Network Topology Design using Analytic Hierarchy Process

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Abstract—The topology of a network seriously affects its cost, reliability, throughput, and traffic pattern, etc., so we need to simultaneously consider these multiple criteria, which have different units, when evaluating network topologies. Moreover, we need to reflect the relative importance of each criterion when evaluating the network topology. However, ordinary methods of network topology design consider only a single criterion. In this paper, we apply AHP (analytic hierarchy process), which has been widely used when making a rational decision with multiple criteria, to network topology design. AHP enables us to reflect the relative importance of each criterion on the results. In network topology design, we need to consider a huge number of candidates, and the differences in the weights among candidates are small. Therefore, the optimum topologies will be determined by just the weighted criteria, and other criteria will not be considered. Therefore, to emphasize the difference of the weights, we propose to use a linear-transformed value of each criterion when constructing weights in AHP.

I. INTRODUCTION

For network carriers operating and managing physical network resources, one important problem is how to design the physical network topology. The physical topology determines many important factors, including the network equipment cost, the network operating cost, and the reliability against node or link failure. Moreover, if we cannot set a logical path such as a label switched path (LSP) or a wavelength path, then the physical topology directly affects the route that packets take and determines the quality perceived by the user, such as packet network delay or throughput. On the other hand, if we use a multiprotocol label switching (MPLS) router or optical cross connect (OXC) and set a logical path on the physical network, we can limit the influence of the physical network topology on the user-perceived quality by appropriately designing the logical topology. In this case, we need to design the logical network topology carefully. Therefore, it is important to investigate the optimum design method for the physical and logical network topologies.

For the backbone network topology, for example, we should carefully consider both the connectivity between any pair of edge nodes and the redundancy to maintain the connectivity in the case of node or link failure. To improve the redundancy, it is desirable to have more routes between each edge node pair by providing more intermediate nodes and links. However, the increase in nodes and links will also increase the equipment cost and the operating cost. For users, it is desirable to avoid congestion at intermediate nodes and have a shorter path length to reduce the packet network delay. From the viewpoint of efficient use of limited resources and reliability, traffic concentration on a specific link should be avoided, and the traffic load should be balanced among all links. However, if we decrease the number of nodes and links to reduce the network cost, then the flexibility of path design is degraded, so it becomes difficult to balance the traffic load among all links and suppress the path length. Hence, when designing the network topology, we need to consider multiple incompatible criteria having different units.

There have been few studies on physical topology design. One proposed a physical topology design minimizing the total physical link count under the condition that connectivity between all pairs of nodes is maintained in the case of a single physical link failure [1]. On the other hand, there are many reports on logical topology design for a given physical network topology. For example, [2] and [3] proposed a logical topology design minimizing the maximum link load in a wavelength-routed optical network. A design method minimizing the average hop count of wavelength paths was proposed in [4], and another method maximizing overall throughput in a wavelength-routed optical network was proposed in [5]. Although a lot of network topology design methods have been proposed, almost all of them consider just a single criterion.

As an approach that considers multiple criteria, the concept of the Pareto frontier is well known [6], and one study applied this concept to logical topology design [7]. Assume that there are $M$ criteria, $V_1, \cdots, V_M$, and let $V_{m,x}$ denote the $m$-th criterion of candidate $x$. We can say that candidate $x$ is better than candidate $y$ in the Pareto sense only if $V_{m,x} \leq V_{m,y}$ for any $m$ and there exists criterion $m$ that satisfies $V_{m,x} < V_{m,y}$. (Assume that smaller values are desirable in all criteria.) All candidates that are surpassed by no other candidates are the optimum solution set, i.e., the Pareto frontier. However, a large number of candidates are regarded as the Pareto frontier, so it is difficult to effectively limit the optimum candidates and select one network topology to use.

As another evaluation method considering multiple criteria simultaneously, data envelopment analysis (DEA) is known [8],[9]. DEA has been developed mainly as a way of evaluating the efficiency of businesses or projects, and it derives the efficiency by emphasizing the strong criteria of each candidate. We applied DEA to the network topology design problem in [10]. However, we need to solve a linear program, and the required calculation time dramatically increases as the
number of topology candidates increases. Moreover, we cannot consider the degree of importance of each criterion because DEA evaluates all criteria with the same weights. We need to reflect the relative importance of each criterion when evaluating the network topology.

