Adaptive Field-oriented Control of Synchronous Motors with Damping Windings

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Two different nonlinear dynamic control algorithms are presented for synchronous motors with damping windings: (i) an adaptive speed-sensorless controller for rotor position tracking in the presence of unknown constant load torque, on the basis of rotor angle, stator and field windings currents measurements; (ii) an adaptive control law for rotor speed tracking in the presence of uncertain constant load torque and motor inertia, which is based on measurements of mechanical variables (rotor angle and speed) and stator windings currents but does not require field current. As in classical field oriented control, the three voltage inputs are designed so that the direct axis component of the stator current vector is driven to zero; the controllers generate, as an intermediate step, the reference signals for the field current and for the quadrature axis component of the stator current vector, which respectively determine the direct axis component of the damping winding flux vector and the electromagnetic torque. Simulation results are provided for a 20-KW synchronous machine, which show the effectiveness of the two proposed control algorithms.

Keywords: Nonlinear control design; output feedback control; output tracking; synchronous motors

1. Introduction

Salient pole wound rotor synchronous motors with damping windings (see [15,16]) consist of a stator containing armature windings (electrically connected to the a.c. supply system) and a rotor equipped with a field winding carrying a d.c. current and one or more damping windings so that oscillations in the power grid are damped. Due to their higher efficiency and overload capability, to their smaller physical size and to their lower maintenance and ventilation requirements along with the absence of brush commutation at stator side (so that no power limitation is imposed), synchronous machines gradually replaced d.c. motors in applications such as: mine-shaft winders, rolling mill and gearless cement-mill drives, ship propulsion drives, large compressors, excavators and belt conveyor systems.

The problem of controlling a salient pole wound rotor synchronous motor is however rather difficult. It is a nonlinear, highly coupled and multivariable problem with three control inputs (stator and excitation windings voltages) and three outputs to be controlled (rotor position or speed, damping winding flux modulus and stator current vector angle), which do not coincide with the measured outputs. Although rotor angle and speed and stator currents can be measured, flux sensors are usually not available and no field current information can be obtained in applications in which slip rings are excluded; moreover, damping winding currents are also not available.

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The basics of field oriented control for synchronous motors with damping windings can be found in [11] (see also [3,4] and references therein), while theoretical and experimental results in [18] show that, due to the rotor salient pole effect, the air-gap flux oriented control does not allow for decoupled control of synchronous motors with dampers. In [17], a magnetizing-flux oriented control method is presented: the stator current vector is divided into two components which independently determine the magnetization and the electromagnetic torque. In [2], a method of estimating the currents in the damping windings is proposed in order to allow their inclusion within a field oriented controller while a vector control theory is developed in [12] for a current-fed salient pole synchronous machine with dampers. A nonlinear control scheme consisting of two nested loops for speed and currents control is experimentally validated in [1]: the references for stator and field currents are computed for desired internal and torque angles (the field oriented control can be obtained as a special case) while unmeasurable damper winding currents are estimated by a reduced order observer. A novel and simple method of achieving a field-oriented self-controlled synchronous motor is presented in [14]: an additional field winding is located in the quadrature axis of the rotor. Two different methods for estimating the field current when brushless excitation is employed are proposed in [5].

The aim of this paper is to design two nonlinear adaptive output feedback controls with stability analysis for synchronous motors with damping windings: rotor position or speed tracking of arbitrary smooth bounded reference signals is to be guaranteed along with tracking for direct axis component of damping winding flux vector; direct axis component of the stator current vector is to be regulated to zero so that field orientation is achieved. Two different adaptive algorithms are proposed: (i) a speed-sensorless rotor position tracking controller, which is based on rotor angle and both stator and excitation windings currents measurements; (ii) a rotor speed tracking control law relying on measurements of both rotor angle and speed and stator windings currents. While the former does not require rotor speed sensors and achieves exponential position tracking for any unknown constant load torque (global exponential position tracking when flux initial values are known), the latter is adaptive with respect to both constant uncertain load torque (within known bounds) and motor inertia (within restricted bounds) and guarantees asymptotic speed tracking for any motor initial condition without requiring the availability of field current measurements. As in classical field oriented control, reference signals for quadrature axis component of stator current vector and field current are generated as an intermediate control step: they are responsible for the tracking of rotor angle or speed and for the tracking of direct axis component of damping winding flux vector, respectively. Numerical simulations with reference to a two pole pairs 20-KW synchronous motor with damping windings illustrate the effectiveness of the two proposed control algorithms.

2. Dynamical Model and Field Orientation

Assuming linear magnetic circuits, the dynamics of a balanced synchronous motor with damping windings in the \((d, q)\) reference frame rotating at the electrical angular speed \(p\omega\) and identified by the electrical angle \(p\theta\) in the fixed \((a, b)\) reference frame attached to the stator, are given by the seventh-order nonlinear model (see for instance [6,11,17])

\[
\dot{\theta} = \omega \\
\dot{\omega} = \frac{3p}{2J} [\psi_{sd} i_{sq} - \psi_{sq} i_{sd}] - \frac{T_L}{J} = \frac{T_e}{J} - \frac{T_L}{J} \\
\psi_{sd} = -R_s i_{sd} + p\omega \psi_{sq} + u_{sd} \\
\psi_{sq} = -R_s i_{sq} - p\omega \psi_{sd} + u_{sq} \\
\psi_{wd} = -R_w i_{wd} \\
\psi_{wq} = -R_w i_{wq} \\
\psi_f = -R_f i_f + u_f
\]

in which: \(p\) is the number of pole pairs, \(\theta\) is the rotor angle, \(\omega\) is the rotor speed, \((\psi_{sd}, \psi_{sq})\) are the stator fluxes, \((\psi_{wd}, \psi_{wq})\) are the damping winding fluxes, \(\psi_f\) is the excitation winding flux, \((u_{sd}, u_{sq})\) are the stator voltages, \(u_f\) is the excitation winding voltage, \(J \in [J_m, J_M]\) is the motor inertia while \((R_s, R_w, R_w, R_f)\) are the resistances in stator, damping and excitation windings, respectively. \(T_e\) is the electromagnetic torque while the load torque \(T_L \in [T_{Lm}, T_{LM}]\) is typically an uncertain step disturbance.

The magnetic equations are assumed to be linear and given by

\[
\begin{align*}
\psi_{sd} &= L_{sd} i_{sd} + L_{md} i_{wd} + L_{md} i_f \\
\psi_{sq} &= L_{sq} i_{sq} + L_{mq} i_{wq} \\
\psi_{wd} &= L_{md} i_{sd} + L_{md} i_{wq} + L_{md} i_f \\
\psi_{wq} &= L_{mq} i_{sq} + L_{mq} i_{wq} \\
\psi_f &= L_{md} i_{sd} + L_{md} i_{wq} + L_{mf} i_f
\end{align*}
\]

in which: \((i_{sd}, i_{sq})\) are the stator currents, \((i_{wd}, i_{wq})\) are the damping winding currents, \(i_f\) is the excitation current, \((L_{sd}, L_{sq}), (L_{md}, L_{mq}), L_{mf}\) are the inductances
in stator, damping and excitation windings, respectively, while \((L_{md}, L_{mq})\) are the mutual inductances between stator and rotor windings.

The vectors \(\left(\begin{array}{c} i_{sd} \\ i_{f} \end{array}\right)\), \(\left(\begin{array}{c} i_{sd} \\ i_{f} \end{array}\right)\), \(\left(\begin{array}{c} i_{sd} \\ i_{f} \end{array}\right)\), \(\left(\begin{array}{c} i_{sd} \\ i_{f} \end{array}\right)\), \(\left(\begin{array}{c} i_{sd} \\ i_{f} \end{array}\right)\), \(\left(\begin{array}{c} i_{sd} \\ i_{f} \end{array}\right)\), \(\left(\begin{array}{c} i_{sd} \\ i_{f} \end{array}\right)\), \(\left(\begin{array}{c} i_{sd} \\ i_{f} \end{array}\right)\) are obtained multiplying the corresponding vectors \((\psi_{sa}, \psi_{sb})\), \((\psi_{wa}, \psi_{wb})\), \((\psi_{sa}, \psi_{sb})\), \((\psi_{sa}, \psi_{sb})\), \((\psi_{sa}, \psi_{sb})\), \((\psi_{sa}, \psi_{sb})\) in the fixed \((a, b)\) reference frame by the matrix:

\[
R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}.
\]

