Modeling and Verification for Cross-Department Collaborative Business Processes Using Extended Petri Nets

Qing Tian Zeng, FaMing Lu, Cong Liu, Hua Duan, and ChangHong Zhou

Abstract—Recently, cross-department business processes are becoming more and more complex. Different kinds of coordination patterns exist among different departments, which make modeling and analysis work more difficult. To our best knowledge, there is no formal method to give systematic modeling and verification for the cross-department processes when considering different kinds of coordination patterns among different departments. This paper proposes such a method based on Petri nets. The WF-net model extended with resource and message factors, RM_WF_Net for short, is first introduced. Then, the formal model of tasks is proposed and its coordination relations are given. Next, RM_WF_Net modeling for intradepartment processes is investigated and cross-department coordination patterns, including message interaction pattern, resource interaction pattern, task collaboration pattern, procedure abstract, service outsourcing pattern, and process activation pattern, are formally defined. The soundness of the RM_WF_Net is verified based on the reachability graph. A running case of the cross-department medical diagnosis business process is given to validate our proposed method.

Index Terms—Coordination patterns, correctness verification, cross-department business processes, Petri nets, process modeling, resource and message.

I. INTRODUCTION

WITH the ever-accelerating development of society informatization, today’s information environment is becoming increasingly complex. For example, a typical cross-department medical business process is usually dispersed in different medical information systems and involves different departments, which will exert negative effects on its process monitoring, reconstructing, and optimization. Lack of effective workflow management in the medical domain will lead to low work efficiency and business process flexibility. By applying the workflow management techniques to a medical environment, medical operation efficiency can be improved and corresponding medical costs will be reduced remarkably. Considering diversity of coordination patterns among different departments, it is of great significance to perform modeling, analysis, verification, and optimization. However, the traditional workflow modeling and analysis techniques [1], [2], [5], [6], [11]–[13], [16], [21], [23], [25] cannot be applied directly due to its cross-department features.

As a tool to model and analyze discrete event systems, Petri nets [7], [14], [15], [20], [22], [28] have shown their great power in dealing with concurrences and conflicts. Because of this advantage, they have been widely used to model, analyze, control, and verify workflows [1], [4]–[6], [8], [11]–[13], [16], [21], [23], [25]. In this paper, the modeling and verification methods for cross-department business processes are investigated on the basis of Petri nets. A cross-department medical diagnosis business process is studied as an example scenario to validate our proposed approaches. Some of the excellent existing research results in the cross-organization workflow field are properly referred. The main contributions of our work contain the following.

1) RM_WF_Net, a WF-net extended with resource and message factors, is introduced to model the cross-department business processes.

2) A formal model of tasks is proposed and its coordination relations are: message transmission relation, message broadcasting relation, resource sharing relation, task collaboration relation, and task abstract relation.

3) Cross-department coordination patterns, including message interaction pattern, resource interaction pattern, task collaboration pattern, procedure abstract, service outsourcing pattern, and process activation pattern, are formally defined. Moreover, their corresponding RM_WF_Net models are obtained.

4) Soundness of the RM_WF_Net is defined and verified on the basis of the reachability graph.

The remainder of this paper is organized as follows. Section II discusses the related work. Section III introduces a scenario of cross-department medical diagnosis business process. In Section IV, modeling approaches for intradepartmental business

processes are addressed. Section V discusses the coordination patterns and their corresponding RM_WF_Net models. Section VI investigates the integrated modeling approaches of the cross-department business process. In Section VII, correctness verification of the obtained RM_WF_Net is performed. Finally, Section VIII draws concluding remarks.

II. RELATED WORK

In this section, we mainly summarize the work related to modeling and analysis of cross-organization business processes and medical business processes.

A. Modeling and Analysis of Cross-Organization Business Processes

van der Aalst [3] considered that workflows distributed over a number of organizations. Two important questions were well addressed in his work: 1) the minimal requirements of interorganizational workflow and 2) how to decide whether an interorganizational workflow, modeled with Petri nets, is consistent with an interaction structure specified through a message sequence. In [18], Schulz and Orłowska focused on three aspects to support the execution of cross-organizational workflows that have been modeled with a process view approach: 1) communication between the entities; 2) their impact on an extended workflow engine; and 3) the design of cross-organizational workflow architecture. A Petri-net-based state transition approach that binds states of private workflow tasks to their adjacent workflow view-task was introduced. The concepts are demonstrated by a scenario, run by two extended workflow management systems. Jiang et al. [9] described a timed colored Petri net and process-view combined approach to construct cross-organizational workflows, and a models framework was proposed to realize the interoperability of cross-organizational workflows. Finally, a case study on the collaborative development of a motorcycle was presented to verify the validity of our approach.

The above-mentioned literature on modeling and analysis of cross-organization workflow only focuses on message interaction among different organizations. However, other interaction patterns, such as the resource sharing, task collaboration, procedure abstract, service outsourcing, etc., are of vital importance to characterize modern business process in a more accurate and detailed manner.

B. Modeling and Analysis of Medical Business Processes

An integrated method of formal specification for healthcare information systems workflow was proposed in [24] on the basis of Petri net theory and workflow technology. This method provides a theory base for analyzing, verifying and optimizing the healthcare information systems workflow. To effectively fulfill the integration of medical information and to solve the problem of interoperability among medical information systems, a workflow model of integrated healthcare process based on Petri Net was established in [27]. By using the platform independent Petri net editor (PIPE2), the reachability graph of the Petri net model is generated, and the correctness verification approaches is given. In [17], Liu et al. introduced a methodology for the application of process mining techniques that leads to identification of regular behavior, process variants, and exceptional medical cases. The methodology is implemented in a tool that integrates the main stages of process analysis. An approach for mining clinical care pathways correlated with patient outcomes that involve a combination of clustering, process mining and frequent pattern mining was presented in [19]. This approach was implemented as a set of interactive tools in the business process insight platform, a collaborative software as a service platform, that provides an event-driven process-aware analytics toolset. Experimental results showed that the tools can provide new insights to facilitate the improvement of existing clinical care pathways.

