Cooperative Collision Warning Through Mobility and Probability Prediction

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ABSTRACT

The past decade has witnessed the confluence of Intelligent Transportation Systems (ITS) and Vehicular Ad hoc Networks (VANET) that promises to revolutionize incident detection and the timely dissemination of traffic-related information to the various interested parties. One of the key components is expected to be a Cooperative Collision Warning System (CCWS). Our main contribution is to derive analytical expressions for key CCWS metrics that rely on mobility information exchanged by various players. The feasibility of CCWS has been demonstrated in previous work; this paper analyzes the mobility parameters and derives the conditional probability of a collision. We begin by proposing a set of fundamental parameters for CCWS: conditional probability of collision, headway distance, driver reaction time, relative velocity and acceleration. Preliminary simulation results have demonstrated the effectiveness of our analytical derivations.

I. INTRODUCTION

In the past few years, Vehicular Adhoc NETworks (VANETs) have received a huge amount of well-deserved attention in the literature. Indeed, because of their unmistakable societal impact that promises to revolutionize the way we drive, various car manufacturers, government agencies and standardization bodies have spawned national and international consortia devoted to promoting the idea of a confluence between ITS and VANET. Examples include the Car-2-Car Communication Consortium [1], the Vehicle Safety Communications Consortium [2], and Honda’s Advanced Safety Vehicle Program [3], among many others.

The original impetus for VANETs was provided by the need to inform fellow drivers of actual or imminent road conditions, delays, congestion, hazardous driving conditions and other similar concerns. Therefore, most concerns of agencies and standardization bodies have spawned national and international consortia devoted to promoting the idea of a confluence between ITS and VANET. Examples include the Car-2-Car Communication Consortium [1], the Vehicle Safety Communications Consortium [2], and Honda’s Advanced Safety Vehicle Program [3], among many others.

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Huang et al. [4] proposed a Kalman filter technique to find errors in location estimation and trajectory prediction, incorporates communication errors. Abuelela et al. [5] stated statistical processing method to detect collisions. Due to the high mobility of vehicles, vehicles are constantly changing topology. Collision detection based on location errors will be challenging. Chang et al. [6] applied data fusion system to vision or GPS coordinate to predict vehicle collisions. However, an obvious limitation of their paper is that the dynamics of vehicle mobility are hard to capture by using data fusion alone. [7], [8] assessed the feasibility of CCWS in VANET. Young et al. [9] presented an accident reconstruction technique using an Event Data Recorder, a device that is very much like the notorious black boxes on board commercial aircraft. Sebastian et al. [10] proposed a graph model to capture the complex interactions between multiple roads and vehicles. The model can predict collisions using vehicle vicinity information along with mobility parameters including vehicle position, speed, and direction of travel. However, they did not consider at all vehicular acceleration and deceleration, two fundamental mobility parameters. Also, they do not compute the probability of a collision. Yang et al. [11] proposed an inter-vehicular communication protocol for CCWS. Misener et al. [12] applied sensors to predict collision. But sensors are affected by geometries. For example, cameras work well in good lighting conditions but they work poorly at night. These attempts in literature share a common shortcoming that ignores the large scale of mobility of vehicles and relies on a single parameter to predict vehicle collisions which can be caused by multiple factors. Therefore, we address analytical expressions include mobility parameters (relative distance, speed and acceleration) and conditional probability prediction of a collision. We state two intelligent collision warning algorithms using these expressions.
III. VEHICULAR MOBILITY OF A COLLISION

We assume that the vehicles are endowed with wireless transceivers which communicate with other vehicles and with whatever roadside infrastructure is in place. The exchanged information may include both mobility information and geographic information which can be used to predict a collision.

The idea to select metrics for a collision is to cover the common cases. With this in mind, in this paper we investigate two types of collision warning parameters: 1) mobility parameters including

- headway distance which is a distance between two consecutive vehicles, driver reaction time
- relative acceleration
- relative velocity
- warning time which is a duration of time when a collision can be alerted and avoided.

2) and, the conditional probability parameter that is, the conditional probability of a collision given headway distance, velocity and acceleration.