By eliminating the unmeasured variables \((\psi_{sd}, \psi_{sq}, \psi_{q})\) in Eq. (1) by using Eq. (2), the dynamics of the synchronous motor with damping windings in the \((d, q)\) reference frame are given by [note that \(L_{sd} - L_{sq} = L_{md} - L_{mq}\) according to [11]]

\[
\begin{align*}
\dot{\theta} &= \omega \\
\dot{\omega} &= \frac{3p}{2T_f} [\beta_{\omega 1} - \beta_{\omega 2}] i_{sq} i_{sd} + \beta_{\omega 1} i_{f} i_{sq} \\
&+ \frac{L_{md}}{L_{wd}} (\psi_{wa} i_{sq} - \psi_{wa} i_{sd}) - \frac{T_L}{T_f} \\
\dot{i}_{wd} &= -\frac{R_{wd}}{L_{wd}} i_{wd} + \frac{R_{wd} L_{md}}{L_{wd}} (i_{sd} + i_{f}) \\
\dot{i}_{wq} &= -\frac{R_{wq}}{L_{wq}} i_{wq} + \frac{R_{wq} L_{md}}{L_{wq}} i_{sq} \\
\frac{d}{dt} \begin{bmatrix} i_{sd} \\ i_{f} \end{bmatrix} &= L^{-1} \begin{bmatrix} \Xi_d \\ \Xi_f \end{bmatrix} \\
\frac{d}{dt} \begin{bmatrix} i_{sq} \end{bmatrix} &= \beta_{\psi 1} \left[ -L_{wd} (R_s L_{wq}^2 + R_{wq} L_{mq}^2) i_{sq} \\
&- (L_{sd} L_{wd} - L_{md}^2) \frac{R_{wd} i_{sd}}{L_{wq}} \\
&- L_{md} (L_{wd} - L_{md}) \frac{R_{wd} L_{md} L_{mq}}{L_{wq}^2} \psi_{wq} \\
&- L_{md} \frac{R_{wd} \psi_{wd} + L_{wd} i_{sd}}{L_{wq}} \right] \\
\Xi_d &= -\left( R_s L_{wd}^2 + R_{wd} L_{md}^2 \right) \frac{L_{wd}^2}{L_{wd}^2} i_{sd} - \frac{R_{wd} L_{md}^2}{L_{wd}^2} i_{f} \\
&+ \frac{L_{sq} L_{wq} - L_{mq}^2}{L_{wq}} \frac{R_{wd} L_{md} L_{mq}}{L_{wq}^2} \psi_{wq} \\
&+ \frac{R_{mq} \psi_{wq} + u_{sd}}{L_{wq}} \\
\Xi_f &= -\left( R_s L_{wd}^2 + R_{wd} L_{md}^2 \right) \frac{L_{wd}^2}{L_{wd}^2} i_{f} - \frac{R_{wd} L_{md}^2}{L_{wd}^2} i_{sd} \\
&+ \frac{R_{wd} L_{md}}{L_{wd}^2} \psi_{wd} + u_{f}
\end{align*}
\]

in which \((\theta, \omega, i_{sd}, i_{sq}, i_{f}, \psi_{sda}, \psi_{sq})\) constitute the state variables and \((u_{sd}, u_{sq}, u_{f})\) are the control inputs. To simplify notation, in Eq. (3) we have used the reparametrization:

\[
\begin{align*}
\beta_{\omega 1} &= L_{md} - \frac{L_{md}^2}{L_{wd}} > 0 \\
\beta_{\omega 2} &= L_{mq} - \frac{L_{mq}^2}{L_{wd}} > 0 \\
\beta_{\psi 1} &= \frac{L_{wq}}{(L_{sq} L_{wq} - L_{mq}^2) L_{wd}} > 0 \\
L &= \frac{(L_{md} L_{wd} - L_{md}^2) L_{wd}}{L_{sd} L_{wd} - L_{md}^2} \frac{L_{md} L_{wd} - L_{mq}^2}{L_{wd}} > 0.
\end{align*}
\]

According to [11], the torque per ampere can be maximized by imposing the stator current to be all in the \(q\)-axis, i.e., \(i_{sd} = 0\): this condition is usually referred to as field orientation. Even though one drawback of choosing \(i_{sd} = 0\) is the inescapable fact that the terminal power factor will be lagging (which may be unacceptable in large hp drives), this choice is advantageous in terms of decoupling the transient response of the system. In fact, if field orientation is performed, the electromagnetic torque is

\[
T_e = \frac{3p}{2} \left[ \beta_{\omega 1} i_{f} + \frac{L_{md}}{L_{wd}} \psi_{wd} \right] i_{sq}
\]

while the dynamics of the damping wind flux vector \(d\)-component are given by

\[
\dot{\psi}_{wd} = -\frac{R_{wd}}{L_{wd}} \psi_{wd} + \frac{R_{wd} L_{md}}{L_{wd}} i_{sq}
\]

so that, while in induction machines the electromagnetic torque production requires the stator currents to contain both the flux- and the torque-producing components, in a synchronous machine the excitation is provided by the field current and the main torque is produced by its interaction with the quadrature axis component of the stator current vector, leading to a complete decoupling of the \(d\)-damper circuit from the stator windings.

If we rewrite, according to Eq. (2), the torque equation as

\[
T_e = \frac{3p}{2} \left[ L_{md} (i_{f} + i_{sd}) i_{sq} - L_{mq} i_{sq} i_{sd} \\
+ (L_{wd} - L_{mq}) i_{sd} i_{sq} \right]
\]

it is divided in the three torque components:

- \(T_{e(d)} = L_{md} (i_{f} + i_{sd}) i_{sq}\) (field plus \(d\)-axis damper winding torque);
- \(T_{eq} = L_{mq} i_{sq} i_{sd}\) (\(q\)-axis damper winding torque);
- \(T_{er} = (L_{wd} - L_{mq}) i_{sd} i_{sq}\) (reluctance (saliency) torque).
Under steady-state conditions and field orientation, \( \psi_{ad}, \psi_{aq} \) are constant and \( i_{sd}, i_{sq} \) are zero, so that the torque expression reduces to

\[
T_e = \frac{3p}{2} L_{mld} f_{bq}
\]

which is similar to the one of the compensated separately excited d.c. machine.

3. Rotor Position Tracking

In this section, assuming that rotor angle and both stator and field current measurements are available for feedback, a fifth order dynamic nonlinear adaptive control algorithm is designed for the seventh order nonlinear model (Eqs. 3–5) of a synchronous motor with damping windings: asymptotic rotor position tracking of arbitrary smooth bounded reference signals, damping winding flux vector \( d \)-axis component tracking of smooth bounded reference signals belonging to a certain class of time functions (including positive constant reference values) and stator current vector \( d \)-axis component regulation to zero are to be guaranteed in the presence of unknown constant load torque. The controller includes an observer and an estimator for both the unmeasured rotor speed and damping winding fluxes and for the unknown load torque and guarantees, when the damping winding flux estimator is properly initialized, boundedness and exponential tracking and estimation for any initial condition and for any unknown constant load torque; a local result holds when flux initial values are not exactly known. The solution presented in this section is suitable in positioning applications such as mine-shaft winders and excavators, in which it is desirable to eliminate speed sensors. In fact, in addition to cost considerations, the two common methods for obtaining velocity information (direct tachometer readings or differentiation of the rotor position measurements) may fail to provide accurate and noise-free velocity estimates allowing speed and may deteriorate the positioning performance.

3.1. Problem Statement

Let us denote by \( \theta^*(t) \) and \( \psi^*(t) \) the smooth bounded reference signals with bounded time derivatives \( \theta^{(i)}(t), \psi^{(i)}(t), 1 \leq i \leq 3, 1 \leq j \leq 2 \), for the output variables to be controlled, which are the rotor position \( \theta(t) \) and the damping winding flux vector \( d \)-component \( \psi_{wd}(t) \), respectively and assume that \( \psi^*(t) \) and \( \psi^{(1)}(t) \) satisfy, for all \( t \geq 0 \), the following inequality:

\[
\psi^*(t) + \frac{(L_{wd} - L_{md})}{R_{wd}} \psi^{(1)}(t) \geq c_{\psi} > 0 \quad (6)
\]

in terms of a positive scalar \( c_{\psi} \).

Following the field oriented control strategy [11], our goal is to design a dynamic output feedback compensator [only \( (\theta, i_{sd}, i_{sq}, i_{sd}) \) measurements are available]

\[
\begin{bmatrix}
    u_{sd} \\
    u_{sb} \\
    u_f
\end{bmatrix} =
\begin{bmatrix}
    \cos \theta & -\sin \theta & 0 \\
    \sin \theta & \cos \theta & 0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    u_{sd} \\
    u_{sq} \\
    u_f
\end{bmatrix}
\]

so that the asymptotic properties

\[
\lim_{t \to +\infty} [\theta(t) - \theta^*(t)] = 0
\]

\[
\lim_{t \to +\infty} [\psi_{wd}(t) - \psi^*(t)] = 0
\]

\[
\lim_{t \to +\infty} [i_{sd}(t)] = 0
\]

are guaranteed for any unknown constant load torque \( T_L \).