The analytic hierarchy process (AHP) is known as a way to make a rational decision considering multiple criteria [11],[12]. Using AHP, we can reflect the relative importance of each criterion on the evaluation result. It grasps all the related factors as a hierarchical structure and quantifies qualitative factors, such as the importance of each criterion, using paired comparison. In this paper, we apply AHP to network topology evaluation. When AHP is used, the number of decision candidates is normally up to around seven [13]. However, we can construct a huge number of topology candidates for even a reasonably sized network, so the differences in the weights among candidates vanish. Therefore, we propose to use the linear-transformed value of each criterion as the weight to emphasize the difference of weights among candidates.

In Sec. II, we summarize the problem of network topology design. In Sec. III, we describe the method of topology design using AHP. We present the results of a numerical evaluation in Sec. IV and conclude with a brief summary in Sec. V.

II. NETWORK TOPOLOGY DESIGN

A. Definition of Problem

The problem of designing a network topology can be classified into physical and logical topology design. Here, we summarize both topology design problems.

Physical Topology Design

As general components of a network, we consider three components: edge nodes, core nodes, and links. Edge nodes include source and destination nodes of traffic and accommodate end hosts either directly or through an access network. On the other hand, a core node provides just a relaying function and does not accommodate end hosts. A link connects any edge or core node. Assume that the traffic matrix between any pair of edge nodes is given. Network carriers already have conduits and buildings, which were constructed considering the geographical conditions, so it is unrealistic to redesign these fixed properties. Hence, we assume that the positions of all edge nodes as well as the positions where we can put core nodes or links are given.

A realistic problem is how to select the positions for setting optical fibers or routers within the existing infrastructure, according to the network requirements. In this paper, therefore, we define the physical topology design as selecting the locations of core nodes and links from the given candidate positions. Edge nodes are put at all the candidate locations for edge nodes. Because the physical topology is designed before the actual network resources, such as optical fibers or routers, are provided, restrictions on network resources, e.g., link capacity, are not considered in the physical topology design.

Logical Topology Design

Using MPLS routers or OXCs, we can construct a virtual path, i.e., an LSP or wavelength path, on the physical network and explicitly set the route via which traffic travels between each pair of edge nodes. It is difficult to modify the physical topology after it has been designed, so the time scale of physical topology design is a year or more. On the other hand, the logical topology can be modified easily, and flexible management is possible by modifying the logical topology within a short time period, e.g., a few minutes to hours, according to changes in traffic pattern [14]. The logical topology design sets logical paths on the given physical topology and the traffic matrix. Because the physical network resources are given, we need to consider various restrictions such as link capacity.

If we can set logical paths on the physical network, we can simultaneously design both the physical and logical topologies. The time scale of the logical topology design is much shorter than that of the physical topology, so only the logical topology is redesigned after the operation has been started.

Hereafter, we choose physical topology design for backbone networks as an example and assume that the logical paths are set simultaneously at the time of the physical topology design, without considering resource restrictions. However, our design method can be easily applied to logical topology designs with various resource restrictions.

B. Evaluation Criteria

Now, let us enumerate criteria that should be considered when designing the physical topology. For the physical topology of backbone networks, we should consider both connectivity between any pair of edge nodes and redundancy to maintain the connectivity in the case of node or link failure. Hence, we consider two restrictions: (i) achieving connectivity between any pair of edge nodes and (ii) maintaining the connectivity even when a single link failure occurs. Although there are various evaluation criteria that we could choose, we use the following four as examples, hereafter. (Let $V_i$ denote the $i$-th criterion.)

1) Total node count

This is the total number of edge and core nodes. Letting $N_e$ and $N_c$ denote the numbers of edge nodes and deployed core nodes, respectively, we have

$$V_1 = N_e + N_c.$$ (1)

From the viewpoint of the network equipment cost and the network operating cost, smaller $V_1$ is desirable.

2) Total link length

This is the sum of all link lengths. Let us define $d_i$ as the length of link $l$ and $E_a$ as the deployed link set. We have

$$V_2 = \sum_{l \in E_a} d_i.$$ (2)

From the viewpoint of the network equipment cost and the network operating cost, smaller $V_2$ is also desirable.