Most of the existing work about medical business process modeling only involves single department, and therefore, lacks coordination and collaboration among different medical departments. In addition, most of their analysis work is based on software simulation which lacks theory basis. Therefore, integrated modeling methodology for cross-department medical business processes is badly needed.

C. Summary of Related Work

Recently, research into medical business processes [10], [17], [24], [27] has drawn much public attention and enjoyed an accelerated flush. However, more effort is required to address their formal modeling and verification issues when considering different kinds of coordination patterns among different departments. In this paper, we pay much attention to the modeling approaches for cross-department business processes with coordination patterns. Correctness verification perspective is also involved. As the first complete framework with the regard to the analysis of coordination patterns in cross-department business process management, we are sure that our work can improve the state-of-art greatly.

III. RUNNING EXAMPLE OF CROSS-DEPARTMENT MEDICAL DIAGNOSIS BUSINESS PROCESSES

Recently, cross-department medical diagnosis has been needed to meet the requirement of medical diagnosis information integration. A typical scenario of cross-department medical diagnosis business process usually involves the following departments: the receptionist department, surgical department, X-ray department, charge office, cardiovascular department, and pharmacy. This scenario includes the following steps:

1) When a patient arrives, the outpatient medical staff performs preexamination triage that is composed of three specific tasks, including register, preexamination and triage. After finishing these tasks, the medical history is generated and sent to the surgical medical staff.

2) The surgical medical staff takes admissions and presents the reservation application, and then generates the reservation form.

3) X-ray department receives the reservation form and performs reservation. Next, acceptance notice is sent to the surgeon.
4) The surgeon makes imaging plan according to the notice receiving time, and sends a photo form to the X-ray department.

5) Medical staff of the X-ray department registers the imaging application information and generates the cost bill that is send to the charge office.

6) The charge office informs the patient to pay for the X-ray, and charges the expenses.

7) After receiving the expenses, charge office instructs the X-ray department to start imaging.

8) X-ray department performs machine operation, image processing, backup, and writes imaging reports which is send to the surgeon.

9) The surgeon diagnoses patients according to the imaging reports. Then, the surgeon applies consultation by sending consultation form to the cardiovascular internists.

10) The internists receive the form and arrange consultation.

11) Surgeons and internists consult to make a prescription together.

12) The internists makes the consultation summary.

13) Medical staff in the pharmacy accounts the cost according to the prescription and generates its cost bill.

14) Charge office informs the patient to pay for the medicine bill, and sends medicine taking message to the pharmacy after payment.

15) Medical staff in the pharmacy offers the medicine.

According to the above-mentioned process descriptions, we can conclude several features of cross-department medical business processes as follows.

1) The whole medical diagnosis business process involves six departments, and each department has its respective business process and task set. For example, the business of receptionist department is composed of three sequence tasks which are register, preexamination, and triage. Surgical department contains tasks, such as admissions, reservation application, imaging planning, diagnosis, consultation application, consultation, and give prescription.

2) Each medical task contains several components. For example, to perform the task admissions in the surgical department, medical history and a diagnosis room are needed. Generally speaking, a medical task is usually composed of task content, required message set, sent message set, required resource set, released resource set, and sub-task set. Taking the cross-department medical diagnosis business process as an example, tasks and their corresponding elements are illustrated in Table I.

IV. MODELING APPROACHES FOR INTRADEPARTMENT BUSINESS PROCESSES

To analyze the coordination patterns among different departments, formal model of task is needed. In this section, we...
first propose the formal specification of tasks. And then, RM_WF_Net, a kind of WF-net extended with resource and message factors, is introduced. Next, intradepartment business process modeling with RM_WF_Net is fully addressed.

A. Formal Specifications of Tasks

In traditional business process modeling, tasks are usually composed of workflow task name, also referred to as task content, executors (or organizations), required resources, etc. Our work aims to analyze the coordination patterns for cross-department process, therefore, task elements, such as task content, required message set, sent message set, required resource set, released resource set, and sub-task set, are fully considered. Formal specification of a task is first introduced in the following.

Definition 1: A task is a seven-tuple Task = <Name, Organization, MessagesReq, MessagesSent, ResourceReq, ResourceRel, Sub-tasks>, where: 1) Name is the name of a task, representing the content of a task; 2) Organization represents the executing organization of a task; 3) MessagesReq is the message set that is required to execute a task; 4) MessagesSent is the message set that is sent when a task finishes; 5) ResourceReq is the resource set that is required to execute a task; 6) ResourceRel is the resource set that is released when a task finishes; and 7) Sub-tasks is the task set of its corresponding sub-process.

Take the medical scenario in Table I as an example. Task admissions is formalized as $t_2 = <$admissions, surgical department, {medical history}, $\emptyset$, {diagnosis room}, {diagnosis room}, $\emptyset$>, and task preexamination triage is denoted as $t_1 = <$preexamination triage, receptionist department, $\emptyset$, {medical history}, $\emptyset$, $\emptyset$, $\{t_{1.1}, t_{1.2}, t_{1.3}\}$>, where sub-tasks are listed as: $t_{1.1} = <$register, receptionist department, $\emptyset$, $\emptyset$, $\emptyset$, $\emptyset$>, $t_{1.2} = <$preexamination, receptionist department, $\emptyset$, $\emptyset$, $\emptyset$, $\emptyset$, $\emptyset$>, $t_{1.3} = <$triage, receptionist department, $\emptyset$, $\emptyset$, $\emptyset$, $\emptyset$>.