A. Mobility Constraints and Five Second Rule

1) Safety Distance: In current practice, a safety distance is widely used for the traffic, e.g. at 60 miles per hour about 225 feet, or 68.58 meters [14], [15]. This represents the absolute minimum stopping distance given that the vehicle travels at 60 miles per hour, regardless of the driver reaction time or skill level. As it turns out, this is roughly the distance traversed by a vehicle doing 60 miles per hour in three seconds. Therefore, in the ITS literature the safety distance is computed by the “Three Second Rule” [16]. In this paper, we use this value as the absolute Safety Distance.

2) Driver Reaction Time: In an emergency, the driver needs a certain amount of time to react. Accumulated empirical evidence suggests that in highway scenarios the average driver reaction time is about two seconds [14]. Actually, statistical data suggests that most drivers need 1.5 seconds just to start applying the brakes. Nonetheless, in this paper, we use two seconds as our reaction time. The reader should note that while the Three Second Rule is an absolute value (the actual distance required by the laws of mechanics to bring the vehicle to a halt), the Two Second driver reaction time is purely subjective and represents the reaction time of an average driver.

3) Warning Distance: Due to the above constrains, we define the collision warning distance, as the Five Second Rule: namely the sum of three second for stopping plus the two Second reaction time, i.e. five-second × velocity. In other words, we have to send out a warning message before the distance between two vehicles is less than or equal to warning distance.

B. Headway Distance

The headway distance $X_i$ is a distance between two consecutive vehicles. The headway distance is log-normal distribution [17]. We are interested about link distance $X$, i.e. the distance between a sender and a receiver. We write $X = \sum_i^n X_i$, shown in Figure 1. Since $X_1, X_2, ..., X_m$ are independent, $X$ can be approximated by log-normal distribution [17].

![Fig. 1. Headway distance and link distance.](image1)

C. More on the safety distance

The main goal of this section is to derive analytical expressions for safety distance at time $t$. Assume, without loss of generality, that at time $t_0 = 0$, two vehicles $i$ and $j$ move in the same direction or in the opposite direction, with $j$ ahead of $i$. Referring to Figures 2 let the random variable $X$ denote the distance separating the two vehicles moving in the same direction.

![Fig. 2. Illustrating the same direction scenario.](image2)

We only consider the collision involving vehicles traveling in the same lane. Mindful of the Five Second Rule, we have the safety distance

$$D_w = 5 \times v_{m}$$

where $D_w$ is the safety distance constrain and $v_{m}$ is the speed limit. We assume $X > D_w$ at time $0$. At time $t$, vehicles $i$ and $j$ have their own speeds $v_i(t)$ and $v_j(t)$ and accelerations $a_i(t)$ and $a_j(t)$ respectively. Therefore, the distance that two vehicles collide on the mobility condition at time $t$, called collision distance $X_c$ of $i$ and $j$, is defined by the Five Second Rule

$$X_c = 5 \times |v_j(t) - v_i(t)|$$

Assuming that $X_c \leq D_w$ we refer to $D_w - X_c$ as the warning zone $X_w$:

$$X_w = D_w - X_c$$

Similarly, we can specify a collision zone when two vehicles have a headway distance shorter than $X_c$. The relationship between the distances discussed above are shown in Figure 3. Assume that we are in a warning zone at time $t$.

![Fig. 3. Warning distance.](image3)
We assume that the speed limit on the roadway is $v_m$ and that no vehicle will run faster than $v_m$. For $t \geq 0$, we define $a(t)$, the acceleration of the vehicle at time $t$ as follows:

- if $a(0) = 0$, then $a(t) = 0$ for all $t \geq 0$;
- if $a(0) > 0$, then
  
  $$a(t) = \begin{cases} 
  a(0) & \text{for } t \leq \frac{v_m - v(0)}{a(0)} \\
  0 & \text{otherwise} 
  \end{cases}$$  
  (4)

- if $a(0) < 0$, then
  
  $$a(t) = \begin{cases} 
  a(0) & \text{for } t \leq \frac{-v(0)}{a(0)} \\
  0 & \text{otherwise} 
  \end{cases}$$  
  (5)

In other words, (4) and (5) indicate that as long as the vehicle has not reached the maximum speed $v_m$ or has not stopped (in case $a(0) < 0$), its acceleration remains $a(0)$. However, once the vehicle reaches the speed limit its speed (or has stopped), its acceleration becomes 0.