3) Sum of path lengths weighted by path traffic

Because the propagation delay between edge nodes is proportional to the path length, a shorter path length is desirable. Therefore, we can consider the sum of path lengths weighted by path traffic, $\sum_{s,d \in N_e, s \neq d} D_{sd} T_{sd}$, as the third criterion, where $T_{sd}$
is the amount of traffic on $P_{sd}$. $P_{sd}$ is the path from edge node $s$ to $d$, $D_{sd}$ is the length of $P_{sd}$ ($D_{sd} = \sum_{i \in \Phi_{sd}} d_i$), and $\Phi_{sd}$ is the link set on $P_{sd}$. Moreover, let $t_i$ denote the total amount of traffic on link $l$. $\zeta$ can also be obtained from $d_i$ and $t_i$ as $\zeta = \sum_{i \in E_a} d_i \times t_i$. So, we choose $V_3$ as

$$V_3 = \sum_{i \in E_a} d_i \times t_i.$$  \hspace{1cm} (3)

Smaller $V_3$ is also desirable.

4) **Amount of traffic on maximally loaded link**

When traffic concentrates on a specific link, the quality of paths on the bottleneck link is seriously degraded. Because the link transmission capacity can be adjusted in discrete units, i.e., optical fiber or wavelength, the link load should be balanced among all links from the viewpoint of effectively utilizing the network resources.

We can consider the variance of link load or the amount of traffic on the maximally loaded link as the criterion indicating the degree of load balancing. Here, we choose the latter as the fourth criterion and set

$$V_4 = \max_{l \in E_a} t_l.$$  \hspace{1cm} (4)

Smaller $V_4$ is also desirable.

C. **Making Topology Candidates**

If we freely select the locations to put core codes and links among the given positions, the obtained physical topology set includes ones that do not satisfy the restrictions mentioned in Sec. II-B. Therefore, we first construct an initial physical topology that does satisfy the restrictions and then make the physical topology candidates by adding core nodes or links to it. As a result, we can choose any physical topology from the candidate set without considering the restrictions because they have already been satisfied at the stage of initial topology construction.

Let $z$ denote the number of positions where we can put a link, except the ones where links are deployed in the initial physical topology. The number of topologies obtained by putting links at any possible position is $2^z$. For each obtained physical topology, we put core nodes at the possible locations with degree of three or more. However, no switching function is required at the possible locations with degree of two, so we do not put core nodes at these positions; instead, the two links are directly connected. All topologies having core node positions with degree of one are invalid topologies, so we eliminate them from the candidate set.

Next, for each topology candidate, we set logical paths between each pair of edge nodes using the given traffic matrix. Here, we assume that a single logical path $P_{sd}$ is provided from edge node $s$ to $d$, and logical paths are deployed using a greedy algorithm in descending order of path traffic. $T_{sd}$, the path design method affects the path length and the degree of load balancing among the four criteria mentioned in Sec. II-B. Hence, the route having the minimum value of $\sum_{i \in \Phi_{sd}} d_i \times t_i'$ is selected when each path route is allocated. Here, $t_i'$ is the amount of traffic on link $l$ at the stage of target path accommodation. After setting all the paths, we also eliminate from the candidate set all the topologies with links that do not accommodate any paths.

D. **Network Model**

The network model (the Japanese archipelago model [15]) used as an example in this paper is shown in Fig. 1. Closed circles represent edge nodes and dotted circles and lines represent positions where we can put core nodes and links, respectively. The link lengths between nodes $i$ and $j$, $d_{ij}$ (km), presented in [15] are used. Moreover, the population accommodated by edge node $k$, $U_k$, is shown in Table I. The amount of traffic from edge node $s$ to $d$, $T_{sd}$, was set to be proportional to the product of $U_s$ and $U_d$. That is,

$$T_{sd} = \frac{T \cdot U_s U_d}{\sum_{m,n \neq m} U_m U_n},$$  \hspace{1cm} (5)

where $T$ is the total amount of traffic within the network, and we set $T = 10$ Tbps.

A loop topology is a typical and simple topology in which connectivity between all pairs of edge nodes is maintained in the case of a single link failure. It is often used in backbone networks. Here, we assumed a loop topology as the initial physical topology and always put links at the locations shown by solid lines in Fig. 1. The degree of all possible positions of core nodes on the loop was two, so no switching function was necessary at these positions. In the initial physical topology, we did not put any core nodes on the loop.

