Harmonic Signatures of Static Eccentricities in the Stator Voltages and in the Rotor Current of No-Load Salient Pole Synchronous Generators

Claudio Bruzzese, Member, IEEE, and Gojko Joksimovic, Member, IEEE

Abstract—This paper shows that a static eccentricity makes rise a double fundamental frequency ripple in the rotor current of salient-pole synchronous machines. This ripple leads, under conditions ruled by the stator windings, to precise signatures in the no-load voltage spectrum. Both rotor current ripple and voltage harmonics can be used for diagnosis. The fault-related voltage harmonics are theoretically previewed in this paper, through analysis of the windings. Simulations performed by using the winding function approach confirm the theoretical predictions. A four pole 15kVA generator was used for experiments, featuring an innovative flange with exact and easy predictions. A winding function approach confirm the theoretical sensitivity. Experiments also showed an additional rotor-rotation frequency ripple in the rotor current, in case of mixed-type fault.

Index Terms—Fault diagnosis, rotor current analysis, machine voltage analysis, rotor eccentricities, synchronous generator

 NOMENCLATURE

\[ b \] breadth of the pole-shoe
\[ B_{\text{polar,healthy}} \] rotor air-gap flux density of the centered-rotor machine with dc excitation
\[ B_{\mu v} \] peak value of rotor flux density \((\mu, v)\) component
\[ D \] mechanical frictioanal damping
\[ \delta_s, \delta_d \] fixed orientations of eccentricities
\[ \Delta g_s, \Delta g_d \] maximum amplitudes of eccentricities
\[ f = \omega/2\pi \] fundamental frequency
\[ g \] actual air gap length function
\[ g_0 \] air gap length function of the healthy machine
\[ g_m \] minimum air gap of the healthy machine
\[ [i_s], i_r \] stator and rotor currents
\[ J \] coupled rotor-engine inertia
\[ K_c \] Carter's coefficient for slotted machine
\[ l \] machine axial length
\[ [L_s], L_r \] stator and rotor self-inductances
\[ [L_{\text{sr}}] \] stator-rotor mutual inductances
\[ M_{\text{ac}}, M_{\text{dc}} \] rotor magnetomotive force for ac/dc excitation
\[ M_{\text{r}}^{(\text{st.ecc.})} \] peak amplitude of \(v^\text{th}\) rotor MMF harmonic
\[ n_r \] rotor turn function
\[ N \] set of natural integers
\[ N_r \] rotor winding function
\[ p \] machine pole pairs
\[ P_{\text{healthy}} \] air gap permeance of centered-rotor machine
\[ P_{\text{rr}} \] peak value of \(f^\rho\) permeance space harmonic
\[ \rho \] additional permeance due to static eccentricity
\[ \rho_{st}, \rho_{rr} \] stator and rotor flux linkages
\[ \rho_{st.ecc.} \] stator and rotor ohmic resistances
\[ \rho_{s}, \rho_{r} \] static eccentricity degree, \(0 \leq \rho < 1\)
\[ \rho, \rho_s \] stator bore inner radius
\[ \rho_m \] mean radius for surface integration
\[ T_p \] electromagnctic torque
\[ T_{\text{pm}} \] prime mover (engine) torque
\[ \theta \] rotor mechanical angular position
\[ \theta_s, \theta_r \] mechanical angles in stator and rotor frames
\[ [u_s], [u_r] \] stator and rotor voltages
\[ \omega_m = \omega/p \] mechanical angular speed
\[ \langle x \rangle \] mean value of \(x\) on \(2\pi\) mechanical radians


I. INTRODUCTION

THE production of electric energy from primary sources mainly relies on synchronous generators (SGs), which however are subject to failures. Faults determine economic losses due to service interruption and machine damage, which are proportional to machine importance and rating. Power generators can operate under asymmetrical conditions due to mechanical and/or electrical faults, summarized as follows:

1) stator insulation fault resulting in shorting of one or more circuits and core defects [1]-[4];
2) short circuit in rotor windings with overheating and unbalanced magnetic pull, and eventual rotor bending [1];
3) rotor eccentricity due to bearing bad positioning or weariness, or to line faults; a stator-rotor rub can result in damages to stator and rotor windings and cores [5]-[7].

Classic tools for fault diagnosis include electrical, mechanical, chemical, and magnetic techniques, which are the basis for developing on-line and off-line condition monitoring systems [8]-[11]. No established technique can be mentioned...
for specific detection and quantification of rotor eccentricities in power generators. Many papers deal with eccentricities in induction machines \([12]-[19]\), and very few in power SGs. Large vertical-axis hydro-generators \([5], [6]\) and heavy-duty machines (e.g., ship generators) \([20]\) are examples of SGs needing attention to rotor misalignment. A fiber optics system for measurement and analysis of air gap dynamics in hydro-generators was presented in \([5]\). The technique was effective, but invasive due to sensor installation on the rotor. In \([6]\), air-gap measurements were achieved through the modulated capacitive current between a metallic plate fixed on the stator and the field poles. Theoretical investigations \([21], [22]\) by using the winding function approach (WFA) showed the influence of dynamic rotor eccentricities on the stator current spectra. Papers \([23], [24]\) extended the WFA to static and mixed-type eccentricities, showing their effect on the voltage harmonics of no-load SGs. No-load voltages were studied in \([7]\) too, by using finite element methods.