Given a generic vehicle with initial speed $v(0)$, the instantaneous speed $v(t)$ at time $t$ is defined as

$$v(t) = v(0) + \int_0^t a(u) du,$$  
(6)

where for all $u \in [0, t]$, $a(u)$ is the instantaneous acceleration at time $u$ defined above.

Now, (4) and (5) and (6), combined imply that

- if $a(0) = 0$, then $v(t) = v(0)$ for all $t \geq 0$;
- if $a(0) > 0$, then
  
  $$v(t) = \begin{cases} 
  v(0) + a(0)t & \text{for } t \leq \frac{v_m - v(0)}{a(0)} \\
  v_m & \text{otherwise} 
  \end{cases}$$  
  (7)

- if $a(0) < 0$, then
  
  $$v(t) = \begin{cases} 
  v(0) + a(0)t & \text{for } t \leq \frac{-v(0)}{a(0)} \\
  0 & \text{otherwise} 
  \end{cases}$$  
  (8)

Similarly, the distance that our generic vehicle travels in the time interval $[0, t]$ is

$$S(t) = \int_0^t v(x) dx,$$  
(9)

where $v(x)$ was defined above.

We now return to our vehicles $i$ and $j$. To simplify the notation, we write $v_i = v_i(0)$, $a_i = a_i(0)$ and $v_j = v_j(0)$, $a_j = a_j(0)$. The instantaneous speeds and accelerations $v_i(t)$ and $a_i(t)$, respectively, $v_j(t)$ and $a_j(t)$ are obtained by suitably instantiating (4), (5), (7), (8).

Now, (9) guarantees that the distances traversed in the time interval $[0, t]$ by vehicle $i$ and $j$ are, respectively,

$$S_i(t) = \int_0^t v_i(x) dx$$  
(10)

and

$$S_j(t) = \int_0^t v_j(x) dx.$$  
(11)

Assuming that at connection setup (i.e. time 0) the distance between the two vehicles was $x$, it follows that the distance between $i$ and $j$ at time $t$ can be written as

$$S_j(t) - S_i(t) + X.$$  
(12)

Given the safety distance constrain $D_w$, we can write:

$$X_c \leq S_j(t) - S_i(t) + X \leq D_w$$

D. Collision Time

We are interested about a time $t_m$ how long warning messages can be repeatedly sent. If we have this time, we can figure out how frequently we can send out warning messages. Let time $t_w$ be the time when two vehicles enter the warning zone, i.e. (13). We write:

$$S_j(t_w) - S_i(t_w) + X \leq D_w$$  
(13)

Let time $t_c$ be the time when two vehicles is in a collision zone. For $t_c$, we can compute it through equation:

$$S_j(t_c) - S_i(t_c) + X \leq X_c$$

Therefore, we write:

$$t_m = t_c - t_w$$  
(14)

We will solve $t_w$ as an example of how to obtain the $t_c$ and $t_w$. For the full detail of all the cases discussion, please refer [18].

IV. PROBABILITY OF A COLLISION

We are not only interested in mobility parameters, but also in the conditional probability of a collision given current mobility parameters of both vehicles. Since we notice that mobility parameters are correlated each other, it is complex to compute the conditional probability. We will use a statistical method that does not require that the mobility parameters be independent random variables.

Let $D_{ij}$ be the distance between two cars $i$ and $j$ , $D_w$ be the safety distance between two cars, then we can define a binary random variable $Y_1$, to be:

$$Y_1 = \begin{cases} 
  1 & \text{if } \log D_{ij} < \log D_w \\
  0 & \text{if } \log D_{ij} \geq \log D_w 
  \end{cases}$$

We take log operation because the distance $D_{ij}$ is log-normal distribution and we want to obtain normal distribution variables.

Let $v_i(t)$ be the velocity of car $i$ and $v_j(t)$ be the velocity of car $j$ at time $t$, $v_r$ be the safety relative velocity between two cars. Similar to the distance definition, we can define a binary random variable $Y_2$ to be

$$Y_2 = \begin{cases} 
  1 & \text{if } v_j(t) - v_i(t) > v_r \\
  0 & \text{if } v_j(t) - v_i(t) \leq v_r 
  \end{cases}$$