Besides the links used in the initial physical topology, 17 possible positions were left for links, so we were able to construct $2^{17}$ physical topologies in total. However, after removing all topologies with core nodes of degree 1 or unused links, as mentioned in Sec. II-C, we had 83,868 candidate topologies. The time needed to obtain these candidates was about 75 seconds using a PC with a 3.2-GHz CPU and 1 GB of memory.

![Network model](image)

**Fig. 1.** Network model.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$U_k (10^3)$</th>
<th>$k$</th>
<th>$U_k (10^3)$</th>
<th>$k$</th>
<th>$U_k (10^3)$</th>
<th>$k$</th>
<th>$U_k (10^3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5683</td>
<td>4</td>
<td>41,059</td>
<td>5</td>
<td>26,313</td>
<td>7</td>
<td>16,292</td>
</tr>
<tr>
<td>2</td>
<td>11,822</td>
<td>6</td>
<td>9,847</td>
<td>6</td>
<td>15,310</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table I**

**EDGE NODE POPULATIONS**
III. NETWORK TOPOLOGY EVALUATION USING AHP

A. Overview of AHP

In a decision-making problem, there are normally three kinds of elements, i.e., problem $P$, evaluation criteria $V$, and alternative plan $A$. As shown in Fig. 2, AHP grasps the relationship among these elements as the hierarchical structure. AHP links related elements\(^1\), and evaluation criteria $V$ can take multiple layers, $V^1$, $V^2$, \ldots. By calculating the relative strength (weight) for each pair of related elements, AHP derives the score $S_i$ of each alternative plan $A_i$.

We need to quantify the relative importance of each criterion $V$ against the problem $P$. AHP achieves this by comparing the elements on each level in pairs. For two elements $X_i$ and $X_j$ in layer $c$, the value shown in Table II is set to $a_{ij}$, the relative importance of $X_i$ against $X_j$. Defining $w_i$ as the true weight of $X_i$, we have $a_{ij} = w_i/w_j$ ideally. Let $A$ and $w$ denote a matrix of pairwise comparisons $a_{ij}$ and a vector of $w_i$, respectively. By multiplying $A$ by $w$, we get $Aw = nw$, where $n$ is the number of elements in the layer. Therefore, $w$ is the principal eigenvector and $n$ is the maximum eigenvalue.

In practice, it is difficult to consistently set $a_{ij}$ for all pairs of elements, so we need to judge the degree of inconsistency. Letting $\lambda_{\text{max}}$ denote the maximum eigenvalue of $A$, we have $\lambda_{\text{max}} \geq n$ \cite{11,12}. So, we can judge the degree of inconsistency using the consistency index $\text{C.I.}$ defined by

$$\text{C.I.} = \frac{\lambda_{\text{max}} - n}{n - 1}. \quad (6)$$

$\lambda_{\text{max}}$ decreases as the consistency index increases, and $\lambda_{\text{max}} = n$ when $A$ is a consistent matrix. Hence, the degree of consistency increases as $C.I.$ decreases, and the pairwise comparison is normally accepted when $C.I.$ is smaller than or equal to 0.1.

Let $w_{ij}^c$ denote the weight of the $i$-th element in layer $c$ against the $j$-th element in layer $c - 1$. We also define $\Phi_i^c$ as the element set related to element $V_c^i$. Then, $S_i^c$, the score of $V_i^c$ against the problem $P$, is derived as

$$S_i^c = \sum_{j: V_j^{c-1} \in \Phi_i^c} w_{ij}^c X_j^{c-1}. \quad (7)$$

In layer 1, $S_i^1$ is equal to the weight of each element against $P$. We can recursively obtain $S_i^c$ in the order of $c = 2, 3, \ldots$ and finally derive $S_i$, the score of alternative plan $A_i$. Plans with large $S_i$ are desirable.

<table>
<thead>
<tr>
<th>Numerical values</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equally important or preferred</td>
</tr>
<tr>
<td>3</td>
<td>Slightly more important or preferred</td>
</tr>
<tr>
<td>5</td>
<td>Rather more important or preferred</td>
</tr>
<tr>
<td>7</td>
<td>Much more important or preferred</td>
</tr>
<tr>
<td>9</td>
<td>Very much more important or preferred</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>Intermediate values to reflect compromise</td>
</tr>
<tr>
<td>Reciprocals</td>
<td>Used to reflect dominance of the second alternative over the first</td>
</tr>
</tbody>
</table>

\(^1\)In the figure, only some of the links are shown for simplicity.