This paper applies and expands the Machine Voltage Signature Analysis (MVSA) technique, \([7], [23], [24]\), but a novel technique called Field Current Signature Analysis (FCSA) is also introduced and experimented in this work.

This paper shows that a static rotor eccentricity makes rise some particular harmonics in the machine no-load phase and line-line terminal voltages, which are normally smaller in case of centered rotor. These harmonics are not the same for all the machines, since their frequencies are ruled by the stator winding structure. In the 15kVA generator studied in this work, voltage harmonics useful for fault diagnosis were previewed to be the 3\(^{rd}\), 9\(^{th}\), 15\(^{th}\), etc., and their features were tested by computer simulations and practical experiments.

The growth of these voltage harmonics is also strengthened by a 2\(f\) frequency component raised in the rotor current by the static eccentricity. This ac current component is a sensitive fault-indicator itself, at least for low pole number machines, as proved by experiments.

This paper theoretically carries out and explains the no-load voltage harmonics raised by a static eccentricity, taking in account side-effects such as the 2\(f\) rotor current ripple and the saturation, through analysis of internal MMF and air gap permeance space harmonics.

Simulations performed through WFA, also considering damping cage and parallel paths in the stator, confirm the previsions. Experiments with eccentricities of increasing gravity were easily performed on a laboratory four pole 15kVA machine equipped with an original fully adjustable five-ring mechanical flange, appositely constructed.

The experiments showed that a dynamic fault component is almost unavoidable in practical machines. The tests proved that an \(fp\) component is superimposed to the 2\(f\) component in the current of rotor with mixed-type eccentricity. A theoretical explanation was also furnished for this additional signature.

Section II of this paper furnishes theoretical tools to preview no-load voltage harmonics of SGs with static eccentricity; in Section III the model and the simulations performed for the experimental SG are shown; finally, Sections IV, V, and VI describe the adjustable eccentric flange used for tests and the experimental results.

II. THEORETICAL ANALYSIS OF STATIC ECCENTRICITIES IN SGs

Voltage harmonics are induced at the stator terminals of a no-load SG by various flux density waves traveling in the machine air gap. Air gap irregularities affect directly these field waves due to changes in the permeance distribution, and also indirectly due to some current ripple induced in the rotor circuit, which in turn affects the rotor magneto-motive force (MMF) sustaining those field waves. The analysis of these phenomena is presented in four steps as follows.

A. Symmetric Machine with DC Excitation Current

The MMF developed by the rotor field winding of a salient pole machine consists of all odd space harmonics:

\[ M^{(dc)} = \sum_{\nu = 1, 3, 5...} M^{(dc)}_\nu \cos \nu \theta_s \]  

(1)

The shape of (1) only depends on the rotor circuit disposition, while coefficients \(M^{(dc)}_\nu\) are proportional to the excitation current, and are constant for dc current. In particular, terms in (1) are not dependent on air-gap shape or on iron saturation.

The air gap permeance of salient pole machines with centered rotor is the sum of a constant component and of even-order space harmonics rotating at synchronous speed:

\[ B^{(healthy)} = \sum_{p = 0, 2, 4, \ldots} P^{(healthy)} \cos \mu \theta_s \]  

(2)

Terms in (2) depend on the field current only when the iron core comes in saturation. The rotor air gap flux density is:

\[ B^{(dc, healthy)} = M^{(dc)} \cdot B^{(healthy)} \]  

(3)

and it consists of a double infinite summation whose generic term is split in two components:

\[ B^{(dc, k)} = B^{(dc, k') \mu} \cos \nu p + \mu \theta_s + B^{(dc, k') \nu} \cos (\nu p - \mu \theta_s) \]  

(4)

Flux distributions in (4) are stationary in the rotor frame, but they become traveling waves in the stator frame due to the rotor motion. By using \(\theta_s = \theta_s + \omega t / p\), from (4) we obtain:

\[ B^{(dc, k') \mu} \cos \left( \frac{v + \mu}{p} \theta s - \left( v + \frac{\mu}{p} \right) \theta_s \right) \]  

(5)

\[ B^{(dc, k') \nu} \cos \left( \frac{v - \mu}{p} \theta s - \left( v - \frac{\mu}{p} \right) \theta s \right) \]  

(6)

with indexes \(v = 1, 3, 5, 7, \ldots\), \(\mu / p = 0, 2, 4, 6, \ldots\).

The waves above induce electromotive forces (EMFs) of frequencies \((v + \mu / p) \omega / 2\pi\) on each conductor in the stator slots; however, due to the internal connections in practical windings, some frequency usually disappears in the terminal EMFs. A flux density wave from (5), (6) induces EMFs in the rotor current with frequency 

\[ \omega / p \]  

(7)

and its multiples; hence it produces an additional harmonic in the EMF of the machine.