Let $a_i(t)$ be the acceleration of car $i$ and $a_j(t)$ be the acceleration of car $j$ at time $t$, $a_r$ be the safety relative acceleration between two cars. Similar to the distance definition, we can define a binary random variable $Y_3$ to be

$$Y_3 = \begin{cases} 
  1 & \text{if } a_j(t) - a_i(t) > a_r \\
  0 & \text{if } a_j(t) - a_i(t) \leq a_r 
  \end{cases}$$
Note that $Y_i, i = 1, 2, 3$ are binary random variables with distribution functions

$$F(Y_i) = \begin{cases} 0 & \text{if } Y_i < 0 \\ 1 - p_i & \text{if } 0 \leq Y_i < 1 \\ 1 & \text{if } Y_i \geq 1 \end{cases}$$

where $p_i$ is the probability of $Y_i = 1$, i.e. $Pr(Y_i = 1)$.

Let $C$ denote the binary random variable of a collision will occur between cars $i$ and $j$, and $E[C] = P_c$. Our interest is to find the conditional probability of $C_{ij}$ given $Y_i$'s, $i = 1, 2, 3$.

$$P(C|Y_1, Y_2, Y_3) = \frac{P(C, Y_1, Y_2, Y_3)}{P(Y_1, Y_2, Y_3)} \quad (15)$$

The joint distribution of discrete marginal distribution is not unique and hard to get, but a fully specified distribution function can be constructed using Gaussian copula [19].

A. The Gaussian Copula – a Refresher

Constructing multivariate distributions by means of copula has proved popular in recent years, many research have done by [20],[21],[19] and [22]. The motivation for the copula approach is probably rooted in the aim of forming multivariate non-normal distributions by combining marginal models with dependence patterns.

If $X = (X_1, X_2, \cdots, X_m)' \sim H$ where $H$ is a $m$-dimensional distribution function with margins $H_1, H_2, \cdots, H_m$, then the copula is of the form

$$C_H(u_1, u_2, \cdots, u_m) = H\{H_1^{-1}(u_1), \cdots, H_m^{-1}(u_m)\}$$

$u_i \in (0,1), i = 1, 2, \cdots, m$.

When $X \sim N_m(0,\Gamma)$ with standardized margins and $H_i = \Phi$, then we have an important special case, called Gaussian copula which is denote by $C_\Phi(u|\Gamma)$ and its density is given by

$$C_\Phi(u|\Gamma) = |\Gamma|^{1/2} \exp\{-\frac{1}{2} q^T \Gamma^{-1} q + \frac{1}{2} q^T q\}$$

$$= |\Gamma|^{1/2} \exp\{-\frac{1}{2} q^T (I_m - \Gamma^{-1}) q\} \quad (16)$$

where $q = (q_1, q_2, \cdots, q_m)'$ with normal scores $q_i = \Phi^{-1}(u_i), i = 1, 2, \cdots, m$. By complementing the copula $C_H$ with given margin $F_1, F_2, \cdots, F_m$ a new multivariate distribution can be obtained by

$$G(y) = C_\Phi\{F_1(y_1), F_2(y_2), \cdots, F_m(y_m)|\Gamma\} \quad (17)$$

When marginal distributions are discrete, a multivariate probability is obtained by taking Radon–Nikodym [23] derivation for (17),

$$g(y) = P(Y_1 = y_1, Y_2 = y_2, \cdots, Y_m = y_m)$$

$$= \sum_{j_1=0}^1 \cdots \sum_{j_m=0}^1 (-1)^{m+\sum_{i=1}^m j_i} C_\Phi(u_1^{j_1}, u_2^{j_2}, \cdots, u_m^{j_m}|\Gamma)$$

where $u_i^1 = F_i(y_i)$, $u_i^0 = F_i(y_i-)$. $F_i(y_i-)$ is the left-hand of $F_i$ at $y_i$ which is equal to $F_i(y_i - 1)$ when the support of $F_i$ is an integer set.