B. RM_WF_Net

Modeling approaches for a single department business process have been studied for decades, and some excellent models such as, WF-net, XPDL, BPEL, BPMN, etc., have been widely used. Whatever models are used, the process is totally determined by tasks and their dependency relations. Our work is based on WF-net, which is a subclass of Petri nets, and it is in a dominating position in this area. We assume that readers are familiar with the basic concepts of Petri nets [7], [14], [15], [20], [22], [28], [30], [31] and WF-net [4]. Some of the essential terminologies and notations are listed as follows.

Definition 2 [31]: A Petri net is a 4-tuple $\Sigma = (P; T; F, M_0)$, where: 1) $P = \{p_1, p_2, \ldots, p_n\}$ is a finite set of places; 2) $T = \{t_1, t_2, \ldots, t_k\}$ is a finite set of transitions; 3) $F \subseteq (P \times T)$ and $(T \times P)$ is a finite set of arcs (flow relation); 4) $M_0: P \rightarrow \{0, 1, 2, 3, \ldots\}$ is the initial marking; and 5) $P \cap T = \emptyset$ and $P \cup T \neq \emptyset$.

For all $x \in P \cup T$, the set $x = \{y \in P \cup T \land (y, x) \in F\}$ is the preset of $x$, and $x' = \{y \in P \cup T \land (x, y) \in F\}$ is the postset of $x$. An initial marking is denoted by $M_0$. $p$ is marked by $M$ iff $M(p) > 0$. A transition $t \in T$ is enabled under $M$, if and only if $\forall p \in t: M(p) > 0$, denoted as $M[t>]$. If $M[t>]$ holds, $t$ may fire, resulting in a new marking $M'$, denoted as $M[t>]$, such that $M'(p) = M(p) - 1$ if $\forall p \in t \land \exists M'(p) = M(p) + 1$ if $\forall p \in t \land \exists M'(p) = M(p)$.

Definition 3 [35]: A Weighted Petri net is a 5-tuple $\Sigma = (P; T; F, M_0, W)$, where: 1) $\Sigma = (P; T; F, M_0)$ is a Petri net and (2) $W: F \rightarrow \{1, 2, 3, \ldots\}$ is a weight function.

A Petri net which models a workflow process definition is called the workflow net (WF-net).

Definition 4: A Petri net $\Sigma = (P; T; F, M_0)$ is a WF-net if: 1) there is one source place $i \in P$ such that $i = \emptyset$; 2) there is one sink place $o \in P$ such that $o = \emptyset$; 3) $\forall x \in P \cup T$ is on a path from $i$ to $o$; and 4) $\forall p \in P$, $M_0(p) = 1$ if $p = i$, and otherwise $M_0(p) = 0$.

In a WF-net, the transition set $T$ is used to represent the tasks, and source place and sink place represent the start and end of the process, respectively. To model the intradepartment process, we propose the RM_WF_Net, which is a WF-net extended with resource and message information.

Definition 5: An Extended Petri net $\Sigma = (P; T; F, M_0, W)$ is an RM_WF_Net if the following hold.

1) $P = P_L \cup P_R \cup P_M$, $P_L \cap P_R = \emptyset$, $P_R \cap P_M = \emptyset$, $P_L \cap P_M = \emptyset$; $P_L$ represents logic places corresponding to the ordinary places in the WF-net, and $P_R$ and $P_M$ represent the resource set and message set, respectively.

2) $F \subseteq F_L \cup F_R \cup F_M$, where: a) $F_L = (P_L \times T) \cup (T \times P_L)$ represents the logical structure of the model; b) $F_R = (P_R \times T) \cup (T \times P_R)$ shows the required and released resource relations; and c) $F_M = (P_M \times T) \cup (T \times P_M)$ shows the required and sent message relations.

3) $\forall p \in P$, $M_0(p) = 1$ if $p \in P_L \land p = \emptyset$, $M_0(p) = \#_p$ if $p \in P_R$, and otherwise $M_0(p) = 0$, $\#_p$ represents the initial available amount of resource $p$.

4) $P_L, T_L: F_L, M_0|_{P_L}$ is a WF-net, where $M_0|_{P_L}$ is the projection of $M_0$ on $P_L$. $W: F \rightarrow Z^+$ is the weighted function, such that: $\forall f \in F$, $W(f) = 1$ if $f \in F_L \lor f \in F_M$, $W(f) = \#_{rl}(t, r)$ if $(f \in F_R \cap P_R \times T) \land (f = (t, r))$, and $W(f) = \#_{rl}(t, r)$ if $(f \in F_R \cap T \times P_R) \land (f = (r, t))$, $\#_{rl}(t, r)$ represents the amount of required resource $r$ to execute task $t$, and $\#_{rl}(t, r)$ represents the amount of released resource $r$ when task $t$ finishes.

The RM_WF_Net is used to model the intradepartment business process with message and resource information. Logic structure and connection relations among tasks can be represented by the $(P_L, T_L; F_L, M_0|_{P_L})$. Compared with the traditional WF-net, the main differences of an RM_WF_Net defined in Definition 5 are as follows.

1) Three components are involved in the place set, i.e., $P = P_L \cup P_R \cup P_M$. Resource set ($P_R$) and message set ($P_M$) are separated from the normal place set ($P_L$).

2) The weighted function is introduced to represent the amount of required or released resource when a task starts or finishes. All resources are well-prepared and allocated properly before use. All messages are generated during the process execution, so each message place does not contain any token in the model before process execution.
C. Modeling Intradepartment Business Processes

Modeling approaches for the intradepartment process based on the RM_WF_Net contain two phases: 1) modeling single task with RM_WF_Net and 2) modeling control structure with RM_WF_Net. Detailed discussions for these two phases are given in the following.