In the case of eccentricity, the magnetic field is altered due to the air gap variation, which also changes the permeance of the machine. Therefore, the rotor EMF is modified and so is the stator EMF. This is the principle behind the diagnosis of faults through MMF and stator current analysis.
been computed for the 15kVA SG described in Section VI (winding scheme and data are in Appendix B). This MMF distribution shows all-even space harmonics, since the machine has 2 pole pairs; however, some line is missing (such as sixth, twelfth, etc.). For this machine, the rotor flux density wave (5) for \( v=1 \) and \( \mu=2p \), having \((v+\mu)/p=1+2\times 6=6\) pole pairs, can not induce any EMF at the stator terminals at frequency \((v+\mu)/p f = 3f\). However, for \( v=3 \) and \( \mu=2p \) the wave (5) produces stator EMFs at frequency \(5f\), because it has \((3+2)\) \(p=10\) pole pairs which corresponds to a line in the spectrum of Fig. 1. Some frequencies and pole orders of waves (5), (6) are listed in Table I. Since in Fig. 1 are missing all the lines of order multiple of \(3p\), from Table I descends that in the healthy 15kVA generator no third harmonic is present in the no-load EMFs, so remaining the 1\(s\), 5\(s\), 7\(s\), 11\(s\), 13\(s\), etc.. Obviously, a different winding arrangement could produce a stator MMF distribution with different spectral pattern, where some other harmonic is missing. The missing harmonics are very useful to detect faulty conditions since they re-appear when the rotor is eccentric, as shown in the following points.

### B. Machine with Static Eccentricity and DC Excitation Current

The air gap permeance distribution of a salient-pole synchronous machine with static eccentricity is described by adding a new series of terms \(P_{\text{sat.ecc.}}\) to (2) (see Appendix C):

\[
P_{\text{dc, s.e.}}(\theta) = P_{\text{healthy}} + P_{\text{sat.ecc.}} = \sum_{\mu=0,2p,4p,\ldots}^{\mu_{\text{max}}} P_{\mu}^{(\text{h})} \cos(\mu \theta_s) + \sum_{\mu=1,2,3,\ldots}^{\mu_{\text{max}}} P_{\mu}^{(\text{sat.ecc.})} \cos(\mu \theta_s) \tag{7}
\]

where a static displacement of the rotor center is assumed with maximum gap reduction located at \(\theta_s=0\). Terms of \(P_{\text{sat.ecc.}}\) in (7) define permeance waves stationary in the stator frame; the rotor MMF (1) built up by a dc field current acts through the new terms giving rise to two new series of flux density waves:

\[
B_{\text{dc, s.e.}}^{(\text{h}, \text{s.e.})}(\theta) = \omega t - \left( \frac{\mu}{p} \right) \theta_s.
\]  

From (8) for \( v=3 \) and \( \mu=2p \) follows that previously missing 3f frequency components rise in the terminal EMFs, since the generating field waves have opporute pole orders. A 9f frequency also rises for \( v=9 \) and \( \mu=4 \). Table II shows that only eccentricity permeance components for \( \mu=2,4,6,\ldots \) (i.e., non-dominant components) generate new frequencies in phase or line-line stator voltages of the 15kVA generator. The same is true for any machine with \( p\geq2 \). Only machines with two poles and fractional-slot windings can benefit of EMF frequency signatures from (8), in conditions of static eccentricity, with \( \mu=1 \), i.e., from the first (dominant) harmonic of eccentricity.

### C. Effect of the Eccentricity-Induced AC Excitation Current Component

In case of static eccentricity, the rotor winding experiences a time-varying air gap distribution, and the rotor inductance depends on rotor position. Any rotor pole faces a minimum and a maximum-length gap through one rotor revolution: since the rotor has \( 2p \) coils in series or parallel, the field circuit experiences \( 2p \) cycles each round, i.e. rotor inductance changes with frequency \( 2pf/p=2f \). (a proof is given in Appendix D). So, as far as the rotor winding is supplied from a dc voltage source, a 2f frequency current component will be superimposed to the rotor dc current. The MMF produced by this ac component is (9), which must be summed to (1):

\[
M_{\text{ac}}^{(\theta)} = \sum_{v=1,3,5,\ldots} M_{\text{ac}}^{(v \theta)} \cos(2\omega t + v \phi \theta_s) \cos(2\omega t) = \sum_{v=1,3,5,\ldots} M_{\text{ac}}^{(v \theta)} \cos(2\omega t + v \phi \theta_s) + \cos(2\omega t - v \phi \theta_s) \tag{9}
\]

When the MMFs in (9) are considered acting through the air gap permeance of the healthy machine in (7), four additional series of flux density waves can be carried out.
acts through a 2

increasing eccentricity in the unsaturated machine with respect (10)
machine. So, the voltage spectra of the experimental 15kVA

D. Additional Effects

1) Machine Non-Idealities: Due to constructive
imperfections in practical machines, the filtering action of the
stator winding can not prevent that a little amount of missing
harmonics appear in the external EMFs of the healthy
machine. So, the voltage spectra of the experimental 15kVA
generator in Section VI usually showed small harmonics 3, 9,
15, etc. in healthy conditions too.