B. The Probability: $P(C, Y_1, Y_2, Y_3)$

In our case, $m = 4$, according to the four-variate probability function is of the form

$$Pr = P(C, Y_1, Y_2, Y_3)$$

$$= C_\Phi(w^1, u_1^1, u_2^1, u_3^1) - C_\Phi(w^0, u_1^1, u_2^1, u_3^0) - C_\Phi(w^1, u_1^0, u_2^1, u_3^1) + C_\Phi(w^1, u_1^0, u_2^0, u_3^1) + C_\Phi(w^0, u_1^0, u_2^0, u_3^1)$$

$$- C_\Phi(w^0, u_1^0, u_2^1, u_3^0) - C_\Phi(w^1, u_1^0, u_2^0, u_3^0)$$

$$+ C_\Phi(w^0, u_1^0, u_2^1, u_3^0) + C_\Phi(w^0, u_1^1, u_2^0, u_3^0)$$

$$+ C_\Phi(w^0, u_1^1, u_2^1, u_3^0) - C_\Phi(w^0, u_1^0, u_2^0, u_3^0)$$

$$- C_\Phi(w^1, u_1^0, u_2^1, u_3^0) + C_\Phi(w^0, u_1^0, u_2^0, u_3^0)$$

$$- C_\Phi(w^0, u_1^1, u_2^0, u_3^0) + C_\Phi(w^0, u_1^0, u_2^0, u_3^0). \quad (19)$$

where

$$w^1 = F(C), w^0 = F(C - 1)$$

$$u_i^1 = F_i(y_i), u_i^0 = F_i(y_i - 1)$$

for $i = 1, 2, 3$. So far, we obtain the numerator of the fraction in equation (15).

C. The Probability: $P(D, A, V)$

We are interested to compute the probability of a certain distance, acceleration and velocity, i.e. $P(D, A, V)$. $P(D, A, V)$ is the denominator of the fraction in equation (15).

We assume the velocity of vehicles is normally distributed, i.e. velocity random variable $V \sim N(\mu_v, \sigma_v)$. Similarly, we assume the acceleration of vehicles is normally distributed, i.e. acceleration random variable $A \sim N(\mu_a, \sigma_a)$. As we know that the distance between vehicles is log-normally distributed, i.e. headway distance $D \sim logN(\mu_d, \sigma_d)$ [24].

We can construct the joint distribution of $D$, $A$ and $V$ by the same model as 16. Since $D$, $A$ and $V$ are continuous random variables, a multivariate dispersion model ([19],[25] and [26]) can be equivalently defined by the density of the following form

$$P(D, A, V; \Gamma) = c_\Phi\{F_1(D), F_2(A), F_3(V)|\Gamma\}$$

$$\cdot f_1(D)f_2(A)f_3(V) \quad (20)$$

where

$$c_\Phi(u|\Gamma) = |\Gamma|^{1/2} \exp\{-\frac{1}{2} q^T \Gamma^{-1} q + \frac{1}{2} q^T q\}$$

$$= |\Gamma|^{1/2} \exp\{\frac{1}{2} q^T (I_3 - \Gamma^{-1}) q\}$$

D. The Probability: $P(C|D, A, V)$

We are interested to compute the probability of an accident given distance, acceleration and velocity $D, A, V$, i.e.
threshold, as shown in table I. We will introduce two
Each of them can be computed and compared with a certain
threshold, as shown in table I. We will introduce two

\[
P(C|D, A, V) = \frac{P(C, D, A, V)}{P(D, A, V)}
\]

By definition (15), we can obtain

\[
P(C|D, A, V) = \frac{P(C, D, A, V)}{P(D, A, V)}
\]

Based on the previous discussion, we can compute
\(P(C|D, A, V)\) by using (19) and \(P(D, A, V)\) by using
(20) separately. Therefore, we can compute the value of
\(P(C|D, A, V)\).

V. DISCUSSION

We have obtained a set of collision prediction parameters. Each of them can be computed and compared with a certain
threshold, as shown in table I. We will introduce two

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Thresholds</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safety distance (X)</td>
<td>(5 \times v_m, 5 \times v_r)</td>
<td>(12)</td>
</tr>
<tr>
<td>Relative velocity (v_r)</td>
<td>(v_m)</td>
<td>(v_j(t) - v_i(t)) (21)</td>
</tr>
<tr>
<td>Relative acceleration (a_r)</td>
<td>(\frac{v_m}{2s})</td>
<td>(a_j(t) - a_i(t))</td>
</tr>
<tr>
<td>Warning time (t_m)</td>
<td>-</td>
<td>(14)</td>
</tr>
<tr>
<td>Collision probability (P_c)</td>
<td>(p_c)</td>
<td>(21)</td>
</tr>
</tbody>
</table>

collision prediction examples: weighted collision prediction and priority based prediction.

