On the Throughput Potential of Two-Dimensional Wireless Multi-hop Networks Using Directional Antennas

Invited Paper

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Abstract—The goal of this paper is to establish the rate region of a wireless multi-hop, one-sender, two-receiver transmission chain. We examine and evaluate the advantages of utilizing directional antennas in this context. In particular, we consider a two-dimensional communication network with a single source node $S$, two destination nodes $D_1$, $D_2$, and $2 \times (N-1)$ intermediate nodes placed equidistantly between them. Multi-hop transmission is an extension of single-hop transmission that can take advantage of the reduced attenuation between closely spaced relay nodes, as well as the opportunity of spatial reuse. In two-dimensional multi-hop half-duplex transmission, each node utilizes capacity-achieving point-to-point codes to forward the most recently decoded message to its nearest neighbor in the direction of $D_1$ or $D_2$. The transmission is performed over two parallel chains and it is a mixture of two techniques, “broadcast” communication (one transmitter to two relay nodes) and interference channel. First we describe our network and the assumptions made in the scope of this study. We further evaluate the end-to-end rate region of this topology under our transmission constraints and describe the effects that take place after replacing omni-directional antennas with perfectly directional ones. Our results indicate a 5-fold end-to-end sum rate improvement over a 10-hop network with the use of 30 degrees directional beams, as compared to omni-directional transmission.

Index Terms—Directional antennas, wireless networks, multi-hop networks, reuse factor, power, superposition coding, interference channel, throughput, rate region.

I. INTRODUCTION

Our starting point for this paper is our recently submitted work [1]. We investigated there the improvement in the throughput of a wireless linear multi-hop network with a single source and destination node using directional antennas instead of omni-directional ones. The main advantage of omni-directional antennas is that they are simple in use and propagate signals without any spatial preference. They are mainly used when coverage is required in all directions. Their disadvantage is, though, that they also require a lot of power to operate. This may result in a waste of energy, as the signal reaches areas which may not need to be covered.

For a two-dimensional network, with one source node, multiple relays are organized in a $2 \times N$ grid, with final aim two destination nodes (see Fig. 1). For the first-hop single-source, two-destinations network, we use the superposition coding method to establish the throughput region of the broadcast channel for two destinations ($F_{1,1}, F_{2,1}$). We cluster the remaining nodes as $2 \times 2$ networks and derive the throughput region for common information multiple-access channel (MAC). Finally, to get the end-to-end rate region, we combine these separate rate regions by taking their cut.

The results are compared with the case of directional antennas. Using directional antennas we can benefit in multiple ways. First, we have a better utilization of the available power and thus an increase in the achievable rate. (Of course this throughput gain depends on the beamwidth of the selected antennas.) Additionally, these antennas can be aimed at a certain direction. This helps to target specific nodes in a known topology and avoid causing unintended interference.

Finally, the elimination of possible interference in some neighboring nodes can also help exploit better the available bandwidth and permit denser parallel transmissions. In this paper, we will attempt to quantify the impact of these effects on the end-to-end rate regions and show the improvement obtained with directional antennas.

II. SYSTEM MODEL: SIMO MULTI-HOP NETWORK

The communication system under consideration is illustrated in Fig. 1. It consists of one source node $S$ (potentially a base station or access point), which transmits the signal over two separate parallel chains, aiming at two destinations (and potentially users). Each branch consists of $N-1$ intermediate relay node pairs, placed equidistantly on the line from the first relay node to the destination. This case is a rather straightforward extension of the SISO case and our previous work [1]. Our purpose is to derive the end-to-end throughput...
region of the entire network using both omni-directional and directional antennas and to compare the two cases.

The nodes share a band of radio frequencies allowing for a signaling rate of W complex-valued symbols per second. The objective of the system is the reliable delivery of bits generated at the source node S at a bandwidth-normalized rate (just called rate) of R bits per second per Hertz to the destination nodes using coded transmission and consuming the available for each node transmit power P. We assume perfect directional antennas with high gain and directivity (eliminating the back- and sidelobes).

The nodes comprising the system operate in half-duplex mode, i.e. they are incapable of simultaneous transmission and reception. Additionally, we assume that the source only transmits (does not receive) and that the destination pair only receives (does not transmit). We also require that the multi-hop transmission is based entirely on point-to-point coding for the Additive White Gaussian Noise (AWGN) channel. This implies that each node fully decodes the original message based on the signal received from the preceding code, re-encodes it, and forwards it to the following node ("decode and forward"). All interference from nodes transmitting simultaneously with an active node is regarded as additional white Gaussian noise. All receiving nodes are assumed to have the same additive Gaussian thermal noise of PSD $N_0/2$ and the channels between the different nodes are modeled as i.i.d. circularly symmetric complex Gaussian random values.