1) Modeling Single Tasks With RM_WF_Net: In the RM_WF_Net model, a task is represented by a transition which has one input logic place and one output logic place representing the ready and end states, respectively. In addition, according to the involved (i.e., required, released or sent) resource/message, corresponding places are added with flow relations. More specifically, RM_WF_Net-based task model is illustrated in Fig. 1, where \( P_{\text{ready}} \) is the task ready place, \( P_{\text{end}} \) is the task end place, \( P_{\text{resourceReq}} \) and \( P_{\text{resourceRel}} \) are resource places which show the required/released resources when a task starts/ends, \( P_{\text{messageReq}} \) and \( P_{\text{messageSent}} \) are message places which show the required/sent messages when a task starts/ends. To distinguish the logic place, resource place and message place properly, they are drawn as normal circle, double circle with full line, and double circle with broken line accordingly. Moreover, directed arc drawn with full line represents occupation/release of resources and while directed arc drawn with broken line represents receiving/sending messages.

Take the task medicine accounting in the pharmacy as an example. It is formalized as \( T_2 = \langle \text{medicine accounting, pharmacy,} \{\text{prescription}\}, \{\text{cost bill2}\}, \{\text{charge system}\}, \{\text{charge system}\}, \emptyset \rangle \). Its corresponding RM_WF_Net model is shown in Fig. 2. It should be noted that there exists a two-way arc between the transition (medicine accounting) and the place (charge system), which means that task medicine accounting occupies the charge system when starts and releases it when finishes.

2) Modeling Control Structure With RM_WF_Net: In this section, task dependencies of the intradepartment process are investigated. Definitions and WF-net models of basic control structures, such as sequence structure, AND-split structure, AND-join structure, XOR-split structure, XOR-join structure, start structure, end structure, and loop structure, are illustrated in Table II.

The intradepartment business process can be modeled with the following steps.

Step 1) Based on the modeling approaches of basic control structures in Table II, the WF-net model to express the task dependencies is constructed.

Step 2) Add the resource places, message places and corresponding flow relations to each task of the obtained WF-net, in this way, an RM_WF_Net is constructed to model the intradepartment business process.

Take the process in surgical department as an example. This medical process contains tasks, such as admissions, reservation application, imaging planning, diagnosis, consultation application, consultation, and prescription given. Based on the Step 1 of constructing intradepartment business process, its WF-net is shown in Fig. 3.

It is obvious that only sequence structures are involved in this medical process. By adding message and resource information to tasks, the RM_WF_Net model of surgical department medical process is illustrated in Fig. 4.

According to Fig. 4, we can see that the medical history is required by the task admissions, task image planning needs acceptance notice, and imaging reports is used by task diagnosis. Also, some messages are released when certain task finishes. For example, the photo form is released by task image planning and diagnosis report is released when task diagnosis finishes. Moreover, admissions and consultation need to occupy resource diagnosis room. Here, we assume that there are two available diagnosis room.

V. COORDINATION PATTERNS AND MODELING APPROACHES OF CROSS-DEPARTMENT BUSINESS PROCESS

In this section, formal specifications of coordination relations are first addressed. Then, different coordination patterns are defined and modeled with RM_WF_Net.

A. Formal Specification of Coordination Relations Among Cross-Department Tasks

In this subsection, six coordination relations: message transmission relation, message broadcasting relation, resource sharing relation, task collaboration relation and task abstract relation, among cross-department tasks are formally defined.

1) Message Transmission Relation: If two tasks affiliate to different departments, one sends a message, and the other needs to read this message when executes. Therefore, message transmission relation exists between them. The formal definition of message transmission relation is presented in Definition 6.

Definition 6: Let \( O_1 \) and \( O_2 \) be two departments, and \( <T_1, O_1, MessageReq_1, MessageSent_1, ResourcesReq_1, ResourcesRel_1, SubTasks_1> \) and \( <T_2, O_2, MessageReq_2, MessageSent_2, ResourcesReq_2, ResourcesRel_2, SubTasks_2> \) be two tasks. If: 1) \( O_1 \neq O_2 \) and 2) \( MessageSent_1 \cap MessageReq_2 \neq \emptyset \) or \( MessageSent_2 \cap MessageReq_1 \neq \emptyset \) are satisfied, then message transmission relation exists between these two tasks.
TABLE II
BASIC CONTROL STRUCTURES AND ITS WF-NET MODEL

<table>
<thead>
<tr>
<th>Control Structure</th>
<th>Meaning</th>
<th>WF-net model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequence Structure</td>
<td>$t_2$ is enabled after $t_1$ finishes</td>
<td><img src="image" alt="Sequence Structure Diagram" /></td>
</tr>
<tr>
<td>AND-Split Structure</td>
<td>${t_{21}, \ldots, t_{2n}}$ are enabled concurrently after $t_1$ finishes</td>
<td><img src="image" alt="AND-Split Structure Diagram" /></td>
</tr>
<tr>
<td>AND-Join Structure</td>
<td>$t_2$ is enabled after all tasks in ${t_{11}, \ldots, t_{1n}}$ finish</td>
<td><img src="image" alt="AND-Join Structure Diagram" /></td>
</tr>
<tr>
<td>XOR-Split Structure</td>
<td>one of the task in ${t_{21}, \ldots, t_{2n}}$ is enabled after $t_1$ finishes</td>
<td><img src="image" alt="XOR-Split Structure Diagram" /></td>
</tr>
<tr>
<td>XOR-Join Structure</td>
<td>$t_2$ is enabled after one of the task in ${t_{11}, \ldots, t_{1n}}$ finishes</td>
<td><img src="image" alt="XOR-Join Structure Diagram" /></td>
</tr>
<tr>
<td>Start Structure</td>
<td>$t_2$ is enabled when the process starts</td>
<td><img src="image" alt="Start Structure Diagram" /></td>
</tr>
<tr>
<td>End Structure</td>
<td>The process comes to an end after $t_1$ finishes</td>
<td><img src="image" alt="End Structure Diagram" /></td>
</tr>
<tr>
<td>Loop Structure</td>
<td>${t_{11}, \ldots, t_{1m}}$ are enabled again after ${t_{21}, \ldots, t_{2n}}$ finish</td>
<td><img src="image" alt="Loop Structure Diagram" /></td>
</tr>
</tbody>
</table>

Fig. 3. WF_Net model of surgical department medical process.