2) Sensitivity Lowering Due to Saturation: The iron
saturation due to rated field current lowers the amplitude of all
permeance components in (7), so affecting the field waves (8),
(10)-(12) and the EMF amplitudes. In Section VI it is shown
that third harmonics show higher relative increases with
increasing eccentricity in the unsaturated machine with respect
to saturated conditions; so, the saturation actually reduces the
fault sensitivity of fault-related harmonics.

3) Frequency Shift Due to Saturation: The saturation also
acts through a 2f frequency modulation of all permeance
amplitudes in (7), due to periodic alignment of poles of the
statically displaced rotor with the position of minimum gap,
where the flux density is stronger. This effect is represented by
using modulated permeance amplitude expressions such as:

\[
B_{p0}^{\text{ac, h}} \cos \left( v + \frac{\mu}{p} \pm 2 \omega t \right) \left( v + \frac{\mu}{p} \right) p \theta_s \right) \text{ (10)}
\]

\[
B_{p0}^{\text{ac, h}} \cos \left( v - \frac{\mu}{p} \pm 2 \omega t \right) \left( v - \frac{\mu}{p} \right) p \theta_s \right) \text{. (11)}
\]

From (10), (11) for v=1 and \( \mu/p=0 \) follows that 3f frequency
components rise in the terminal voltages through dominant
components of MMF and permeance, as well as from (11) for
v=1 and \( \mu/p=2 \). Also from (10) a 9f component follows for
v=5, \( \mu=2p \), and a 15f component for v=7, \( \mu=6p \). Frequencies
and pole orders for (10), (11) are listed in Table III. Note that
frequencies in Table III are as in Table I, but with pole order
changed of \( \pm 2p \); so, any given frequency filtered out in a
generic healthy machine has chance to reappear in faulty
conditions, due to pole-order shifting.

By applying the MMFs in (9) to the permeance terms of
\( P^{(p, Ecc)} \) in (7), other four series of flux density waves are
obtained, which complete the calculation:

\[
B_{p0}^{\text{ac, x.e}} \cos \left( v \pm 2 \omega t \right) \left( v \pm \frac{\mu}{p} \right) p \theta_s \right) \text{. (12)}
\]

Table IV shows that other third harmonics of voltage are
induced in the stator winding of the 15kVA machine, through
non-dominant permeance terms (similar arguments as for (8)
apply).

Conclusively, based on the points A, B, C above, the
15kVA machine affected by static eccentricity is expected to have a 2f frequency component superimposed to the dc field
current, while new frequencies (3rd, 9th, 15th, etc.) normally
missing in the healthy machine should appear in the no-load
voltages, together with a variation in other frequency
components already present in the healthy case.

D. Additional Effects

1) Machine Non-Idealities: Due to constructive
imperfections in practical machines, the filtering action of the
stator winding can not prevent that a little amount of missing
harmonics appear in the external EMFs of the healthy
machine. So, the voltage spectra of the experimental 15kVA
generator in Section VI usually showed small harmonics 3, 9,
15, etc. in healthy conditions too.

2) Sensitivity Lowering Due to Saturation: The iron
saturation due to rated field current lowers the amplitude of all
permeance components in (7), so affecting the field waves (8),
(10)-(12) and the EMF amplitudes. In Section VI it is shown
that third harmonics show higher relative increases with
increasing eccentricity in the unsaturated machine with respect
to saturated conditions; so, the saturation actually reduces the
fault sensitivity of fault-related harmonics.

3) Frequency Shift Due to Saturation: The saturation also
acts through a 2f frequency modulation of all permeance
amplitudes in (7), due to periodic alignment of poles of the
statically displaced rotor with the position of minimum gap,
where the flux density is stronger. This effect is represented by
using modulated permeance amplitude expressions such as:

\[
P^{(h)}_\mu = P^{(h)}_{\mu} + P^{(h, \gamma)}_{\mu} \cos 2 \omega t
\]

\[
P^{(x,e)}_\mu = P^{(x,e)}_{\mu} + P^{(x,e, \gamma)}_{\mu} \cos 2 \omega t
\]

The ac terms in (13) produce additional EMFs in the stator
terminals, easily obtained through a \( \pm 2 \omega \) frequency shift in all
the entries of Tables II-IV. No new frequency conclusively
appears, however additional components of missing harmonics
with different pole orders enrich the voltage spectra of the
saturated faulty machine.

4) Superimposed Dynamic-Type Faults: No ac component
can be developed in the field current in case of pure dynamic
eccentricity; however, a mixed-type eccentricity develops both
the 2f component as well as an \( f/p \) frequency sub-harmonic
component, as proved in Appendix D, point C, and as shown
by experiments in Section VI.

