also in multi-channel transmission systems.

A. Laser Characteristics

The fluctuations of the number of carriers and number of photons become important, if laser operates in steady state. These random fluctuations induce the time-dependent changes of power and phase of optical source having significant impact on time and spectral characteristics of laser. This impact can be seen in Fig. 3. From this figure, we can observe that changes of optical phase are too slow in comparison with power changes, because the influence of stimulated emission is dominant in steady state compared to the case, when the dominance of spontaneous emission is obvious during transients.

In Fig. 5, we can see the dependence of average optical power on wavelength for three important telecommunication wavelengths.

To obtain the maximal output power of presented laser source model, it is highly recommended to operate at shorter wavelengths, what is traditional situation of optical transmission system operating at wavelength with zero dispersion (around 1310 nm) in case of using standard single-mode optical fiber (ITU-T G. 652). But on the other hand, these output powers for longer wavelengths are currently sufficient from transmission point of view, because the most preferred transmission window for single- and multi-channel transmission systems is located around 1550 nm (C-band).

B. Modeling Efficiency

Fig. 7 shows the original and re-sampled spectrum. The implementation of re-sampling technique (numerical reconstruction) is necessary to enhance the computational efficiency of our model of CW DFB laser. In Fig. 7, the black curve corresponds to the obtained spectrum of our laser after solving the coupled rate equations by mentioned numerical technique. The time of numerical simulation without employing the re-sampling technique is approximately 30 seconds. According to [14], we have derived the new re-sampling relation describing new spectral form of laser (the red curve in Fig. 7), which can be expressed as follows:
with its corresponding frequency of spectral component follows:

$$S_l(f) = \sum_{k} S_l(k) \text{sinc} \left( \frac{f - f(k)}{\Delta f} \right), \quad -\infty < f < \infty \quad (9)$$

where $\Delta f$ denotes frequency step and $S_l(k)$ is sampled spectrum of complex envelope of laser before re-sampling with its corresponding frequency of spectral component $f(k)$ of $S_l(f)$, where $S_l(f)$ is Fourier transform (FT) of the complex envelope $s_l(t)$. The derived relation between spectrum before and after using the re-sampling technique can be expressed as follows:

$$S_j(k) = \left[ S_j(f) \right]_{f \rightarrow f(k)} \quad (10)$$

By including and employing the re-sampled equation into the presented model of laser source, we have obtained significant improvement of computational efficiency. The time of numerical simulation of proposed CW DFB laser after using re-sampling technique has been reduced to the value of approximately 2 seconds. This is obvious decrease of computational effort of our numerical model of CW DFB laser.

Our presented approach enhances the efficiency and also the total accuracy of final laser characteristics, so the final shape of optical spectrum is smoother. The spectral width of presented optical source is approximately 35 MHz. This value is in good agreement with [3] and also this value fulfills the laser linewidths requirements for modern coherent transmission systems.

The achieved enhancement of computational efficiency is important and significant from the point of view of implementation of presented optical source into the model of transmission systems using well-known numerical techniques for optical pulse propagation as Split-Step Fourier Method (SSFM) and Finite Difference Method (FDM).

Our presented optical source has successfully been implemented in our in-house propagation tool [16], where we have used the presented optical source to investigate fiber-channel impairments with emphasis on nonlinear polarization-mode dispersion (PMD), also including other degradation effects such as chromatic dispersion (CD), losses and polarization-induced nonlinear effects of self-phase modulation (Pol-SPM) and cross-phase modulation (Pol-XPM). The obtained results have shown good agreement with theoretical analysis and our approach is more realistic in comparison with traditional monochromatic approach, which includes only one wavelength of optical pulse instead of realistic spectral range of optical pulse.

IV. CONCLUSION

In this paper our attention has been focused on numerical investigation of optical source for coherent transmission systems. The developed numerical model represents semiconductor CW DFB laser with good agreement with other approaches. The presented model has showed fundamental characteristics and properties, which are important to implement it in further numerical models. The technique of re-sampling has been included and employed into the model to improve computational efficiency. We have showed significant reduction of it for presented numerical model. The simulation time has been decreased approximately from 30 seconds to 2 seconds. The employed re-sampling approach also improves the accuracy of presented numerical model of laser source in sense to better estimate the laser bandwidth, which in our case has been estimated at 35 MHz. This value is in good agreement with sub-MHz laser requirements.

REFERENCES