A. Weighted Collision Prediction

When two vehicles are in the same lane, the basic idea of weighted collision prediction is the following:

\[
CI = w_1 \frac{X}{5 \times v_m} + w_2 P_c + w_3 \frac{v_m - v_r}{v_m} + w_4 \frac{v_m 2a_r}{v_m}
\]

where \(CI\) is the collision index which is a value, \(X = \frac{v_m - P_c}{v_m}\) is the fraction representing headway distance effect, \(w_1\) is the fraction representing relative speed effect, \(w_2\) is the fraction representing relative acceleration effect. We write \(1 \geq w_1 \geq w_2 \geq w_3 \geq w_4 \geq 0\). If \(CI\) is small, the likelihood of collision is high. If \(CI\) is big, the likelihood of collision is low. The mobility parameters will be periodically exchanged among vehicles. The collision warning message will be regenerated on new mobility data during the time \(t_m\).

Advantages of this algorithm include: 1) state parameters such as safety headway distance, collision probability, etc. 2) dynamic parameters such as relative velocity, relative acceleration. We combine all of these parameters to predict the collision to improve the accuracy of prediction. For example, vehicles \(i\) and \(j\) are close each other at 30m at speed 60mph but they are driving on the same speed and acceleration. In this case we will not send out annoying collision warning message. If \(i\) moves closer to \(j\) with a high speed and acceleration and then decreases the speed and acceleration to the same ones as \(j\) has, we will only send one warning message instead of repeatedly sending annoying message.

B. Priority Based Prediction

We assume the priority has the following relationship: \(X > P_c > v_r > a_r\) and two vehicles are in the same lane. The basic idea of priority base prediction is the following:

1) Check \(X\). If \(X\) indicate a collision, we send out collision warning; otherwise, do step 2;
2) Check \(P_c\). If \(P_c\) indicate a collision, we send out collision warning; otherwise, do step 3;
3) Check \(v_r\). If \(v_r\) indicate a collision, we send out collision warning; otherwise, do step 4;
4) Check \(a_r\). If \(a_r\) indicate a collision, we send out collision warning; otherwise, no collision;

The advantage of the priority based prediction: 1) parameters are sequentially checked on the basis of priority; 2) simple to implement.

VI. NUMERICAL RESULTS

In this section, we will use MATLAB to show the numerical results. We are interested in the probability of a collision given a certain value of distance, speed and acceleration. Assuming that the relative speed \(v_r\) satisfies \(v_j(t) - v_i(t) > v_r\), we varied the value of relative acceleration \(a_r\) and the value of \(\log D_w\). As expected, the probability of collision becomes bigger when \(\log D_w\) becomes smaller, shown in Figure 4. This is because smaller \(\log D_w\) means that there is less space for collision warning. When the safety relative acceleration \(a_r\) becomes larger, the value of acceleration difference \((a_j(t) - a_i(t)) - a_r\) becomes smaller. The acceleration is more close to the safety acceleration. Therefore, the probability of collision becomes smaller, shown in Figure 4.

![Fig. 4. The probability of collision vs. \(\log D_w\) and \(a_r\).](image)

We are also interested to examine the affect of relative speed to the value of probability of collision. We varied the value of safety relative speed and acceleration, assuming \(\log D_{ij} < \log D_w\). As expected, the probability of collision decreases when the value of \(a_r\) or \(v_r\) increases, shown in Figure 5. This is because the real speed or acceleration is
more close to the safety speed or acceleration requirement. Therefore, the probability of collision is smaller.

![Graph showing the probability of collision vs. vr and ar.]

**Fig. 5.** The probability of collision vs. vr and ar.

**VII. CONCLUDING REMARKS**

In this work we have derived analytical expressions of mobility parameters and conditional probability which can be used for collision prediction. This paper follows a statistical approach to characterize the conditional probability of a collision given certain mobility parameters and states a mobility analysis to uncover the expressions of driver reaction time. As a discussion, the possible algorithms of using these metrics are also shown in this paper.

In the future, we plan to supplement our analytical results by extensive simulation such as importing real traffic data and simulating the performance of collision prediction. In addition, the cooperative collision warning can integrate the sensor-based methods to enhance the robustness and correctness of prediction.

**ACKNOWLEDGMENT**

Work supported in part by NSF grant CNS 0721586.

**REFERENCES**