III. THE 15KVA GENERATOR MODEL AND SIMULATIONS

A. No-Load Generator Dynamic Model

The 15kVA generator was preliminarily simulated to
obtain confirmation of the theory exposed. Electric and
magnetic equations of the machine were written according to
the multiple coupled circuit theory as in (14)-(17):
\[ u_s = r_{ss} i_s + \frac{d}{dt} [\psi_s] \]  \tag{14} \\
\[ u_r = r_i i_r + \frac{d}{dt} [\psi_r] \]  \tag{15} \\
\[ [\psi_s] = L_{ss} [i_s] + L_{sr} [i_r] \]  \tag{16} \\
\[ [\psi_r] = L_{rs} [i_s] + L_{ir} [i_r] \]  \tag{17} \\

Model (14)-(17) was re-arranged with \([i_i] = 0\) in no-load condition. The time derivative of \([i_i]\) was carried out as in (18). The mechanical equations (19), (20) complete the model. The prime-mover torque \(T_{pm}\) and the field supply voltage \(u_c\) are input variables to the dynamic model.

\[ \frac{di_r}{dt} = \frac{1}{L_r} \left[ u_r - r_i i_r - \omega_m i_r \right] + \frac{dL_r}{dt} \frac{d\theta}{dt} \]  \tag{18} \\
\[ \frac{do_m}{dt} = \frac{1}{J} \left[ T_{pm} + \frac{1}{2} \frac{dL_r}{dt} \frac{d\theta}{dt} - D o_m \right] \]  \tag{19} \\
\[ \frac{d\theta}{dt} = \omega_m . \]  \tag{20} \\

Eq. (14) was used to obtain the terminal no-load voltages, once the rotor current and its derivative have been carried out:

\[ [u_s] = \left[ L_{sr} \right] \frac{di_r}{dt} + \omega_m i_r \frac{d}{dt} \left[ L_{sr} \right] \frac{d\theta}{dt} \]  \tag{21} \\

Note the first term on the right of (21) is zero in healthy machines, and it appears when an ac ripple rises in the rotor current for eccentric rotor.

**B. Winding Function Approach and Air Gap Modeling**

The inductances of the fractional-slot-winding 15kVA SG (machine data are in Appendix B) have been carried out through WFA. WFA has been mainly used in the past for induction machine modeling [25], [26] and simulation with broken bars [13], stator faults [13], [27], and eccentricity [14], [16]. Extensions of this method to SGs have been attempted for modeling of stator faults [28] and rotor eccentricities [20]-[22], [24]. Unfortunately, the rotor saliences pose serious difficulties when the air gap length function has to be defined in the inter-pole region, where a unique magnetic path is not predictable when the armature reaction is present [24]. The inter-pole air gap modeling may not be neglected here, since field space harmonics are of special interest. In no-load conditions, and without parallel loops in the stator windings, the method suggested in [24] can be used as follows.

The rotor flux above the pole face has been retained radial as usual, whereas the infra-pole flux path has been modeled by straight lines joining the pole shoe extremity \(A\) and the generic point \(P\) on the stator bore, Fig. 2. Finite element analyses showed that such approximation is rather general and accurate [24]. The air gap length function can be arranged as follows: let define a local angle \(\theta_r\) in the rotor frame, bounded between \(-\pi/2p\) and \(\pi/2p\) around the first pole center-line (first sector) for every (unbounded) value of the rotor abscissa \(\theta:\)

\[ \theta_r = \theta - \frac{\pi}{p} \left[ 2 \frac{\theta_r + \pi/2p}{2 \pi/2p} \right] . \]  \tag{22} \\

The gap wave-shape of the non-eccentric machine \(g_0\) can thus be easily computed for every rotor position as:

\[ g_0(\theta_r) = \begin{cases} 0 & \forall \theta_r \in \left[-\frac{\pi}{2p}, -\frac{\pi}{2p}\right] \\
\frac{g_m}{\cos \rho \theta_r} & \forall \theta_r \in \left[-\frac{\pi}{2p}, -\frac{\pi}{2p}\right] \cup \left[-\frac{\pi}{2p}, \frac{\pi}{2p}\right] \end{cases} \]  \tag{23} \\

where \(g_m\) is the minimum gap on the top of a pole and \(2\theta_e\) is the angle embraced by one pole shoe in the healthy machine. The fixed point \(A\) and the variable point \(P\) have the following rectangular coordinates \((b, \rho)\) defined in Fig. 2:

\[ A = \left( \frac{b}{2}, \frac{b}{2} \tan \theta_A \right), \quad P = \rho \left( \cos \theta_r', \sin \theta_r' \right) . \]  \tag{24} \\

The gap length variation due to a static eccentricity can be described by the following function:

\[ g(\theta_r, \theta) = -g_m \cos \theta r + g_r(\theta_r) \]  \tag{25} \\

The parameter \(\rho\) in (25) is the eccentricity degree, \(0 \leq \rho < 1\). The final value of gap length is obtained by adding (25) to (23), and then applying the Carter coefficient \(K_r\), to account for the slotted stator:

\[ g(\theta_r, \theta) = K_r \left[ g_0(\theta_r) + g_1(\theta, \theta) \right] . \]  \tag{26}
Fig. 3. Air gap permeance functions in the rotor frame for $\rho = 0$ and $\rho = 0.5$, calculated for two rotor positions: a) with a pole at minimum distance from the stator surface; b) with two poles equidistant from the stator.