Finally we allow spatial reuse: To facilitate parallel transmission of several packets through the network, the available bandwidth is reused between transmitters, with a minimum separation of K hops between simultaneously transmitting nodes ($2 \leq K \leq N$). When decoding the message, nodes $F_{ij}$ and $D_{ij}$ regard all signals not originating from the preceding node as Gaussian interference. The achievable end-to-end rate is the minimum of the rates achieved at each hop. In our analysis, we do not consider fading and thus each channel is time-invariant with a constant link gain representing the signal path loss. This problem can be divided into two different and independent sub-problems.

### A. Broadcast Channel

The first stage consists of the communication between the source node and its 1-hop neighbors, corresponding to a classic broadcast channel. This “downlink” communication features a single transmitter communicating information simultaneously to several receivers. We consider the case, in which the transmitter is sending independent information to each “user” (in our case first-hop node). This problem has been studied in recent literature [3]–[7]. In current broadcast systems, the separation of user data is achieved by using orthogonal schemes in which the users are separated in time and / or frequency. A better transmission scheme for the broadcast channel is to superimpose the user signals and use interference cancellation at each receiver. We will follow this method - superposition coding - as it was proposed by D. Tse in [3].

In superposition coding the transmitted signal is obtained by superposition of the independent user signals.

We have assumed that the channel coefficients $h_1$ are independent and identically-distributed complex random variables (i.i.d) with zero mean and unit standard deviation. Especially when $h_1 << h_2$ (or $h_1 >> h_2$) superposition coding helps a lot. The transmit signal is the linear superposition of the signals of the two users. Thus, each instant $k$, the transmitted signal is given by the sum of the two users signals ($x(k) = x_1(k) + x_2(k)$) and has an average power constraint of $P$ Joules / sec, whereas the equivalent received signals by each user are $z_1(k)$ and $z_2(k)$. The transmitter encodes the information for each user using an i.i.d. Gaussian code spread over the entire bandwidth and powers $P_1$, $P_2$, respectively, with $P_1 + P_2 = P$. Each user decodes its data separately. The main idea is that user 1 (the one with the weak channel) treats the signal of user 2 as noise and decodes its data from $\{z_1\}$. Finally user 2, performs Successive Interference Cancellation (SIC); it decodes the data of user 1 (and hence the transmit signal corresponding to user 1’s data) and then proceeds to subtract the transmit signal of user 1 from $\{z_2\}$ and decodes the data. The rates are bounded from the following equations:

$$R_1 = \log \left(1 + \frac{P_1 |h_1|^2}{P_2 |h_1|^2 + N_0} \right) \text{ bits/s/Hz} \quad (1)$$

$$R_2 = \log \left(1 + \frac{P_2 |h_2|^2}{N_0} \right) \text{ bits/s/Hz} \quad (2)$$

The achievable rate pair can then be attained with each possible power split of $P = P_1 + P_2$.

### B. Interference Channel

In the second stage, we have the widely-studied interference channel (IC) [9]–[15]. In this paper we consider only the two-user IC (see Fig. 2). An interference channel models the situation where $M$ unrelated senders try to communicate their separate information to $M$ different receivers via a common channel. There is no cooperation between any of the transmitters or receivers. Hence, transmission of information from each sender to its corresponding receiver interferes with the communication with the other senders and their receivers.

To facilitate the study of this sub-problem we will consider the interference channel with common information under
strong interference. The capacity region is known when the interference is “strong” [16]. In this regime, the received interfering signal component carrying the unwanted message is strong enough so that a receiver can also decode the unwanted message. The interference channel then behaves as two multi-access channels, one to each receiver. The rate region \( C_a \) is the set of all pairs \((R_1, R_2)\) such that simultaneously user 1 and user 2 can reliably communicate at rates \( R_1 \) and \( R_2 \), respectively. It is given from the following expression:

\[
C_a = \bigcup \left \{ (R_1, R_2) : R_1 \geq 0, R_2 \geq 0, R_1 \leq \min \left \{ C \frac{P|h_1|^2}{N_0}, C \frac{P|h_{12}|^2}{N_0} \right \}, R_2 \leq \min \left \{ C \frac{P|h_2|^2}{N_0}, C \frac{P|h_{21}|^2}{N_0} \right \}, R_1 + R_2 \leq \min \left \{ C \frac{P|h_1|^2}{N_0} + P|h_{21}|^2}{N_0} \right \}, C \right \}
\]

where \( C(x) = \log(1+x) \) and \( h_1, h_2, h_{12}, h_{21} \) are the channel coefficients, as shown in Fig. 2.

III. OMNI-DIRECTIONAL ANTENNAS VS. DIRECTIONAL ANTENNAS

We will first derive results for the end-to-end throughput of the network using omni-directional antennas. Our main goal is to show the improvement after introducing directional antennas. In the case of omni-directional antennas, the equations for the first hop (1-2) are the same. For the remaining hops the applicable equation is (3), giving the omni-directial case the full benefit of strong interference MAC rate attainment.