3.2. Control Algorithm (I)

The proposed control algorithm consists of a current loop:

\[
\begin{aligned}
    u_{sd} &= -\phi_d - k_d i_{sd} - \frac{5}{4} k p^2 i_{sd} \left[ \frac{L_{mq}^2 \psi_{q}^2}{L_{mq}^2 \psi_{wq}} \right] + \frac{(L_{mq} L_{wq} - L_{mq}^2)^2}{L_{wq}^2} i_{sq} \\
    u_{sb} &= -\phi_f - k_f (i_f - \dot{i}_f) \\
    u_{sq} &= -[\beta_q L_{wd}]^{-1} \left[ \phi_q + k_q (i_q - \dot{i}_q) - \frac{5}{4} k p^2 \beta_q^2 (i_{sq} - \dot{i}_{sq}) \left( (L_{wd} L_{wd} - L_{md})^2 \dot{i}_{sq}^2 + \frac{L_{md}^2 (L_{wd} - L_{md})^2 \dot{i}_{sf}^2 + L_{md}^2 \dot{i}_{wd}^2}{L_{wd}^2} \right) \right]
\end{aligned}
\]

with the known functions

\[
\begin{bmatrix}
    \phi_d \\
    \phi_f
\end{bmatrix} = \begin{bmatrix} \Theta_d \end{bmatrix} - L \begin{bmatrix} 0 \\
    0 \end{bmatrix}
\]

\[
\Theta_d = - \frac{(R_s L_{wd}^2 + R_{md} L_{wd}^2)}{L_{wd}^2} i_{sd} - \frac{R_{wd} L_{md}^2}{L_{wd}^2} i_f + \frac{(L_{mq} L_{wq} - L_{mq}^2)}{L_{wq}} p \hat{\omega} i_{sq} + \frac{R_{wd} L_{md}}{L_{wd}^2} \dot{\psi}_{wd} + \frac{L_{mq}}{L_{wq}} \hat{\psi}_{wd}
\]

\[
\Theta_f = - \frac{(R_s L_{wd}^2 + R_{md} L_{wd}^2)}{L_{wd}^2} i_f - \frac{R_{wd} L_{md}^2}{L_{wd}^2} i_{sd} + \frac{R_{wd} L_{md}}{L_{wd}^2} \dot{\psi}_{wd}
\]
\[
\phi_q = \beta_q \left[ -\frac{L_{md}(R_s L_{mq}^2 + R_{wq} L_{mq}^2)}{L_{mq}^2} i_{sq} 
- (L_{ml} L_{wd} - L_{md}^2) p \hat{\omega} i_{td} 
- L_{md}(L_{wd} - L_{md}) p \omega i_{td} + \frac{R_{wq} L_{wd} L_{mq}}{L_{mq}^2} \hat{\psi}_{wd} 
- L_{md} p \omega \hat{\psi}_{wd} \right] - \frac{d\phi_q}{dt}.
\]

It depends on the intermediate reference signals \((\hat{i}_q, i_{sq})\) for \((i_q, i_{sq})\) (which are responsible for tracking of rotor angle and direct axis component of damping winding flux vector, respectively)
\[
\begin{align*}
\hat{i}_q &= \frac{L_{wd}}{L_{wd} R_{wd}} \frac{R_{wd}}{L_{wd}} \psi^* + \psi^{(1)} \\
\hat{i}_{sq} &= \frac{2J}{3p} \left[ \psi^* + \frac{(L_{md} - L_{wd})}{R_{wd}} (L_{mq} - L_{md}) \hat{i}_{sq} \right]^{-1} \\
&\quad \cdot \left[ -k_{\omega} \hat{\omega} - (k_{\omega} k_{\theta} + 1)(\theta - \theta^*) + k_{\omega} \theta^*(1) \\
&\quad + \theta^{(2)} - k_{\theta}(\omega - \theta^*) + \frac{\hat{T}_L}{J} \right] \\
&= \frac{L_{wd}}{L_{wd} R_{wd}} \frac{R_{wd}}{L_{wd}} \psi^* + \psi^{(1)} \\
&\quad - \frac{L_{md}}{L_{wd}} \hat{i}_{sq} - \frac{L_{md}}{L_{wd}} \hat{i}_{td} \\
&\quad - \frac{\hat{T}_L}{J} + k_2 (\theta - \theta^*) \\
&= -J k_3 (\theta - \theta). 
\end{align*}
\]

The estimates of the unmeasured damping winding fluxes are obtained by the second order open loop observer
\[
\begin{align*}
\hat{\psi}_{wd} &= -\frac{R_{wd} L_{wd}}{L_{wd}} \hat{i}_{td} + \frac{R_{wd} L_{md}}{L_{wd}} (i_{td} + i_j) \\
\hat{\psi}_{wq} &= -\frac{R_{wd}}{L_{wd}} \hat{i}_{td} + \frac{R_{wd} L_{mq}}{L_{wd}} i_{sq}. 
\end{align*}
\]

3.3. Stability Analysis
Let \(y(t) = [\hat{\theta}, \hat{\omega}(t), \hat{\psi}_{wd}(t), \hat{\psi}_{wq}(t), \hat{i}_{sd}(t), \hat{i}_f(t), \hat{i}_{sq}(t), \hat{i}_{q}(t), \hat{e}_p(t), \hat{e}_s(t), \hat{e}_T(t)^T \) be the solution of the closed loop system (Eqs. 3–5 and 7–10) with \(\theta = \theta - \theta^*, \hat{\omega} = \omega - \hat{\omega}^*(1) + k_\theta \theta, \hat{\psi}_{wd} = \psi_{wd} - \hat{\psi}_{wd}^*, \hat{\psi}_{wq} = \psi_{wq} - \hat{\psi}_{wq}, \hat{i}_f = \hat{i}_f^* - \hat{i}_f^*, \hat{i}_{sq} = \hat{i}_{sq} - \hat{i}_{sq}^*, \hat{e}_p = \theta - \theta^*, \hat{e}_s = \omega - \hat{\omega}, \hat{e}_T = \hat{e}_T^* - \hat{e}_T^* \) and on the estimates \((\hat{\omega}, \hat{T}_L)\) for the unmeasured variable \(\omega\) and the unknown load torque \(T_L\), respectively, which are provided by the third order observer including the auxiliary variable \(\hat{\theta}\)
\[
\begin{align*}
\dot{\hat{\theta}} &= \hat{\omega} + k_1 (\theta - \theta) \\
\dot{\hat{\omega}} &= \frac{2p}{J} \left[ (\beta_{\omega 1} - \beta_{\omega 2}) i_{sq} \hat{i}_{sd} + \beta_{\omega 1} i_j i_{td} \\
&\quad + \frac{L_{wd}}{L_{wd}} \hat{i}_{sq} \hat{i}_{td} - \frac{L_{md}}{L_{wd}} \hat{\psi}_{wd} \hat{i}_{sd} \\
&\quad - \frac{\hat{T}_L}{J} + k_2 (\theta - \theta) \right] \\
\hat{T}_L &= -J k_3 (\theta - \theta). 
\end{align*}
\]

The control law (Eqs. 7–10), which is designed by using back-stepping and robust techniques (including the choice of suitable Lyapunov functions as it is shown in the next section) consists of: (i) an inner control loop (Eq. 7) for stator and field currents containing feed-forward actions and suitable stabilizing and robust feedback terms; (ii) two outer control loop (Eq. 8) for rotor angle and damping winding flux vector \(d\)-component; (iii) an observer (Eq. 9) and an estimator (Eq. 10) for the unmeasured rotor speed and damping winding fluxes and the unknown load torque.

The control algorithm (Eqs. 7–10) depends on: the available \((\hat{\theta}, \hat{i}_{td}, \hat{i}_f, \hat{i}_{sq})\) measurements; the smooth bounded reference signals \(\theta^*(t), \psi^*(t)\) and their bounded time derivatives \(\theta^{(i)}(t), \psi^{(i)}(t), 1 \leq i \leq 3, 1 \leq j \leq 2\); the known motor parameters \(p, R_s, R_{wd}, R_{wq}, R_f, L_{sd}, L_{sq}, L_{wd}, L_{md}, L_{mq}, J\); the positive control parameters \(k_\theta, k_\omega, k_d, k_f, k, k_1, k_2, k_3\).