Fig. 4. RM_WF_Net model of surgical department medical process.

Taking the cross-department medical diagnosis scenario in Section III into consideration, task preexamination triage is formalized as $t_1 = <\text{preexamination triage, receptionist department, } \emptyset, \{\text{medical history}\}, \emptyset, \{t_{11}, t_{12}, t_{13}\}>$, and task admissions of surgical department is denoted as $t_2 = <\text{admissions, surgical department, } \{\text{medical history}\}, \emptyset, \{\text{diagnosis room}\}, \{\text{diagnosis room}\}, \emptyset>$. Obviously, the sending message set of $t_1$ intersects with the receiving message set of $t_2$. More, they belong to different departments, thereby they are in a message transmission relation.

2) Message Broadcasting Relation: Assume that one task sends a message, and two tasks in other departments need to read this message to execute. We claim that message broadcasting relation exists. Its formal definition of message broadcasting relation is given in Definition 7.

**Definition 7:** Let $<T_1, O_1, MessageReq_{1}, MessageSent_{1}, ResourcesReq_{1}, ResourcesRel_{1}, SubTasks_{1}>$ and $<T_2, O_2, MessageReq_{2}, MessageSent_{2}, ResourcesReq_{2}, ResourcesRel_{2}, SubTasks_{2}>$ be two tasks. If $MessageReq_{1} \cap$
MessageReq₂ ≠ ∅, then message broadcasting relation exists between them.

Taking the cross-department medical scenario in Section III as an example. Task consultation in the cardiovascular department is formalized as <consultation, cardiovascular department, {diagnosis report}, ∅, {diagnosis room}, {diagnosis room}>, ∅>, and task consultation in the surgical department is denoted as <consultation, surgical department, {diagnosis report}, ∅, {diagnosis room}, {diagnosis room}>, ∅>. Obviously, the reading message sets of the two tasks intersect.

3) Resource Sharing Relation: If the required resource sets of two tasks intersect completely or partially, then they satisfy a resource sharing relation. Its formal definition is given in Definition 8.

Definition 8: Let <T₁, O₁, MessageReq₁, MessageSent₁, ResourcesReq₁, ResourcesRel₁, SubTasks₁> and <T₂, O₂, MessageReq₂, MessageSent₂, ResourcesReq₂, ResourcesRel₂, SubTasks₂> be two tasks. If: ResourceReq₁ ∩ ResourceReq₂ ≠ ∅, then resource sharing relation exists between them.

Considering the cross-department medical scenario in Section III, task admissions of surgical department is denoted as <admissions, surgical department, {medical history}, ∅, {diagnosis room}, {diagnosis room}>, ∅>, task consultation in the cardiovascular department is formalized as <consultation, cardiovascular department, {diagnosis report}, ∅, {diagnosis room}, {diagnosis room}>, ∅>, and task consultation in the surgical department is denoted as <consultation, surgical department, {diagnosis report}, ∅, {diagnosis room}, {diagnosis room}>, ∅>. Obviously, the resource diagnosis room is shared among them.

4) Task Collaboration Relation: If one task needs cooperation between two different departments, i.e., each department shoulders parts of the task, we hold that there exists a task collaboration relation between them. Its formal definition is presented in Definition 9.

Definition 9: Let O₁ and O₂ be two departments, and <T₁, O₁, MessageReq₁, MessageSent₁, ResourcesReq₁, ResourcesRel₁, SubTasks₁> and <T₂, O₂, MessageReq₂, MessageSent₂, ResourcesReq₂, ResourcesRel₂, SubTasks₂> be two tasks. If: 1) T₁ = T₂; 2) O₁ ≠ O₂; 3) SubTasks₁ = SubTasks₂; 4) MessageSent₁ = MessageSent₂; 5) MessageReq₁ = MessageReq₂; and 7) ResourceRel₁ = ResourceRel₂ are satisfied, we argue that task collaboration relation exists between two tasks.

Consider the cross-department medical scenario in Section III, task consultation in the cardiovascular department is formalized as <consultation, cardiovascular department, {diagnosis report}, ∅, {diagnosis room}, {diagnosis room}>, ∅>, and task consultation in the surgical department is denoted as <consultation, surgical department, {diagnosis report}, ∅, {diagnosis room}, {diagnosis room}>, ∅>. Obviously, the required resource set, sent message set, required resource set, released resource set, and sub-task set of these two tasks are same, but executed by different departments.

5) Task Abstract Relation: If one task can be decomposed into several sub-tasks to be executed, i.e., the function of this medical task equals with a process segment, then we hold that this task is an abstract of the corresponding process segment and there is a task abstract relation between them. Its formal definition in Definition 10.

Definition 10: Let <T₁, O₁, MessageReq₁, MessageSent₁, ResourcesReq₁, ResourcesRel₁, SubTasks₁> be a task and {<T₁₁, O₁₁, MessageReq₁₁, MessageSent₁₁, ResourcesReq₁₁, ResourcesRel₁₁, SubTasks₁₁>, ..., <T₁ₖ, O₁ₖ, MessageReq₁ₖ, MessageSent₁ₖ, ResourcesReq₁ₖ, ResourcesRel₁ₖ, SubTasks₁ₖ>} be a task set. If: 1) O₁ = O₁₁ = ... = O₁ₖ; 2) MessageReq₁ = MessageReq₁₁ ∪ ... ∪ MessageReq₁ₖ; 3) MessageSent₁ = MessageSent₁₁ ∪ ... ∪ MessageSent₁ₖ; 4) ResourceReq₁ = ResourceReq₁₁ ∪ ... ∪ ResourceReq₁ₖ; 5) ResourceRel₁ = ResourceRel₁₁ ∪ ... ∪ ResourceRel₁ₖ; and 6) SubTasks₁ = T₁₁ ∪ ... ∪ T₁ₖ are satisfied, we say that task abstract relation exists between two them.