As stated earlier, we are allowed to reuse the existing bandwidth. How close the concurrently transmitting nodes can be, determines the value of the optimal reuse factor \( K \), where \( K \) is the separation of simultaneously transmitting nodes. As it was proved in Sikora [8], the optimal value of \( K \), for the one-dimensional chain which achieves the same rate consuming the least possible energy is \( K = 3 \). For our two-dimensional topology, this result still applies intuitively, so we use again \( K = 3 \). We should also keep in mind that the induced cross-channel interference does not only hurt the performance of our network, but also forces us to use more complex techniques and having better knowledge of the channel in advance, which in general costs in bandwidth and time. Thus, the directional solution is not only substantially better in terms of throughput, but also in terms of feasibility.

The rate region in each \( 2 \times 2 \) pair is hence given by:

\[
C_{omni} = \frac{1}{K} \times C_a
\]
the SISO case regarding the optimality of can be considered as independent, our earlier proof in [1] for the omni SISO case was shown to be strictly optimal in [2] for the omni SISO case. This assumes that the two chains are adequately separated and the beams are narrow enough, so that we have no leakages between the two chains; otherwise interference would hurt our system.

The equivalent equations for the directional transmission are given below:

\[
C_{\text{dir}} = \frac{1}{K} \times \bigcup \left\{(R_1, R_2) : R_1 \geq 0, R_2 \geq 0, \right\}
\]

\[
R_1 \leq \min \left\{ C \left( \frac{2P|h_1|^2 g}{3N_0} \right) \right\},
\]

\[
R_2 \leq \min \left\{ C \left( \frac{2P|h_2|^2 g}{3N_0} \right) \right\}
\]

(5)

where \( g \) is the antenna gain in directional transmission. For our experiments and because the beamwidths were of 30\(^\circ\), the antenna gain is equal to 12.

IV. SIMULATION RESULTS

In this section we evaluate the achievable end-to-end rates of the different cases described in Section III, by running numerical simulations. We consider a linear two-dimensional network topology as in Fig. 1, where the channel has unit bandwidth, and \( \text{SNR} = P/N_0 = 10 \text{ dB} \). We consider the case when the relay channels are, as mentioned earlier, i.i.d. circularly symmetric complex Gaussian random variables, of unit variance. There is one source and 10 pairs of nodes (the last one is actually the destinations’ pair). The throughput bounds are shown for the two scenarios after one realization. In Fig. 4 and 5 the achievable rate regions using omni-directional antennas and directional antennas, respectively, are illustrated for a typical channel realization. Each node transmits with its maximum available power \( P \) and the reuse factor is \( K = 3 \) and \( K = 2 \), respectively.

As is shown in Fig. 4, ten separate throughput regions are derived. The end-to-end rate is the minimum (cut) of all the distinctive throughput regions of the intermediate channels and incorporating also the reuse factor effect, as per equation (4), it is scaled by \( 1/K \). Using the same channel characteristics, we derive the equivalent rate regions in the case of directional
antennas with beamwidths of 30° and with reuse factor $K = 2$. These result in higher rates, as shown in Fig. 5.

The improvement in sum rate capacity between the two cases is depicted in Fig. 6 for the given channel realization. We can see that with directional antennas we can achieve almost 6 times higher sum rate. In this figure we show clearly only the overall end-to-end rate regions, which are the inner bound of all distinctive rate regions of each architecture. We observe that with directional antennas we can achieve much higher throughput, which posits the advantages of having transmission beams.

We further ran more simulations over i.i.d. channel realizations. For 20 realizations we calculated the sum rate throughput in each case and derived their ratio in each repetition. The distribution of this ratio can be found in Fig. 7, which depicts its cdf (cumulative distribution function). We can observe that the sum rate gain due to directionality is always higher than 3.5, with mean value 5.2 and median 4.8. We have run simulations for 1000 realizations and the cdf curve was not substantially changed.

In our analysis we have assumed that the considered network consists of nodes that receive omni-directionally but transmit directionally. We could further have beams for both transmission and reception. In this case, we would achieve higher efficiency, as the gains of each antenna would be multiplied. That means that we could further decrease the transmitted power, saving our resources, without affecting the throughput of the network.

V. CONCLUSIONS

In this paper, we extended our previous work on the throughput gain of SISO relay networks due to the directivity of relay nodes, to the case of relay networks with multiple destinations. Focusing on the case of two destination nodes, we showed that the end-to-end sum rate of a 10-hop network can admit an average 5-fold gain using 30-degree beams, while the total radiated power in the network is kept constant. In future work, we plan to generalize these results to richer network topologies and channel models.

REFERENCES