3.3. Stability Analysis
Let \(y(t) = [\hat{\theta}, \hat{\omega}(t), \hat{\psi}_{wd}(t), \hat{\psi}_{wq}(t), \hat{i}_{sd}(t), \hat{i}_f(t), \hat{i}_{sq}(t), \hat{i}_{q}(t), \hat{e}_p(t), \hat{e}_s(t), \hat{e}_T(t)^T \) be the solution of the closed loop system (Eqs. 3–5 and 7–10) with \(\theta = \theta - \theta^*, \hat{\omega} = \omega - \hat{\omega}^*(1) + k_\theta \theta, \hat{\psi}_{wd} = \psi_{wd} - \hat{\psi}_{wd}^*, \hat{\psi}_{wq} = \psi_{wq} - \hat{\psi}_{wq}, \hat{i}_f = \hat{i}_f^* - \hat{i}_f^*, \hat{i}_{sq} = \hat{i}_{sq} - \hat{i}_{sq}^*, \hat{e}_p = \theta - \theta^*, \hat{e}_s = \omega - \hat{\omega}, \hat{e}_T = \hat{e}_T^* - \hat{e}_T^* \) where \(r_q\) satisfies the first order differential equation
\[
\dot{r}_q = -\frac{R_{wd}}{L_{wd}} r_q + \frac{R_{wd} L_{mq} \hat{i}_q}{L_{wd}} 
\]
in which
\[
\dot{r}_{sq} = \frac{2J}{3p} \left[ \psi^* + \frac{(L_{md} - L_{wd})}{R_{wd}} \psi^{(1)} \right]^{-1} \\
\cdot \left[ (k_\omega + k_\theta) e_s + (k_\omega k_\theta + 1) e_\theta \\
+ e_T + \theta^{(2)} + \frac{\hat{T}_L}{J} \right].
\]

Define
\[
\begin{align*}
r_{sq} &= i_{sq} - i_{sq}^* \quad \text{with} \quad \psi_{wd}(0), \psi_{wq}(0), \text{then the closed loop system solution } y(t) \text{ of Eqs. (3–5), (7–10) is bounded and tends exponentially to zero:} \\
&\quad \text{(i) for any initial condition } y(0) \in \mathbb{R}^{10}, \\
&\quad \text{(ii) for any time-varying smooth bounded reference signal } \theta^*(t) \text{ with bounded time derivatives } \theta^{(i)}(t), 1 \leq i \leq 3; \\
&\quad \text{(iii) for any time-varying smooth bounded reference signal } \psi^*(t) \text{ with bounded time derivatives } \psi^{(i)}(t), 1 \leq j \leq 2, \text{ satisfying inequality (6);}
\end{align*}
\]

Theorem 1: If the damping winding flux estimator (10) is properly initialized to the damping winding flux values \(\psi_{wd}(0), \psi_{wq}(0)\), then the closed loop system solution \(y(t)\) of Eqs. (3–5), (7–10) is bounded and tends exponentially to zero:

(i) for any initial condition \(y(0) \in \mathbb{R}^{10}, \)

(ii) for any time-varying smooth bounded reference signal \(\theta^*(t)\) with bounded time derivatives \(\theta^{(i)}(t), 1 \leq i \leq 3; \)

(iii) for any time-varying smooth bounded reference signal \(\psi^*(t)\) with bounded time derivatives \(\psi^{(i)}(t), 1 \leq j \leq 2, \text{ satisfying inequality (6);} \)
(iv) for any positive control parameters $k_0, k_\omega, k_d, k_f, k_q, k$;
(v) for any positive control parameters $k_1, k_2, k_3$ such that the polynomial associated to $[1, k_1, k_2, k_3]^{T}$ has all roots with negative real part.

**Proof:** If the damping winding flux estimator is properly initialized, then $\tilde{\psi}_{wd} = \psi_{wd}, \tilde{\psi}_{wq} = \psi_{wq}$ and the closed loop system is

$$
\begin{bmatrix}
\dot{\tilde{\vartheta}} \\
\dot{\tilde{\omega}} \\
\dot{\tilde{\psi}}_{wq}
\end{bmatrix} = \Sigma_{\tilde{z}}
\begin{bmatrix}
\tilde{\vartheta} \\
\tilde{\omega} \\
\tilde{\psi}_{wq}
\end{bmatrix} +
\begin{bmatrix}
0 \\
\frac{R_{d} L_{sd}}{L_{ew} I_{sd}} \vartheta \\
\frac{R_{d} L_{sd}}{L_{ew} I_{sd}} \omega
\end{bmatrix}
$$

$$
\Sigma_{\tilde{z}} =
\begin{bmatrix}
-k_0 & 1 & 0 \\
-1 + \frac{R_{d} L_{sd}}{L_{ew} I_{sd}} g_1 & -k_\omega + \frac{R_{d} L_{sd}}{L_{ew} I_{sd}} g_2 - \frac{3 p L_{sd}}{2 T_{18}} & 0 \\
\frac{R_{d} L_{sd}}{L_{ew} I_{sd}} g_1 & \frac{R_{d} L_{sd}}{L_{ew} I_{sd}} g_2 & \frac{R_{d} L_{sd}}{L_{ew} I_{sd}} g_2
\end{bmatrix}
$$

Define

$$
e = [e_0, e_\omega, e_T]^{T}
$$

and consider the quadratic positive definite function $[P is the symmetric positive definite solution of $A_{c}^{T} P + P A_{c} = -I$]

$$
V_{c} = e^{T} Pe
$$

whose time derivative along the trajectories of the $e$-subsystem satisfies

$$
\dot{V}_{c} = \|e\|^{2}
$$

Eq. (13) and inequality (14) guarantee that $(e_0, e_\omega, e_T)$ tend exponentially to zero from any initial condition which, in turn, implies, according to Eq. (11), boundedness of $r_q$. Consider the quadratic positive definite function

$$
V_l = \frac{1}{2} [i_{sd}, \tilde{i}_f] L [i_{sd}, \tilde{i}_f]^{T} + \frac{1}{2} \tilde{i}_q^{2}
$$

whose time derivative along the trajectories of the $[i_{sd}, \tilde{i}_f, \tilde{i}_q]^{*}$ subsystem satisfies $[L = L^{T}]$

$$
\dot{V}_l \leq -k_d i_{sd}^{2} - k_f \tilde{i}_f^{2} - k_q \tilde{i}_q^{2} + \frac{e_{T}^{2}}{k}
$$

so that $(i_{sd}, \tilde{i}_f, \tilde{i}_q)$ and consequently $\tilde{\psi}_{wd}$ are guaranteed to tend exponentially to zero from any initial condition. Since the quadratic positive definite function

$$
V = \frac{1}{2} (\dot{\vartheta}^{2} + \dot{\omega}^{2}) + \frac{1}{2} \tilde{\psi}_{wq}^{2}
$$

with

$$
\gamma \geq 4 \frac{R_{q} L_{sd}^{2}}{L_{ew}} \sup_{t \geq 0} \left( \frac{g_{1}^{2}(t)}{2 k_{0}} + \frac{g_{2}^{2}(t)}{k_{\omega}} \right)
$$

satisfies the differential inequality of Lemma B.8 in [7], exponential convergence to zero of $(\vartheta, \omega, \tilde{\psi}_{wd})$ can be established.

Even though the previous theorem does not apply when the damping winding flux estimator is not properly initialized (owing to the non-exact knowledge of flux initial values), a local result is however guaranteed according to the following theorem.

**Theorem 2:** Let $\tilde{r}_q$ be the solution of the first order differential equation

$$
\dot{\tilde{r}}_q = \frac{-R_{q} L_{sd}}{L_{ew}} \tilde{r}_q + \frac{2 p R_{q} L_{sd}}{3 p L_{sd}} \left[ \psi^{*} + \frac{(L_{sd} - L_{md})}{R_{d}} \psi^{(1)} \right]^{-1} \left( \theta^{(2)} + \frac{T_{L}}{J} \right)
$$


The origin $\tilde{\theta}, \tilde{\omega}(t), \tilde{\psi}_{wd}(t), \psi_{wq}(t) - \tilde{\psi}_q(t), \tilde{i}_d(t), \tilde{i}_f(t), \tilde{i}_{dq}(t), e_{d}(t), e_{q}(t), e_{r}(t), \psi_{wd} - \tilde{\psi}_{wd}, \psi_{wq} - \tilde{\psi}_{wq}]^T = 0$ of the closed loop error system (Eqs. 3–5 and 7–10) is locally exponentially stable:

(i) for any time-varying smooth bounded reference signal $\theta^*(t)$ with bounded time derivatives $\theta(t)$, $1 \leq i \leq 3$;

(ii) for any time-varying smooth bounded reference signal $\psi^*(t)$ with bounded time derivatives $\psi^*(t)$, $1 \leq j \leq 2$, satisfying inequality (6);

(iii) for any positive control parameters $k_0, k_\omega, k_d, k_f, k_q, k$;

(iv) for any positive control parameters $k_1, k_2, k_3$ such that the polynomial associated to $[1, k_1, k_2, k_3]^T$ has all roots with negative real part.