Take the medical scenario in Table I as an example. Task preexamination triage is denoted as t₁ = <preexamination triage, receptionist department, ∅, {medical history}, ∅, {diagnosis room}, {diagnosis room}>, ∅>, task consultation in the cardiovascular department, {diagnosis report}, ∅, {diagnosis room}, {diagnosis room}, ∅>. Obviously, task preexamination triage and the sub-task set {register, preexamination, triage} satisfy the conditions in Definition 10.

B. Coordination Patterns and their Modeling Approaches

Based on the above-mentioned coordination relations, in this subsection, coordination patterns of cross-department process are defined and modeled with RM_WF_Net.

1) Message Interaction Pattern: If the received/sent message set of two different departments interact, or message transmission/broadcasting relation exists among tasks of different departments, we argue that message interaction pattern exists among them. The prerequisite to define a message interaction is that the involved departments share same message elements. In this paper, we address two kinds of message elements, consumable message (like the medical history, it can be used only by one task), and nonconsumable message (like a kind of broadcast signal that can be received by many tasks). The consumable message exists between tasks which are in a message transmission relation while the nonconsumable message corresponds with the message broadcasting relation. Both the two kinds of relations lead to the message interaction pattern.

Definition 11: Let Σ₁₁ = (P₁₁, T₁₁; F₁₁, M₀₁₁, W₁₁) and Σ₁₂ = (P₁₂, T₂; F₂, M₀₂₂, W₂) be the RM_WF_Net of two business processes, where P₁₁ = P₁₁₁ ∪ P₁₁₂ ∪ P₁₁₃ and P₁₂ = P₁₂₁ ∪ P₁₂₂ ∪ P₁₂₃. If: 1) P₁₁₁ ∩ P₁₁₂ = ∅; 2) P₁₁₁ ∩ P₁₁₃ = ∅; 3) P₁₁₂ ∩ P₁₂₁ = ∅; and 4) T₁₁ ∩ T₂ = ∅ are satisfied, we argue that a message interaction pattern exist between them. Let Σ₇ = (P, T; F, M₀, W) be the integrated model of Σ₁₁ and Σ₁₂ based on message interaction pattern, such that: 1) P = P₁₁ ∪ P₁₂; 2) T = T₁₁ ∪ T₂; 3) F = F₁₁ ∪ F₂; 4) W = W₁₁ ∪ W₂; and 5) M₀ = M₀₁₁ ∪ M₀₂₂.
Consider the cross-department medical scenario in Section III, message element medical history is exchanged between the receptionist department and the surgical department. Therefore, message interaction pattern exists between them. The integrated model based on this message interaction pattern is illustrated in Fig. 5.

In Fig. 5, the process model of each department is presented in a rectangle drawn with broken line, and the exchanged message is shown between them. We will also use this approach to represent different coordination patterns in the following discussions.

2) Resource Interaction Pattern: If the required/released resource set of two different departments interact, or resource sharing relation exists among tasks of different departments, thereby resource interaction pattern exists among them. We assume that the resource elements in this paper are reusable ones. For example, the charge system can be used by different tasks, and it is exclusively occupied by one task and can be used by other tasks only when it is released. The resource elements exist between tasks which are in a resource sharing relation, thus, this relation will lead to the resource interaction pattern.

Definition 12: Let $\Sigma_{w1} = (P_1, T_1; F_1, M_{01}, W_1)$ and $\Sigma_{w2} = (P_2, T_2; F_2, M_{02}, W_2)$ be the RM_WF_Net of two processes, where $P_1 = P_{1L} \cup P_{R1} \cup P_{M1}$ and $P_2 = P_{1L} \cup P_{R2} \cup P_{M2}$. If: 1) $P_{1R} \cap P_{2R} \neq \emptyset$; 2) $P_{1L} \cap P_{2L} = \emptyset$; 3) $P_{1M} \cap P_{2M} = \emptyset$; and 4) $T_1 \cap T_2 = \emptyset$ are satisfied, then a resource interaction pattern exists between them. Let $\Sigma_w = (P, T; F, M_0, W)$ be the integrated model of $\Sigma_{w1}$ and $\Sigma_{w2}$ based on resource interaction pattern, such that: 1) $P = P_1 \cup P_2$; 2) $T = T_1 \cup T_2$; 3) $F = F_1 \cup F_2$; 4) $W = W_1 \cup W_2$; and 5) $M_0 = M_{01} \cup M_{02}$.

Take the medical scenario in Table I as an example. Task imaging register in the X-ray department will send a cost bill to the charge office when it finishes. Similarly, task medicine accounting in the pharmacy department will also send the cost information to the charge office. Therefore, resource charge system is shared between these two tasks, i.e., a resource interaction pattern exists between them. The integrated model based on resource interaction pattern is illustrated in Fig. 6.

3) Task Collaboration Pattern: If the task set of two different departments interacts, or task collaboration relation exists among tasks, then a task collaboration pattern exists between them.