B. Applying AHP to Network Topology Evaluation

Here, we apply AHP to network topology evaluation. As shown in Fig. 3, the target problem $P$, which is choosing optimum network topologies in this case, is located in the top layer (layer 0), the evaluation criteria $V_c$ are located in the middle layer (layer 1), and the candidate topologies are located in the bottom layer (layer 2).

The pairwise comparisons described in Sec. III-A enable us to derive the scores (weights) of the elements in layer 1, that is the evaluation criteria, for the problem $P$. If all the criteria have numerical values, pairwise comparisons are not necessary to obtain the weights of the elements in layer 2, that is topology candidates, for each element in layer 1. Let $V_{ij}$ denote the $j$-th criterion of candidate $i$. Because AHP evaluates elements with higher weights more highly, it derives weights based on $X_{ij}$, which is the reciprocal of $V_{ij}$, i.e., $X_{ij} = 1/V_{ij}$. The weights of the elements in layer 2, $w_{ij}^2$, need to satisfy the normalized condition $\sum_{i=1}^{N_2} w_{ij}^2 = 1$ where $N_2$ is the number of elements in layer 2. AHP usually uses the normalized values of $X_{ij}$ among all the candidates as the weights, so we have $w_{ij}^2 = X_{ij}/\sum_{k=1}^{N_2} X_{kj}$.

Because the number of decision candidates in AHP has been normally up to around seven \cite{13}, this weight setting has been reasonable. However, the number of topology candidates is huge even in a moderately sized network, so the denominator of this equation becomes huge, and the difference in the weights $w_{ij}$ among the candidates becomes far smaller compared with the difference in the weights $w_i^c$ among the evaluation criteria. As a result, AHP tends to simply choose the candidates with desirable values of the weighted criteria. In the case shown in Sec. II-D, for example, the maximum differences in the weights of the candidates for each criterion were $1.48 \times 10^{-5}$ for $V_1$, $7.00 \times 10^{-6}$ for $V_2$, $5.39 \times 10^{-6}$ for $V_3$, and $1.08 \times 10^{-6}$ for $V_4$. As described in Sec. IV-B, the differences among the weights of criteria for the problem $P$ took values with an order of 0.1, so the final score of each candidate will be determined by just the weighted criteria, and other criteria will not be considered.

To solve this problem, we propose to use the normalized value of $Y_{ij}$, that is, a linear-transformed value of $X_{ij}$, rather than $X_{ij}$ itself, for the weights. In other words, we define $Y_{ij}$ as $Y_{ij} = a(X_{ij} + b)$, where $a$ and $b$ are arbitrary real numbers.
The weights $w_{ij}^2$ are derived as

$$w_{ij}^2 = \frac{a(X_{ij} + b)}{\sum_{k=1}^{N^2} a(X_{kj} + b)} = \frac{X_{ij} + b}{\sum_{k=1}^{N^2} X_{kj} + bN^2}. \quad (8)$$

Because of the normalization, $w_{ij}^2$ is independent of $a$, so we set $a = 1$ hereafter. Moreover, to make all the weights take a positive value or zero, we set $b$ to the minimum value of $X_{ij}$ among all the candidates multiplied by -1, i.e., $b = -\min_{i,j} \{X_{ij}\}$. Figure 4 plots the weights of all the 83,868 candidates described in Sec. II-D in descending order for each of the four criteria. $X$ denotes the results when we simply normalized $X_{ij}$, and $Y$ denotes the results when we normalized $Y_{ij}$ using the method described in this paper. We confirmed that the difference in the weights was increased by the linear-transformation. In particular, we could dramatically decrease the weights for candidates with a large value of the criterion, i.e., a small value of $X_{ij}$. Thus, we could avoid choosing topologies having terrible values for some criteria as optimum ones.

![AHP structure in network topology evaluation.](image)

**Fig. 3.** AHP structure in network topology evaluation.

![Weights in descending order for each criterion.](image)

**Fig. 4.** Weights in descending order for each criterion.

### IV. Numerical Evaluation

This section shows the results of applying AHP to the 83,868 topology candidates described in Sec. II-D.