Proof: The result follows on applying arguments similar to those used in the proof of Theorem 1 and by considering that the damping winding flux estimation errors $e_{wd} = \psi_{wd} - \hat{\psi}_{wd}$ and $e_{wq} = \psi_{wq} - \hat{\psi}_{wq}$ satisfy

$$
\dot{e}_{wd} = -\frac{R_{wd}}{L_{wd}} e_{wd},
$$
$$
\dot{e}_{wq} = -\frac{R_{wd}}{L_{wq}} e_{wq},
$$

(17)

while the terms $\omega \psi_{wd} - \hat{\omega} \hat{\psi}_{wd}$ and $\omega \psi_{wq} - \hat{\omega} \hat{\psi}_{wq}$ can be written as

$$
\omega \psi_{wd} - \hat{\omega} \hat{\psi}_{wd} = e_{wd} \hat{\omega} \psi_{wd} + \omega e_{wd},
$$
$$
\omega \psi_{wq} - \hat{\omega} \hat{\psi}_{wq} = e_{wq} \hat{\omega} \psi_{wq} + \omega e_{wq}.
$$

4. Rotor Speed Tracking

The basic assumption that the rotor position reference signal is bounded, which the result of the previous section relies on, fails in speed tracking applications such as rolling mill and gearless cement-mill drives, large compressors and ship propulsion drives. Assuming that the rotor speed is measured along with the rotor angle and the stator currents, an output feedback sixth order nonlinear adaptive rotor speed tracking control is designed in this section for the seventh-order nonlinear model (Eqs. 3–5) of a synchronous motor with damping windings, in which both load torque and motor inertia are constant and uncertain. It includes a closed loop observer for the unmeasured damping winding fluxes and two closed loop identifiers for the uncertain parameters (load torque and motor inertia). Asymptotic rotor speed tracking of arbitrary smooth bounded reference signals, exponential tracking for damping winding flux vector $d$-axis component of smooth bounded reference signals satisfying inequality (6) and stator current vector $d$-axis component regulation to zero (motor field orientation) are guaranteed by the proposed controller for any motor initial condition, for any uncertain constant load torque within known bounds and for any uncertain constant motor inertia within restricted bounds. When motor inertia is known, a simplified version of the controller guarantees the additional properties of local exponential rotor speed tracking and exponential load torque estimation. Compared to the control designed in the previous section, the availability of rotor speed measurements (which is reasonable in speed tracking applications) allows us to: (i) obtain a global result even when initial flux values are not known; (ii) remove the assumption of known motor inertia; (iii) remove the requirement of measuring the field current. Note that, while in synchronous motor drives with slip rings and brushes the measurement of field current is straightforward, in applications in which slip rings are excluded (for instance in potentially explosive atmospheres), brushless excitation is used and field current measurement is not possible (see [5]).

4.1. Problem Statement

Let us denote by $\omega^*(t)$ the arbitrary smooth bounded reference signal with bounded time derivatives $\omega^*(t)$, $1 \leq i \leq 2$, for the rotor speed $\omega(t)$, let $\psi^*(t)$, $\psi^*(t)$, $1 \leq j \leq 2$, be as in the previous section and assume that $\psi^*(t)$ and $\psi^*(t)$ satisfy inequality (6). Following the field oriented control strategy, our goal is to design a dynamic output feedback compensator [only $(\theta, \omega, i_d, i_q)$ measurements are available]

$$
\begin{bmatrix}
u_a \\ \nu_b \\ \nu_f \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_a \\ u_b \\ u_f \end{bmatrix}
$$

so that the asymptotic properties

$$
\lim_{t \to +\infty} [\omega(t) - \omega^*(t)] = 0
$$
$$
\lim_{t \to +\infty} [\psi_{wd}(t) - \psi^*(t)] = 0
$$
$$
\lim_{t \to +\infty} \left[ i_d(t) \right] = 0
$$

are guaranteed for any motor initial condition, for any uncertain constant load torque $T_L$ within known bounds $T_{Lm}$, $T_{LM}$ and for any uncertain constant motor inertia $J$ within restricted positive bounds $J_m, J_M$. 

4.2. Control Algorithm (II)

The proposed control algorithm consists of a current loop:

\[
\begin{align*}
    u_d &= -\phi_d - k_d i_d \\
    u_f &= -\phi_f + \frac{R_w d L_{md}^2}{L_{wd}^2} i_{sd} \\
    u_{sq} &= -\frac{1}{\beta_q} \left[ \phi_q + k_q (i_q - \tilde{i}_{sq}) \right] \\
    \begin{bmatrix}
        \phi_d \\
        \phi_f \\
    \end{bmatrix} &= \begin{bmatrix}
        \Theta_d \\
        \Theta_f \\
    \end{bmatrix} - L \begin{bmatrix}
        0 \\
        \frac{di_f}{dt} \\
    \end{bmatrix}
\end{align*}
\]

with the known functions

\[
\begin{align*}
    \Theta_d &= -\left( \frac{R_s L_{rd}^2 + R_d L_{md}^2}{L_{wd}^2} i_{sd} - \frac{R_w d L_{md}^2}{L_{wd}^2} i_f \right) \\
    &+ \left( \frac{L_{sq} L_{wa} - L_{mq}^2}{L_{wd}} \right) \rho \omega i_{sd} + \frac{R_w d L_{md}^2}{L_{wd}^2} \psi_{wd} \\
    &+ \frac{L_{mq}}{L_{wd}} p_{sd} \hat{\psi}_{wd} \\
    \Theta_f &= -\left( \frac{R_f L_{rw}^2 + R_d L_{md}^2}{L_{wd}^2} i_f - \frac{R_w d L_{md}^2}{L_{wd}^2} i_{sd} \right) \\
    &+ \frac{R_w d L_{md}}{L_{wd}^2} \hat{\psi}_{wd} \\
    \phi_q &= \beta_q \left[ -\frac{L_{wd} (R_s L_{rw}^2 + R_d L_{mq}^2)}{L_{wd}^2} i_{sq} \\
    &- \left( \frac{L_{sd} L_{rd} - L_{md}^2}{L_{wd}} \right) \rho \omega i_{sd} \\
    &- \frac{L_{md} (L_{rw} - L_{md})}{L_{wd}} \rho \omega i_f \\
    &+ \frac{R_w d L_{mq}}{L_{wd}^2} \hat{\psi}_{wd} - \frac{L_{mq}}{L_{wd}} p_{sd} \omega \hat{\psi}_{wd} \right] - \frac{di_{sq}}{dt}
\end{align*}
\]

depending on the reference signals \((\tilde{i}_f, \tilde{i}_{sq})\) for \((i_f, i_{sq})\)

\[
\begin{align*}
    \tilde{i}_f &= \frac{L_{wd}}{L_{md} R_{wd}} \left[ \frac{R_{sd}}{L_{wd}} \psi^* + \psi^{*(1)} \right] \\
    \tilde{i}_{sq} &= \frac{2 \tilde{J}_L}{3p} \left[ \psi^* + \frac{(L_{md} - L_{md})}{R_{wd}} \psi^{*(1)} \right]^{-1} \\
    &\cdot \left[ -k_{\omega} \text{sat}_s(\hat{\omega} - \omega^*) + \omega^* \right] \\
    &+ \frac{2 \tilde{J}_L}{3p} \left[ \psi^* + \frac{(L_{md} - L_{md})}{R_{wd}} \psi^{*(1)} \right]^{-1}
\end{align*}
\]

which are responsible for tracking of rotor speed and direct axis component of damping winding flux vector, respectively; the estimates \((\hat{\psi}_{wd}, \hat{\psi}_{wd})\) for the unmeasured variables \((\tilde{\psi}_{wd}, \tilde{\psi}_{wd})\) are provided by the third order observer which includes the auxiliary variable \(\hat{\omega}\)

\[
\begin{align*}
    \hat{\omega} &= -k_{\omega} (\hat{\omega} - \omega) - k_{\omega \text{sat}} (\hat{\omega} - \omega^*) + \omega^{*(1)} \\
    &- (\hat{\omega} - \omega) \left\{ \frac{k_{\omega}}{4} + \frac{k_{\omega \text{sat}}}{4} \left( -k_{\omega} \text{sat}_s(\hat{\omega} - \omega^*) \right) \right\} \\
    &+ \frac{9\beta_2}{16} (\beta_{\omega 2} - \beta_{\omega 1}) \hat{r}^2_{sq} \\
    &+ \beta_{\omega 1} \left( \hat{r}_{sq} + \hat{r}^2_{sq} \right) + \frac{L_{mq}}{L_{wd}^2} \left( \hat{i}_{sq}^2 + \psi^2 \right) \\
    &+ \frac{L_{mq}^2}{L_{wd}^2} \left( \hat{i}_{sq}^2 + \hat{r}^2_{sq} \right)
\end{align*}
\]