Definition 13: Let $\Sigma_{w1} = (P_1, T_1; F_1, M_{01}, W_1)$, $\Sigma_{w2} = (P_2, T_2; F_2, M_{02}, W_2)$ ... and $\Sigma_{wn} = (P_n, T_n; F_n, M_{0n}, W_n)$ be the RM_WF_Net of $n$ business processes, where $P_1 = P_{1L} \cup P_{R1} \cup P_{M1}$, $P_2 = P_{1L} \cup P_{R2} \cup P_{M2}$ ... and $P_n = P_{1L} \cup P_{Rn} \cup P_{Mn}$. If: 1) $P_{1L} \cap P_{2L} \cap \ldots \cap P_{nL} \neq \emptyset$ and 2) $T_1 \cap T_2 \ldots \cap T_n \neq \emptyset$ are satisfied, we argue that a task collaboration pattern exists among them. Let $\Sigma_w = (P, T; F, M_0, W)$ be the integrated model of $\Sigma_{w1}$, $\Sigma_{w2} \ldots \Sigma_{wn}$ based on
task collaboration pattern, such that: 1) \( P = P_1 \cup P_2 \cup \ldots \cup P_n \); 2) \( T = T_1 \cup T_2 \cup \ldots \cup T_n \); 3) \( F = F_1 \cup F_2 \cup \ldots \cup F_n \); 4) \( W = W_1 \cup W_2 \cup \ldots \cup W_n \); and 5) \( M_0 = M_{01} \cup M_{02} \cup \ldots \cup M_{0n} \).

Considering the cross-department medical scenario in Section III, both the cardiovascular department and the surgical department contain tasks consultation and prescription given. We argue that a task collaboration pattern exists between them. The integrated model based on the synchronized tasks consultation and prescription given is illustrated in Fig. 7.

4) Procedure Abstract and Service Outsourcing Pattern: If one task in department \( D_1 \) and one set of tasks in department \( D_2 \) satisfy a task abstract relation, then a procedure abstract pattern exists between them if \( D_1 \) and \( D_2 \) belong to the same department, and the former is the abstract of the latter. If \( D_1 \) and \( D_2 \) are two different departments, we argue that they satisfy a service outsourcing pattern.

**Definition 14:** Let \( \Sigma_{w_1} = (P_1, T_1, F_1, M_{01}, W_1) \) and \( \Sigma_{w_2} = (P_2, T_2, F_2, M_{02}, W_2) \) be the RM_WF_Net of two business processes, where \( P_1 = P_{11} \cup P_{R1} \cup P_{M1} \) and \( P_2 = P_{12} \cup P_{R2} \cup P_{M2} \). If: 1) \( P_{11} \cap P_{22} = \emptyset \); 2) \( T_1 \cap T_2 = \emptyset \); and 3) \( \exists \ t_1 \in T_1, \exists \ \{t_{11}, \ldots, t_{1k}\} \subseteq T_2: \text{SubTasks}(t_1) = \{t_{11}, \ldots, t_{1k}\} \) are satisfied, we argue that a procedure abstract
Let $\Sigma_{w_1}$ and $\Sigma_{w_2}$ represent two different processes, they satisfy a service outsourcing pattern.

Take the medical scenario in Table I as an example. In Fig. 8, task preexamination triage is denoted as $t_1 = \langle \text{pre-examination triage}, \text{receptionist department}, \varnothing, \{\text{medical history}\}, \varnothing, \varnothing, \{t_{1,1}, t_{1,2}, t_{1,3}\} \rangle$, and tasks, including register, preexamination, and triage, are listed as: $t_{1,1} = \langle \text{register}, \text{receptionist department}, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing \rangle$, $t_{1,2} = \langle \text{pre-examination}, \text{receptionist department}, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing \rangle$, $t_{1,3} = \langle \text{triage}, \text{receptionist department}, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing \rangle$. Obviously, $t_1$ and $\{t_{1,1}, t_{1,2}, t_{1,3}\}$ satisfy task abstract relation. Moreover, all these tasks belong to the receptionist department. So they satisfy a procedure abstract pattern.

5) **Process Activation Pattern:** If the execution of task $t$ can trigger the execution of other processes in the other departments, we claim that $t$ activates the second business process, i.e., a process activation pattern exists between their corresponding departments.

**Definition 15:** Let $\Sigma_{w_1} = (P_1, T_1; F_1, M_{01}, W_1)$ and $\Sigma_{w_2} = (P_2, T_2; F_2, M_{02}, W_2)$ be the RM_WF_Net of two processes, where $P_{2\rightarrow \text{start}}$ represents the source place of $\Sigma_{w_2}$ and $t \in T_1$ is one task in $\Sigma_{w_1}$. If: 1) $P_{1L} \cap P_{2L} = \varnothing$; 2) $T_1 \cap T_2 = \varnothing$; and 3) $\exists p \in P_{1M} \cap P_{2M'}$ such that $p \in t$ and $p \cap P_{2\rightarrow \text{start}} \neq \varnothing$ are satisfied, we argue that $t$ activates $\Sigma_{w_2}$, and process activation pattern exists between $\Sigma_{w_1}$ and $\Sigma_{w_2}$.

Take the medical scenario in Table I as an example. Task medicine accounting in the pharmacy sends a message to the cross-department medical process to run correctly, capacity of the start place in charge office process is two. As for other departments, they only execute once, therefore, the capacity of message place cost bill2 when it finishes, and then task payment notice of medicine reads it. As task payment notice of medicine is the post-task of the source place in the charge office process, task medicine accounting activates the charge office process. Its specific RM_WF_Net model is illustrated in Fig. 9.

In the following, we abstract the business process of one department as a vertex, and use directed arcs to represent the activation relations among different departments. In this way, the activation relations in a cross-department business process can be illustrated by a directed graph, named process activation graph, whose formal definition is given below.

**Definition 16:** Let $\Sigma = \{\Sigma_{w_k} = (P_k, T_k; F_k, M_{0k}, W_k)\}$ be the business process of the $k$th department. $\Sigma$ is a set of business processes involved in a cross-department business process. Its corresponding activation relation set is $\text{ActivationRelation} = \{<T, \Sigma_{w_l}; v \text{ activates } \Sigma_{w_j}>\}$ and directed graph $G = <V, E>$ is the process activation graph, such that $V$ and $E$ correspond with $\Sigma$ and $\text{ActivationRelation}$, respectively.