Note $K_c$ in (26) depends on the actual modified gap ($g_0 + g_s$). So, $K_c$ cannot be pre-defined in the rotor frame, since the gap increment (25) depends on $\theta$, and $K_c$ must be evaluated for every couple ($\theta, \theta_r$), by using look-up tables.

Finally, the air gap permeance function $P$ is defined as:

$$P(\theta_r, \theta) = \frac{\mu_0}{g(\theta_r, \theta)}.$$  \hspace{1cm} (27)

Fig. 3 shows the permeance curves computed from (27) for the 15kVA SG, for two exemplar positions of the rotor.

C. Calculated Turn Functions and Inductances

Machine turn functions, winding functions, and inductance functions were carried out by exploiting calculation formulas such as those reported in [24]. These formulas are not repeated here for brevity, so only the obtained function wave-shapes are shown. Fig. 4 shows the turn function of one phase; as a consequence of the fractional number of slots-per-pole, this function lacks of odd symmetry. The winding function of phase B is shown in Fig. 5 (note the mean value is affected by the eccentricity). Stator self and mutual inductances are reported in Figs. 6, 7. The rotor turn function (Fig. 8) has been modeled as a square-wave, to exploit the accurate air gap modeling performed in the inter-pole regions. Fig. 9 shows the rotor winding inductance, in case of 50% of static eccentricity. For centered rotor, this function is a constant value (8.58H). Finally, Fig. 10 shows the stator-rotor mutual inductances.

Fig. 4. Turn function of phase winding A.

Fig. 5. Winding function of phase B for $\rho = 0$ (blue line) and $\rho = 0.5$ (red line).

Fig. 6. Phase inductance and derivative for $\rho = 0$ (blue lines) and $\rho = 0.5$ (red).

Fig. 7. Mutual inductance between phase A and B (continuous lines) and angular derivative (dot lines) $\rho = 0$ (blue lines) and $\rho = 0.5$ (red lines).

Fig. 8. Rotor turn function.

Fig. 9. Rotor inductance, $\rho = 0.5$.

Fig. 10. Mutual inductance between phase A and rotor winding (continuous lines) and angular derivative (dot lines) for $\rho = 0$ (blue lines) and $\rho = 0.5$ (red).
D. Simulations of Static Eccentricity

Two conditions were simulated, healthy machine ($\rho=0$) and 50% of static eccentricity ($\rho=0.5$), with rated rotor current. Fig. 11 shows no third harmonics in the phase voltage for $\rho=0$, whereas in Fig. 12 harmonics 3, 9, 15 are clearly present, with a general increase in the other odd lines, so confirming the theoretical previsions. Similar considerations hold for the line-line voltage in Figs. 13, 14. The presence of fault-related third harmonics in the line-line voltage proves that these frequencies are not zero-sequence systems, since they do not cancel out. This was predicted in Table III, where, e.g., the third harmonic for $\nu=1, \mu/p=0$ is produced by flux density waves (10), (11) with $p$ pole pairs (the same as the fundamental wave), so resulting in a direct-sequence voltage system. On the contrary, third harmonics in Table I (produced by a dc rotor current) are all zero-sequence systems, so they cannot be measured in the line-line voltages. Finally, Fig. 15 shows the field current for $\rho=0.5$, with the predicted $2f=100\text{Hz}$ ac component superimposed to the dc value.

Fig. 11. Phase voltage waveform a) and spectrum b), $\rho=0$.

Fig. 12. Phase voltage waveform a) and spectrum b), $\rho=0.5$.

Fig. 13. Line-line voltage waveform a) and spectrum b), $\rho=0$.

Fig. 14. Line-line voltage waveform a) and spectrum b), $\rho=0.5$.

Fig. 15. Rotor current waveform and spectrum, $\rho=0.5$ (in healthy state this is a pure dc current, $I_r=2\text{A}$). The rms value of the 100Hz component is 91mA.
E. Simulations of Static Eccentricity Embedding the Effects of Rotor Cage and Parallel Paths in the Stator

The effects of damping currents in the cage and in parallel paths in the stator are shown in Figs. 16-19. A 1950kVA, 60 Hz, 440V, 6-pole generator [20] has been simulated, with a 42-bar cage and 6 parallel poles-per-phase in the stator. The healthy machine lacks harmonics multiple of three in the stator line-line no-load voltage, Fig. 16, which re-appear in conditions of static eccentricity, Fig. 17-a. The rotor current is a dc value in the healthy machine; in the faulty case, the rotor current (shown in Fig. 17-b) has a $2f$ frequency component very much damped, but still present in the spectrum, Fig. 17-c. Large reaction currents are stimulated in both cage and stator windings, Figs. 17-d, 17-e, normally idle in no-load steady-state healthy conditions. The $2f$ component increases if the cage is suppressed, Fig. 18, and is very large when all the reactions are absent, Fig. 19. However other frequencies multiple of $2f$ populate the spectrum in Fig. 17-c, as theoretically previewed by (D7), which constitute potential fault signatures. In particular frequencies $6f, 12f, ...$, are very high for the considered 1950kVA machine.