where the bounded reference signal \(r_q\) for the damping winding flux vector quadrature component \(\hat{\psi}_{wd}\) is designed as

\[
\hat{i}_q = -\frac{R_w d}{L_{wd}} \hat{\psi}_{wd} + \frac{R_w d L_{md}}{L_{wd}} (i_{sd} + \hat{i}_f) \\
+ \frac{1}{\lambda_q} \left[ \frac{R_w d L_{md}^2}{L_{wd}^2} i_{sd} - \beta_d L_{md} \rho \omega (i_{sq} - \tilde{i}_{sq}) \right]
\]

\[
\hat{\psi}_{wd} = -\frac{R_w d}{L_{wd}} \hat{\psi}_{wd} + \frac{R_w d L_{md}}{L_{wd}} i_{sd} + \hat{i}_f \\
+ \frac{1}{\lambda_q} \left[ \frac{R_w d L_{md}}{L_{wd}^2} \psi_{sq} + \frac{L_{mq}}{L_{wd}} \hat{r}_{sq} \psi_{wd} \right]
\]

\[
\hat{r}_q = -\frac{R_w d}{L_{wd}} r_q + \frac{R_w d L_{md}}{L_{wd}} \hat{r}_q \\
+ r_q(0) = 0;
\]

the load torque and motor inertia estimates \((\hat{T}_L, \hat{J})\) are provided by the adaptation laws

\[
\hat{T}_L = \text{Proj}_T \left( \mu^{-1}(\hat{\omega} - \omega) \hat{T}_L \right), \quad T_{LM} \leq \hat{T}_L(0) \leq T_{LM}
\]

\[
\hat{J} = \text{Proj}_J \left[ \lambda^{-1} \left( -k_{\omega \text{sat}_s}(\hat{\omega} - \omega^*) + \omega^{*(1)} \right) \right]
\]

\[
(\hat{\omega} - \omega) \hat{J}, J_m \leq \hat{J}(0) \leq J_M
\]

where \(\text{Proj}_{T[J]}[\xi, \hat{T}_L]\), and \(\text{Proj}_{T[J]}[\eta, \hat{J}]\) are the projection algorithms given in [13], defined in our case by

\[
\text{Proj}_{T[J]}[\xi, \hat{T}_L] = \begin{cases}
    \xi & \text{if } T_{LM} \leq \hat{T}_L \leq T_{LM} \\
    \xi & \text{if } \hat{T}_L < T_{LM} \text{ and } \xi \geq 0 \\
    \xi_T \xi & \text{if } \hat{T}_L > T_{LM} \text{ and } \xi < 0 \\
    \xi_T \xi & \text{if } \hat{T}_L > T_{LM} \text{ and } \xi > 0
\end{cases}
\]

\[
\xi_T = \begin{cases}
    1 - \frac{T_{LM}^2 - \hat{T}_L^2}{T_{LM}^2 - (T_{LM} - \varepsilon)^2} & 0 \leq T_{LM}(T_{LM} - \varepsilon), \quad 0 \leq T_{LM}(T_{LM} + \varepsilon)
\end{cases}
\]
\[ \text{Proj}_{\gamma} [\eta, \dot{J}] = \begin{cases} \eta & \text{if } J_m \leq \dot{J} \leq J_M \\ \eta & \text{if } \dot{J} < J_m \text{ and } \eta \geq 0 \\ \eta & \text{if } \dot{J} > J_M \text{ and } \eta \leq 0 \\ \eta_1 \eta & \text{if } \dot{J} < J_m \text{ and } \eta < 0 \\ \eta_2 \eta & \text{if } \dot{J} > J_M \text{ and } \eta > 0 \end{cases} \]

The control law (Eqs. 18–22) depends on: the available \((\omega, i_{sd}, i_{sq})\) measurements; the bounded reference signals \(\omega^*(t), \psi^*(t)\) and their bounded time derivatives \(\omega^{(i)}(t), \psi^{(i)}(t), 1 \leq i, j \leq 2\); the known motor parameters \(p, R_s, R_wd, R_wq, R_f, L_{sd}, L_{sq}, L_{wd}, L_{wq}, L_f, L_{md}, L_{md}^\prime\); the known bounds \(T_{LM}, T_{LM}^\prime, J_m, J_M\); the positive control parameters \(k_w, \kappa, \kappa_w, \kappa_u, \lambda_d, \lambda_q, k_d, k_q, \mu, \varepsilon, \lambda, \mu, r\); the saturation function \(\text{sat}_\varepsilon(x)\) (a \(C^\infty\) odd function whose derivative is always positive and has the finite limit \(\alpha\) as \(x\) goes to \(+\infty\)).

### 4.3. Stability Analysis

Define
\[
\begin{align*}
\gamma_0 &= 4 L_{wd}^2 R_f + 2 R_wd L_{md}^2 L_{wd} \\
\gamma_1 &= 4 \sqrt{L_{wd}^2 R_f (L_{wd}^2 R_f + R_wd L_{md}^2)}.
\end{align*}
\]

The main result of this section is stated in the following theorem.

**Theorem 3:** The closed loop system solution \(y(t)\) of Eqs. (3–5) and (18–22) is bounded and satisfies \([\delta_1, \delta_2]\) are suitable positive reals
\[
\lim_{t \to \infty} \|y_{11}(t)\| = 0 \\
\|y_{12}(t)\| \leq \delta_1 \|y_{12}(0)\|e^{-\delta_2 t}, \forall t \geq 0
\]

(i) for any initial condition \(y_1(0) \in \mathbb{R}^n\); (ii) for any time-varying smooth bounded reference signal \(\omega^*(t)\) with bounded time derivatives \(\omega^{(i)}(t), 1 \leq i, j \leq 2\); (iii) for any time-varying smooth bounded reference signal \(\psi^*(t)\) with bounded time derivatives \(\psi^{(i)}(t), 1 \leq i, j \leq 2\), satisfying inequality (6); (iv) for any positive control parameters \(k_w, \kappa, \kappa_w, \kappa_u, \lambda_d, \lambda_q, k_d, k_q, \mu, \varepsilon, \lambda, \mu, r\); the uncertain load torque and motor inertia; (v) a damping winding flux vector \(q\)-component reference signal generator (Eq. 21).

### 4.3. Stability Analysis

Define
\[
\begin{align*}
c_1 &= \frac{\gamma_0 - \gamma_1}{2 R_wd L_{md} L_{wd}} \\
c_2 &= \frac{\gamma_0 + \gamma_1}{2 R_wd L_{md}^2 L_{wd}}.
\end{align*}
\]

Let us denote by \(y(t) = [y_1(t), y_2(t), y_3(t)]^T\) the solution of the closed loop system (3–5) and (18–22) with \(y_1' = [y_1', y_2', y_3']^T, y_1_1 = [\omega - \omega', \varphi(t), \omega, \psi] \in F,v_2 = [\psi, \psi_{sd} - q_{sd}, \psi_{sq} - q_{sq}, \psi_{sd} - \psi_{sq}]_v, y_2 = T_L - \dot{T}_L, y_3 = J - \dot{J} \text{ and define } \phi(t) = [\dot{\omega}(t), y_1(t), y_2(t), y_3(t)]^T.\]

\[
\dot{\omega} = -k_w \text{sat}_\varepsilon(\omega + e_\omega) - \frac{\dot{T}_L}{J} + \frac{3 \beta}{2} \left[ (\beta_{\omega} - \beta_{\omega}) \dot{i}_{sd} \dot{i}_{sd} + \beta_{\omega} \dot{i}_{sd} \dot{i}_{sd} + \frac{L_{md} \dot{i}_{sd}}{L_{wd}} \dot{i}_{sd} \right] + \beta_{\omega} \dot{i}_{sd} \dot{i}_{sd} + \frac{L_{wd} \dot{i}_{wd}}{L_{wd}} \dot{i}_{wd} \dot{i}_{wd}
\]

\[
\beta_{\omega} \dot{i}_{sd} \dot{i}_{sd} + \beta_{\omega} \dot{i}_{sd} \dot{i}_{sd} + \beta_{\omega} \dot{i}_{sd} \dot{i}_{sd} = \frac{J}{K_w} \dot{w}_w(\omega) + \omega^{(1)}(t)
\]
whose time derivative along the trajectories of the closed loop subsystem (26) is given by \([L = L^T]\)