Consider the cross-department medical business process in Section III, its corresponding process activation graph is shown in Fig. 10. As the pharmacy process can be activated by both the surgical department and the cardiovascular department, the corresponding charge office process will execute twice. To maintain the cross-department medical process to run correctly, capacity of the start place in charge office process is two. As for other departments, they only execute once, therefore, the capacity of
VI. MODELING CROSS-DEPARTMENT BUSINESS PROCESS WITH RM_WF_NET

Detailed modeling steps for the cross-department business process are obtained in this section. Generally speaking, three steps are involved: 1) constructing formal specifications for all tasks; 2) modeling control structures of intradepartment business processes; and 3) modeling coordination patterns among different departments. More specifically, modeling steps for the cross-department medical diagnosis business process mentioned in Section III are given.

A. Constructing Formal Specifications for Medical Tasks

According to Table I, the cross-department medical diagnosis scenario involves: receptionist department, surgical department, X-ray department, charge office, cardiovascular department, and pharmacy. Their corresponding tasks are formalized in Table III.

B. Modeling Control Structures of Intradepartment Business Processes

Take the receptionist department in Table I as an example. It consists of three sequence tasks, including register, preexamination, and triage. Based on the control structure modeling approaches in Table II, the control structure model of the receptionist department is given in Fig. 12. Similarly, control structure models of other medical departments are shown in Figs. 13–17.

C. Modeling Coordination Pattern

According to the task specifications in Table III, different coordination patterns are summarized in Table IV. Following the modeling approaches for coordination patterns in Section VI, the RM_WF_Net model for the cross-department medical diagnosis process is obtained and shown in Fig. 18.
VII. VERIFICATION FOR CROSS-DEPARTMENT BUSINESS PROCESSES

In this section, we give the soundness definition and analysis of the cross-department business process. According to [23], formal definition of soundness for single department is given.

Definition 17: Let $\Sigma_1 = (P, T; F, M_0)$ be a WF-net, where $i \in P$ is the source place and $o \in P$ is the sink place, $\Sigma_1$ is sound if and only if: 1) for each token in source place $i$, there will be generated one token in the sink place $o$ when finishes; 2) if the sink place $o$ contains a token, there will be no token in other places; and 3) there are no dead transitions in $\Sigma_1$.

As for the cross-department business process, its control structure model should satisfy conditions in Definition 17. Moreover, the whole cross-department process should also be sound when considering the resource and message factors. Therefore, the soundness definition of the cross-department business process is defined as follows.

Definition 18: Let $\Sigma = \{\Sigma_w, \Sigma_w \in (P_k, T_k; F_k, M_{0k}, W_k) \mid \Sigma_w \text{ is the business process of the } k\text{th department}\}$ be a set of business processes, and $\Sigma_I = (P, T; F, M_0, W)$ is the integrated model of these departments based on different coordination patterns. $\Sigma_I$ is sound if the following hold.

1) The WF-net of each department is sound, i.e., $(P_k, T_k; F_k, M_{0k}, W_k)$ is sound.
2) There is no token left in message places when a process finishes.
3) For each department, the number of tokens in the sink place equals with the original number of the source place when a process finishes. Meanwhile, there is no token in other logic places.
4) There are no dead transitions in $\Sigma_I$.

Take the cross-department medical business process in Section III as an example. According to Definitions 17 and 18, we first model the control structure of the intradepartment processes with the WF-net, as shown in Figs. 12–17. By constructing their respective corresponding reachability graph, we can prove that these intradepartment processes are sound. Next, we construct the reachability graph of the $\text{RM}_W\text{F}_N\text{et}$ model in Fig. 18, as shown in Fig. 19 and the final running state of the $\text{RM}_W\text{F}_N\text{et}$ model that corresponds with the final state node $S_{31}$ is illustrated in Fig. 20. In this state, the number of tokens of each message places is 0. For the receptionist department, surgical department, X-ray department, cardiovascular department and pharmacy, the number of tokens in their respective sink place is 1. The token number of the sink place in the charge office is 2. And the number of tokens in other logic place is 0. Therefore, condition (2) in Definition 18 is satisfied. Next by analyzing each state node, it is easy to conclude that condition (3) is also satisfied. Finally, according to the reachability graph in Fig. 19, all tasks have been executed. Thus, condition (4) is also satisfied. In this way, according to Definition 18, the cross-department medical diagnosis business process is sound.

VIII. CONCLUSION

In this paper, we have presented a methodology based on Petri nets for the analysis and verification of cross-department business processes. In general, modeling approaches for intradepartment processes with $\text{RM}_W\text{F}_N\text{et}$ is first addressed. And coordination patterns and their corresponding $\text{RM}_W\text{F}_N\text{et}$ models are then presented. Based on a cross-department medical diagnosis scenario, the integrated modeling approach of the cross-department process is illustrated. Moreover, correctness verification of the obtained $\text{RM}_W\text{F}_N\text{et}$ is performed.

The coordination patterns mentioned in our work are supported explicitly or implicitly by some existing Business Process Management Suite (BPMS). In the following, we take one of the most popular open-source business process management and workflow suites, Bonita BPM [29] as the tool to specify its support toward our proposed coordination patterns. In the Bonita BPM, the modeling of message...
interaction patterns can be realized explicitly in Bonita environment where messages are represented as signal elements. As for the resource factor, the Bonita provides implicit handling for them. More specifically, resource is regarded as an attribute of task in Bonita, thus it can be configured. Then, to verify the soundness of our RM_WF_Net, we use the Platform Independent Petri net Editor 3.0 [32], PIPE 3.0 for short, to generate its reachability graph.

The complexity of constructing the reachability graph might be bad when it is used in a large-scale cross-department medical process. In the future, we plan to use the soundness preservation method by composition of single department via a set of places [19], [23] to verify correctness of the entire cross-department medical process. In addition, our RM_WF_Net model contains resource and message elements, therefore, resource conflict detection and message read/write conflict detection methods and their corresponding resolution strategies will be highly desired.

REFERENCES