Fig. 16. Healthy machine: line-line voltage.

Fig. 17. Machine with $\rho=0.5$: a) Line-line voltage. b) Rotor current. c) Rotor current spectrum. d) Cage loop currents on a pole. e) Internal stator currents.

Fig. 18. Rotor current a) and spectrum b), $\rho=0.5$, cage eliminated.

Fig. 19. Rotor current a) and spectrum b), $\rho=0.5$, cage and stator windings eliminated.
IV. THE 15kVA GENERATOR WITH ADJUSTABLE ECCENTRIC FLANGE

The shaft of the four pole 15kVA machine is mounted on a ball bearing on the opposite-drive side, where the slip rings are allocated, and on a regulate support on the drive side. For easy implementation and regulation of arbitrarily orientated static and dynamic eccentricities, a system of five nested steel rings was designed and realized as shown in the 3D drawing of Fig. 20. The greatest ring of the structure is fixed to the generator frame, whereas the other four rings (two around the bearing and two inside) provide the required regulations. The couple of regulating rings inside the bearing regulates the dynamic eccentricity, whereas the couple outside regulates the static one. The eccentricity amplitude can be varied in each couple of regulating rings, thanks to a small displacement applied in the design of the separating circle between the two contiguous rings. The displacements are evident in Fig. 21, where the front side of the flange is shown. The separating circles are not directly visible in Fig. 21, due to four retaining rings fixed on the front to lock the regulating rings.

Fig. 20. Exploded view drawing of the regulated flange. The retaining rings are not shown.

Fig. 21. Technical drawing of the flange with regulated eccentricities. The graduated retaining rings are shown mounted in place in front of the flange. Each retaining ring is fixed to the correspondent regulating ring. 1) Dynamic eccentricity direction regulating ring. 2) Dynamic eccentricity amplitude regulating ring. 3) Ball bearing. 4) Static eccentricity amplitude regulating ring. 5) Static eccentricity direction regulating ring. 6) Retaining ring fixed to the external ring. 7) External ring coupled to the generator frame.
The retaining rings are provided of degree-graduated scales; the relative eccentricity $\rho$ and the regulated angular displacement $\gamma$ are linked by the following formula:

$$\gamma = 2 \arcsin (\rho / 2.64). \quad (28)$$

Fig. 22 shows the realized structure, complete of retaining rings fixed with screw-bolts. The pieces were machined with a tolerance smaller than $2 \mu m$, so the cumulative error on the shaft displacement is below $12 \mu m$, and the eccentricity of the $0.75mm$ air gap is actually regulated with an overall error smaller than $1.6\%$. The coupling between the steel rings is so tight that the lightest thermal imbalance locks the rings and impedes the regulation. Both static and dynamic eccentricities can be simultaneously implemented, obviously keeping the sum of the two percentage eccentricities below $100\%$.

The generator is coupled to a driving motor through a double flexsteel-type joint which absorbs the heavy vibrations of the misaligned shaft, Fig. 22. Fig. 23 shows the overall test-bed and instruments. The test-bed permits one to produce an inclined eccentricity, since the displacement affects only one side of the shaft. This fact results in smaller signatures in the voltages and in the field current, with respect to a coaxial-type or uniform eccentricity (a reduction of $50\%$ is expected), without altering the experimental results from a qualitative point of view. Simulations in Section IV were obtained for uniform-type eccentricities, so the comparison with the experimental results shown in the following Section must take in account a pre-multiplication by a factor two.

V. EXPERIMENTAL TESTS

All tests shown in Fig.s 24 - 28 were made at rated speed and frequency (1500rpm, 50Hz). Two sets of experiments were carried out, one set with reduced rotor current ($0.5A$, i.e. $25\%$ of the rated value, unsaturated condition), Fig.s 24-26, and the others with rated rotor current ($2A$, saturated machine), Fig.s 27 - 29. Any set comprises tests with healthy machine, $25\%$ and $50\%$ of static eccentricity, plus one test with mixed eccentricity in Fig. 26. The condition denoted as healthy refers to a $0\%$ of eccentricity set on the flange, although the machine actually presented a little natural (intrinsic) static eccentricity on the opposite drive side. The unforeseen natural defect clearly appeared in the rotor current of the healthy machine, Fig.s 24-a, 27-a, and it was a first proof of the sensitivity of the proposed method. Infact, about a $6\%$ of static eccentricity was measured by inserting metal strips over the poles at the opposite-drive side.