\[
\dot{V}_l = -\lambda_d R_{wd} e_{wd}^2 - \lambda_q R_{wq} e_{wq}^2
+ \frac{R_{wd} L_{md}}{L_{wd}} (\lambda_d + 1) \tilde{i}_f e_{wd}
- \frac{(R_f L_{wd} + R_{wd} L_{md})}{L_{wd}^2} \tilde{i}_f^2
- k_d \tilde{i}_d^2 - k_q \tilde{i}_q^2
\]

which is negative definite for any \(\lambda_d\) satisfying the condition (v); Eqs. (27) and (28) imply that \((e_{wd}, e_{wq}, i_{sd}, i_{sq}, i_{fd}, i_{fq})\) tend exponentially to zero from any initial condition. Subsystem (25) may be rewritten as

\[
\dot{x} = \begin{bmatrix}
-\frac{R_{wd}}{L_{wd}} & 0 & R_{wq} & 0 \\
0 & \frac{R_{wq}}{L_{wd}} & -\frac{R_{wd} L_{md}}{L_{wd}^2} & 0 \\
0 & 0 & 0 & \frac{R_{wd} L_{md}}{L_{wd}^2} (15)
\end{bmatrix} x_w
\]

with \(x = [\tilde{\psi}_{wd}, \tilde{\psi}_{wq}, \tilde{i}_d, \tilde{i}_q]^T\) and \(x_w = [\tilde{i}_{sd}, \tilde{i}_{sq}, \tilde{i}_{fd}, \tilde{i}_{fq}]^T\). Since \(x_w\) tends exponentially to zero for any initial condition, subsystem (25) complies with the hypotheses of Lemma III.1 in [8]: \(\tilde{\psi}_{wd}, \tilde{\psi}_{wq}\) tend globally exponentially to zero. Consider the quadratic positive definite function

\[
V_r = \frac{1}{2} e_{wd}^2
\]

By computing the time derivative of function \(V_r\) along the \(e_{wd}\)-trajectories and by completing the squares, we obtain

\[
\dot{V}_r = -k_{wc} e_{wd}^2 + \frac{1}{\lambda_d} \left[ \tilde{L}_L \tilde{i}_d + \tilde{L}_f \tilde{i}_f + 2 \tilde{i}_d^2 + \tilde{i}_f^2 \right]
+ 2 \tilde{i}_q^2 + \tilde{\psi}_{wd}^2 + \tilde{\psi}_{wq}^2
\]

Since \((\tilde{L}_L, \tilde{L}_f, \tilde{i}_d, \tilde{i}_f, \tilde{i}_q, \tilde{\psi}_{wd}, \tilde{\psi}_{wq})\) are bounded [recall that the adaptation laws for \(\tilde{T}_L\) and \(\tilde{J}\) are designed using projection algorithms], from (30) and (31) it follows that \(e_{wd}(t)\) is bounded for all \(t \geq 0\). Since \((i_{sq}, \psi_{wq})\) are bounded, and \((i_{sd}, r_d)\) are bounded, \((i_{sq}, \psi_{wq}(t))\) are bounded and, consequently, since \(e_{wd}\) is bounded, \(\tilde{\omega}(t)\) and \(\dot{e}_{wd}(t)\) are bounded for all \(t \geq 0\); therefore \(e_{wd}(t)\) is uniformly continuous for all \(t \geq 0\). Since \((\tilde{\omega}, \dot{\tilde{\omega}})\) are bounded, \(\dot{\tilde{\omega}}(t)\) is bounded for all \(t \geq 0\). Consider the quadratic positive definite function

\[
V_a = \frac{1}{2} e_{wd}^2 + \frac{1}{2J} \mu T_L^2 + \frac{1}{2J} \lambda \tilde{J}^2
\]
When the excitation current \( i_f \) is measured, \( i_f' \) may be replaced by \( i_f \) in the functions \( \phi_d \) and \( \phi_q \) while the control inputs \( u_f \) and the estimation law for \( \psi_{wd} \) may be modified according to (\( k_f \) is a positive control parameter)

\[
\begin{align*}
\dot{\psi}_{wd} &= -\frac{R_{wd}}{L_{wd}} \psi_{wd} + \frac{R_{wd}L_{md}}{L_{wd}} (i_{sd} + i_f) \\
&\quad + \frac{1}{\lambda_d} \left( \frac{R_{wd}L_{md}}{L_{wd}}^2 \right) \left( i_{sd} + (i_f' - i_f) \right) \\
&\quad - \beta_d L_{md} P_\omega (i_{dq} - i_{dq}')
\end{align*}
\]

so that no restrictions on the positive control parameter \( \lambda_d \) are to be imposed.

When motor inertia is known, a simplified version of the controller, which guarantees the additional properties of exponential load torque estimation and local exponential rotor speed tracking, can be derived according to the following theorem.

**Theorem 4:** The simplified version of the controller, obtained from Eqs. (18)–(22) by setting \( J \equiv J \) and by modifying the rotor speed observer according to

\[
\begin{align*}
\dot{\omega} &= -k_\omega (\bar{\omega} - \omega) - k_\omega \text{sat}_s (\bar{\omega} - \omega^*) + \omega^{(1)} \\
&\quad + \frac{3p}{2J} \left[ (\beta_{\omega 1} - \beta_{\omega 2}) i_{sq} i_{sd} + \beta_{\omega 1} i_f (i_{dq} - i_{dq}') \right] \\
&\quad + \frac{L_{md}}{L_{wd}} \psi^* (i_{dq} - i_{dq}')
\end{align*}
\]

guarantees \( [h(\cdot) \geq 0 \text{ and } (\cdot > 0) \text{ are suitable functions, } \delta_1, \delta_2 \text{ are positive real numbers}] \)

\[
\begin{align*}
\| \omega(t) - \omega^*(t) \| &\leq \delta_1 \| y_{21} (0) \| e^{-\delta_2 t}, \forall t \geq 0 \\
\| [\bar{\omega}(t) - \omega(t), y_{21}(t)] \| &\leq h(\| \varphi(0) \|) e^{-\delta_2 (0 \| \varphi(0) \|)}, \forall t \geq 0
\end{align*}
\]

(i) for any initial condition \( y_{11} (0) \in \mathbb{R}^9 \);

(ii) for any time-varying smooth bounded reference signal \( \omega^*(t) \) with bounded time derivatives \( \omega^{(1)}(t) \), \( 1 \leq i \leq 2 \);

(iii) for any time-varying smooth bounded reference signal \( \psi^*(t) \) with bounded time derivatives \( \psi^{(ij)}(t) \), \( 1 \leq j \leq 2 \), satisfying inequality (6);

(iv) for any positive control parameters \( k_\omega, k_{\omega r}, \kappa_a, \lambda_q, k_d, k_{\varphi}, \mu, \lambda \);

(v) for any positive control parameter \( \lambda_d \) such that

\[
\lambda_d \in (\max \{ 0, c_1 \}, c_2 )
\]

Furthermore, the origin of the resulting closed loop system is locally exponentially stable.

**Proof:** The result follows on applying arguments similar to those used in the proof of Theorem 3 and
invoking the lemma in the appendix for the \([\dot{\omega} - \omega, y_2]\)-subsystem.

5. Simulation Results

We tested by simulation the proposed controllers (7–10) and (18–22) with reference to the model (3–5) of a two pole pairs 20-KW synchronous machine with damping windings, whose parameters are [p. 100, [11]]: 

\[ p = 2, J = 0.2 \text{ N m s}^2, J = 0.18 \text{ N m s}^2 \text{ in the second simulation}, \]

\[ R_s = 0.1 \Omega, \quad R_f = \frac{0.032}{3} \Omega, \quad R_{md} = \frac{0.34}{3} \Omega, \]

\[ R_{mq} = \frac{0.34}{3} \Omega, \quad L_{md} = 0.00489 \text{ H}, \quad L_{mq} = 0.00279 \text{ H}, \quad L_{md} = 0.00439 \text{ H}, \quad L_{mq} = 0.00291 \text{ H}, \quad L_{md} = 0.00410 \text{ H}, \quad L_{mq} = 0.0022 \text{ H}. \]

The references for rotor speed, rotor angle and damping winding flux vector \(d\)-component were generated by using ramp functions which were filtered using linear filters while the necessary derivatives were obtained from the state space realizations of the filters.