#### A. Properties of Candidate Topologies

Table III summarizes the minimum, maximum, average, and coefficient of variation (CV) of the four criteria: $V_1$, the number of nodes, $V_2$, the total link length ($10^3$ km), $V_3$, the total path length weighted by path traffic ($10^3$ km × Gbps), and $V_4$, the amount of traffic on the maximally loaded link (Gbps), for 83,868 candidate topologies. Because the number of topologies increases as $N$ grows, the average of $V_1$ was close to the maximum value. Hence, in many of the topologies among the 83,868 candidates, core nodes and links were put at a lot of possible locations. In these topologies, the flexibility of path allocation was high, and the averages of $V_3$ and $V_4$ were close to the minimum value.

The correlation coefficients of the four criteria calculated for 83,868 candidate topologies are summarized in Table IV. Because the number of deployed core nodes increased as the deployed link count grew, $V_1$ showed the same tendency as $V_2$. In topologies in which paths were allocated more flexibly, the average path length and the maximum link-load were decreased. Therefore, $V_3$ and $V_4$ showed the same tendency. As a result, positive correlation was observed between two criteria belonging to the same criterion group when $V_1$ and $V_2$ were assigned to criterion group A and $V_3$ and $V_4$ were assigned to criterion group B. In particular, a strong positive correlation was observed between $V_1$ and $V_2$. On the other hand, a negative correlation was observed between two criteria belonging to different groups. This is because we need to increase the topological redundancy to improve the criteria belonging to group B, which means that we need to increase the number of links and core nodes deployed. In the physical topology set, the network costs ($V_1$ and $V_2$) and the path optimization ($V_3$ and $V_4$) are incompatible.

169 topology candidates formed a Pareto frontier when we evaluated 83,868 topologies using the concept of the Pareto optimum mentioned in [7]. In this approach, we cannot focus on a small optimum set.

<table>
<thead>
<tr>
<th>$V_1$</th>
<th>$V_2$</th>
<th>$V_3$</th>
<th>$V_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>7</td>
<td>2.31</td>
<td>6.01</td>
</tr>
<tr>
<td>Maximum</td>
<td>19</td>
<td>5.76</td>
<td>9.74</td>
</tr>
<tr>
<td>Average</td>
<td>14.48</td>
<td>4.59</td>
<td>7.13</td>
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<tr>
<td>CV</td>
<td>0.122</td>
<td>0.065</td>
<td>0.072</td>
</tr>
</tbody>
</table>

### B. Scores of Evaluation Criteria

The importance of each criterion depends on the subjectivity of the network designer, so we simply consider four scenarios in which each of the four criteria is regarded as most important. The matrices of pairwise comparisons of the first and forth

<table>
<thead>
<tr>
<th>$V_2$</th>
<th>$V_3$</th>
<th>$V_4$</th>
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</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>0.694</td>
<td>-0.318</td>
</tr>
<tr>
<td>$V_2$</td>
<td>*</td>
<td>-0.530</td>
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<tr>
<td>$V_3$</td>
<td>*</td>
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</table>
scenarios are shown in Fig. 5. As mentioned in Sec. IV-A, $V_1$ and $V_2$ have a positive correlation, and $V_3$ and $V_4$ also have a positive correlation. Therefore, we also set a high weight to $V_2$ when $V_1$ was regarded as the most important (scenario 1), for example. The pairwise comparison matrixes for the second and third scenarios are also given in the same fashion.

The derived scores of the criteria, $S_1^1$, were $S_1^1 = 0.467$, $S_2^1 = 0.277$, $S_3^1 = 0.095$, and $S_4^1 = 0.160$ in scenario 1, for example. The consistency index C.I. was 0.01 in all four scenarios, so we can judge that the matrixes of pairwise comparisons shown in Fig. 5 were consistent.

**Scenario 1: $V_1$ is important.**

<table>
<thead>
<tr>
<th></th>
<th>$V_1$</th>
<th>$V_2$</th>
<th>$V_3$</th>
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<td>2</td>
<td>4</td>
<td>3</td>
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<td>1/4</td>
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<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>$V_4$</td>
<td>1/3</td>
<td>1/2</td>
<td>2</td>
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**Scenario 4: $V_4$ is important.**

<table>
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</tbody>
</table>

Fig. 5. Example of scenarios for criteria comparison.