5.1. Control Algorithm (I)

We tested the controller (7–10) by simulation with control parameters (the values are in SI units) \(k_d = 90, \quad k_\omega = 180, \quad k_d = k_q = 90, \quad k_f = 1, \quad k = 0.1, \quad k_1 = 24,

\[ k_2 = 171, \quad k_3 = 324. \]

All initial conditions of the motor and of the controller are set to zero excepting for \(\psi_{md}(0) = 0.0001 \text{ Wb}\), so that the damping winding flux estimator (10) is not properly initialized. Fig. 1 illustrates a low-speed application: (i) the position reference trajectory \(\theta^e(t)\), which starts from 0 rad at \(t = 0\) and grows up to the constant value 180 rad; (ii) the reference for damping winding flux vector \(d\)-component, which starts from 0.01 Wb at \(t = 0\) and grows up to the constant value 0.4 Wb; (iii) the load torque (106 Nm), which is applied at \(t = 0.12\) s. Fig. 2 shows the time histories of the rotor angle, damping winding flux vector \((d, q)\)-components and of the load torque estimate, while Fig. 3 shows the tracking errors of the controlled outputs. The \((a, b)\)-components of stator current and voltage vectors and the excitation winding current and voltage are reported in Figs. 4 and 5. The closed loop tracking performance and robustness with respect to model parameter uncertainties are illustrated by Fig. 3: the outputs to be controlled track the corresponding reference signals while negligible steady-state tracking errors appear when, for \(t \geq 10\) s, motor inertia is 20% smaller than its nominal value and the resistances in stator, damping and excitation windings are 20%,
Fig. 2. Rotor angle, damping winding flux vector \((d,q)\)-components and load torque estimate.

Fig. 3. Rotor angle, damping winding flux vector \(d\)-component and stator current vector \(d\)-component tracking errors.
Fig. 4. Stator current vector \((a, b)\)-components and excitation winding current.

Fig. 5. Stator voltage vector \((a, b)\)-components and excitation winding voltage.
Fig. 6. Rotor speed, damping winding flux vector $d$-component reference signals and applied load torque.

Fig. 7. Rotor speed, damping winding flux vector $(d,q)$-components and load torque estimate.
Fig. 8. Rotor speed, damping winding flux vector $d$-component and stator current vector $d$-component tracking errors.

Fig. 9. Stator current vector $(a, b)$-components and excitation winding current.
50% and 20% greater than their nominal values, respectively.

5.2. Control Algorithm (II)

We tested the controller (18–22) by simulation with control parameters (the values are in SI units) $k_\omega = 180$, $k_{\omega e} = 90$, $k_d = k_q = 60$, $\kappa_a = 0.1$, $\mu^{-1} = 15000$, $\lambda_d = 130$, $\lambda_q = 200$, $\lambda^{-1} = 0.012$, $\kappa = 30$, $T_{Lm} = -20$, $T_{LM} = 130$, $\epsilon = 1$, $J_m = 0.155$, $J_M = 0.23$, $\rho = 0.002$. All initial conditions of the motor and of the controller are set to zero excepting for $\psi_{ad}(0) = 0.0001$ Wb and $\tilde{J}(0) = 0.2$. The reference for damping winding flux vector $d$-component starts from 0.01 Wb at $t = 0$ and grows up to the constant value 0.4 Wb. The reference for rotor speed is zero until $t = 0.3$s and grows up to the constant value 189 rad/s. A constant load torque (102 Nm) is applied at $t = 0.63$s. The references for rotor speed and damping winding flux vector $d$-component along with the applied load torque are reported in Fig. 6. The time histories of rotor speed, damping winding flux vector $(d,q)$ components and load torque estimate are reported in Fig. 7, while the tracking and regulation errors are reported in Fig. 8. The $(a,b)$-components of stator current and voltage vectors and the excitation winding current and voltage are reported in Figs. 9 and 10. Fig. 8 shows good tracking and estimation performance in nominal conditions while small steady-state tracking and estimation errors appear when, for $t \geq 1.2$s, the resistances in stator, damping and excitation windings are 20%, 50% and 20% greater than their nominal values, respectively.

6. Conclusions

We have addressed the output feedback field-oriented control of synchronous motors with damping windings whose dynamics are given by the seventh-order nonlinear model (3–5). Two different nonlinear dynamic control algorithms have been presented in Sections 3 and 4 along with their closed loop stability proofs: (i) a speed-sensorless position tracking controller (7–10) feeding back only rotor angle, stator and field windings currents measurements, which is adaptive with respect to unknown constant load torque and is suitable in positioning applications; (ii) a speed tracking control law (18–22) based on measurements of mechanical variables (rotor angle and...
speed) and stator windings currents, which is adaptive with respect to both constant uncertain load torque (within known bounds) and motor inertia (within restricted bounds) and is suitable for applications in which rotor speed tracking is required and field current measurement is not available for feedback. Numerical simulation results illustrate the closed loop system performance and show the effectiveness of the two proposed control algorithms.

Acknowledgments

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References

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Appendix

Lemma: Consider the system

\[ \dot{x} = f_1(t, x, z, w) = A(t)x + B(t)z + C(t)w \]

\[ \dot{z} = f_2(t, x, w) = D(t)x + E(t)w \] (37)

where \( x(t) \in \mathbb{R}^n, z(t) \in \mathbb{R}^p, w(t) \in \mathbb{R}^m, f_1(t, x, z, w) \) and \( f_2(t, x, w) \) are continuous and \( B(t) \) is continuous and differentiable with respect to time for all \( t \geq t_0 \). Assume that, for all \( t \geq t_0 \),

1. \( A(t), B(t), C(t), D(t), E(t) \) are bounded, with \( \|A(t)\| \leq A_M, \|B(t)\| \leq B_M, \|C(t)\| \leq C_M, \|D(t)\| \leq D_M, \|E(t)\| \leq E_M \);
2. there exist two positive reals \( T, k \in \mathbb{R}^+ \), such that the persistency of excitation condition

\[ \int_{t_0}^{t_0+T} B^T(\tau)B(\tau) d\tau \geq kI \] (38)

is satisfied;
3. there exist a smooth proper function \( V(x, z, t) \) and suitable positive reals \( a_1, a_2, a_3, a_4, a_5 \in \mathbb{R}^+ \), such that

\[ a_1(\|x\|^2 + \|z\|^2) \leq V(x, z, t) \leq a_2(\|x\|^2 + \|z\|^2) \]

\[ \dot{V} \leq -a_3\|x\|^2 + a_4\|w\|^2 + a_5\|w\|\|z\| \] (39)

4. \( w(t) \) is a bounded exponentially decaying signal;

then \( x(t) \) and \( z(t) \) tend exponentially to zero from any initial condition.
Proof: As in [10], consider the class of radially unbounded functions

\[ W(x, z, w, t) = V(x, z, w, t) + p_c \|Q z - B^T x\|^2 \]

where \( Q(t) \) is generated by the filter \( dQ(t)/dt = -Q(t) + B^T(t)B(t), \) with \( Q(t_0) = e^{-T_0}kI. \)

By virtue of assumption 2, \( Q(t + T) \geq e^{-T} \int_t^{t+T} B^T(\tau)B(\tau)d\tau \geq e^{-T}kI > 0. \) Since \( \|B(t)\| \leq B_M, \) it is straightforward to deduce that

\[ B^2_M I \geq Q(t) > ke^{-2T}I. \]

By computing the time derivative of \( W(x, z, w, t), \) by virtue of assumption 3, we have

\[
\dot{W} \leq -a_3\|x\|^2 + a_4\|w\|^2 + a_5\|w\|\|z\| + 2p_c(Q z - B^T x)^T [QE - B^T C]w + 2p_c(Q z - B^T x)^T [QD - B^T A - B^T - \dot{B}]x.
\]

so that by adding and subtracting \( 2p (Q z - B^T x)^T B^T x \) in the right hand side of the previous inequality, we obtain

\[
\dot{W} \leq -a_3\|x\|^2 + a_4\|w\|^2 + a_5\|w\|\|z\| - 2p_c\|Q z - B^T x\|^2 + 2p_c(Q z - B^T x)^T [QE - B^T C]w + 2p_c(Q z - B^T x)^T [QD - B^T A - B^T - \dot{B}]x.
\]

By hypothesis 1, there exist two positive reals \( a_6, a_7 \in \mathbb{R}^+, \) such that \( a_6 \geq \|QD - B^T A - B^T - \dot{B}\|^2, \) and \( a_7 \geq \|QE - B^T C\|^2, \) so that, by completing the squares, we obtain

\[
\dot{W} \leq -\left[a_3 - p_c(4a_6 + B^2_M)\right]\|x\|^2 - \frac{p_c}{4}k^2e^{-4T}\|z\|^2 - \frac{p_c}{2}\|Q z - B^T x\|^2 + \left[a_4 + \frac{a_5^2e^{4T}}{p_c^2k^2} + 4p_c a_7\right]\|w\|^2.
\]

For any \( p_c < a_3/(4a_6 + B^2_M), \) Lemma B.5 in Ref. [7] implies the thesis.