### C. Results of Applying AHP

Tables V and VI summarize the values and normalized scores of all the four criteria for the top six topologies with the highest final scores in scenarios 1 and 4, respectively. NS (normalized score) is the relative value among all the candidates. It is defined as $(X_{ij} - \text{Min}X)/(\text{Max}X - \text{Min}X)$, where $\text{Max}X$ and $\text{Min}X$ are the maximum and minimum values of $X_{ij}$. The NS of the topology with the minimum value of $X_{ij}$ is zero, and that of the topology with the maximum value of $X_{ij}$ is unity. Figures 6 and 7 illustrate these topologies in scenarios 1 and 4, respectively. In scenarios 1, topologies with desirable values in the weighted criterion, i.e., $V_1$, were chosen. We observed the same results in scenarios 2 and 3 as well. So, the results for scenario 2 were almost the same as those for scenario 1 because criteria 1 and 2 have a strong positive relationship. In scenario 4, however, topologies with desirable values for all the criteria were chosen.

To see the cause of this, we plotted a scattergram of the NS of each criterion against that of other criteria, as shown in Fig. 8. In the figures, the results for a criterion with a positive correlation, e.g., $V_2$ for $V_1$, are omitted. We confirmed that some topologies with a desirable value for $V_1$, $V_2$, or $V_3$ also had desirable values for other criteria. On the other hand, no topologies with a desirable value for $V_4$ had desirable values in other criteria. This means that we avoided selecting topologies with terrible values in other criteria even when we selected topologies with a desirable value in the weighted criterion, when $V_1$, $V_2$, or $V_3$ was regarded as most important.

In scenario 4, on the other hand, when we selected topologies with a desirable value in $V_4$, the other criteria of the selected topologies were terrible. Thus, in scenarios 1 and 2, the chosen topologies had few core nodes and links, whereas in scenario 4, the chosen topologies were well-balanced ones that could effectively provide alternative paths between the two big cities, Tokyo (node 3) and Osaka (node 5), with a small number of links and core nodes.

To clarify the availability of this weight setting method, Table VII summarizes the properties of the top six topologies when we simply used the normalized values of $X_{ij}$ as the weights in scenario 4. These top six topologies are also shown in Figure 9. As mentioned in Sec. III-B, when we apply the simple normalized values of $X_{ij}$ to the weights, AHP chooses topologies with desirable values in the weighted criteria. We confirmed that topologies with a desirable value for $V_4$ were chosen even though $V_1$ and $V_2$ of these topologies were terrible. Our weight setting method enabled AHP to select desirable topologies automatically according to the properties of criteria.

**Table V**

<table>
<thead>
<tr>
<th>Rank</th>
<th>$V_1$ Value (10^3 km)</th>
<th>$V_2$</th>
<th>$V_3$ Value (10^3 km x Gbps)</th>
<th>$V_4$ Value (Gbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>3.31</td>
<td>1.00</td>
<td>4.67</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>3.54</td>
<td>0.85</td>
<td>3.89</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>3.61</td>
<td>0.80</td>
<td>3.56</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>3.84</td>
<td>0.68</td>
<td>3.37</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>3.46</td>
<td>0.90</td>
<td>3.62</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>3.48</td>
<td>0.89</td>
<td>3.77</td>
</tr>
</tbody>
</table>

**Table VI**

<table>
<thead>
<tr>
<th>Rank</th>
<th>$V_1$ Value (10^3 km)</th>
<th>$V_2$ Value (10^3 km x Gbps)</th>
<th>$V_3$ Value (Gbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>4.00</td>
<td>0.59</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>3.84</td>
<td>0.59</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>4.16</td>
<td>0.52</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
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</tr>
<tr>
<td>5</td>
<td>9</td>
<td>4.17</td>
<td>0.52</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
<td>4.00</td>
<td>0.59</td>
</tr>
</tbody>
</table>

**V. CONCLUSION**

When designing the topology of a network, we need to consider multiple criteria, i.e., cost, throughput, reliability, and transmission delay, simultaneously. It is also important to reflect the relative importance of each criterion in the result. In this paper, we applied AHP, which is known as a way to
make a rational decision by understanding related elements as a hierarchical structure, to network topology evaluation. AHP assumes that the number of elements in each layer is small. However, the number of topology candidates in the topology design problem is huge, so the differences in the weights among candidates are small. So, we proposed to use the normalized value of the linear-transformed criterion as the weights. Through numerical evaluation, we clarified that AHP enabled us to choose desirable topologies according to the relative importance of each criterion.

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REFERENCES