The adjustable flange was undoubtedly free of defects (below $1.6\%$), since the $100\%$ of eccentricity was always reachable with the regulation angles theoretically furnished by (28), i.e. $\pm 45\%$, confirmed by the contact of stator and rotor. Being known amplitude and direction of the natural defect, attempts were done for compensation on the regulated side by means of a contrary static displacement. However, these attempts were fruitless; in fact, while decreasing the static-type fault, a little signature of dynamic eccentricity appeared in the rotor current, in form of a $25Hz$ ripple superimposed to the $100Hz$ and dc components, analogous to that in Fig. 26-a. This additional disturb was probably stimulated by the inclined position of the shaft, and it is still visible in Fig.s 24-b, c, 27-b, c, where more heavy static eccentricities were regulated on the flange, respectively $25\%$ and $50\%$. The test in Fig. 26 was appositely done to confirm the nature of the $25Hz$ ripple, by regulating an artificial $25\%$ of dynamic eccentricity on the inner rings of the flange together with a $50\%$ of static fault on the outer rings. Comparison of Fig.s 24-c and 26-a shows that the $100Hz$ ripple is quite unchanged, whereas the $25Hz$ is increased from the natural level to a much higher one.

The rms value of the $100Hz$ ripple in Fig. 24-c, multiplied by four (taking in account the reduced rotor current) and then by two (taking in account the inclined one-side eccentricity) is about $81mA$, and can be well compared with the simulated one in Fig. 15 ($91mA$), for rated rotor current and uniform eccentricity. However, the clear increases of the $100Hz$ ripple in Fig. 24 and in Fig. 27 concomitant with the static-type fault increments prove the validity of the proposed indicator.

The phase voltage spectra of the unsaturated machine in Fig. 25 show clear increments in the 'missing' harmonics, i.e. third, ninth, fifteenth, etc., as theoretically predicted for linear
Fig. 24. Field current ripple. a) 6%, b) 25%, c) 50% of static eccentricity. Mean rotor current = 0.5A. Vertical axis: 5mA/div; horizontal axis: 10ms/div. Ripple rms amplitude: a) 1.8mA, b) 6.5mA, c) 10.2mA.

Fig. 25. Phase voltage spectrum. a) 6%, b) 25%, c) 50% of static eccentricity. Mean rotor current = 0.5A. Vertical range: -30dB to 50dB, 10dB/div; frequency range: 0Hz to 2500Hz, 250Hz/div. Harmonic amplitudes are reported in Table V.

(unsaturated) conditions. General increases in the other harmonics are present, too. The third harmonics appear quite sensitive to the static eccentricity level, but not so much to the dynamic-type fault level, as evident by comparison of Fig.s 25-c and 26-b, so showing some property of fault selectivity. The added dynamic eccentricity however push up some other higher order harmonic (such as 13th, 17th, and 19th, [21]).

The phase voltage spectra of the saturated machine in Fig. 28 show as well increases in the third harmonics, but less pronounced than in unsaturated condition. E.g., the third harmonic rises only of 16%, instead of 115%, with the static-type fault rising from 6% to 50%.

Finally, Fig. 29 shows the 2f frequency ripple for a different rotor speed (1000rpm). Compared with Fig. 27-b (1500rpm), ripple frequency changes to 66.7Hz, whereas ripple amplitude is just slightly decreased. In fact, the rotor circuit impedance is mainly inductive, and decreases with frequency, as well as the EMF due to inductance fluctuation; neglecting the rotor resistance, the relative values of current ripple and inductance ripple are linked by the following approximate relationship (from (18)), not dependent on rotor speed:

\[
\frac{\Delta I_{\text{ripple}}}{I_{\text{mean}}} \approx \frac{\Delta L_{\text{ripple}}}{L_{\text{mean}}}.
\] (29)

Fig. 26. a) Field current ripple. b) Phase voltage spectrum. (50% of static plus 25% of dynamic eccentricity; same axis scales as in Fig.s 24, 25. Ripple rms amplitude in a): 11.2mA). Harmonic amplitudes are reported in Table V.
VI. CONCLUSIVE REMARKS ON THE EXPERIMENTAL RESULTS

The measured voltage harmonics have been resumed in Table V. The amplitude of harmonics as simulated in Sub-section IV-D should be compared with those measured for unsaturated machine, i.e. the values in Table IV for 25% of rated rotor current, multiplied by eight. The following conclusions can be drawn from the laboratory tests:

1. Table V shows that third harmonics (3, 9, 15,..) reports increases concomitant with the rise of static eccentricity, in the unsaturated machine, as predicted in Section III. In the saturated machine under rated conditions, the thirds harmonics increase too, but at reduced rate.

2. Voltage harmonics other than thirds (i.e., 5, 7, 11, etc.) generally increase with the eccentricity degree both in unsaturated and saturated conditions. They appear more fault sensitive with respect to the third harmonics in the saturated machine, and less in the unsaturated condition.

3. The 2f frequency ripple in the rotor current appears, in the 15kVA machine, as a selective and sensitive indicator of static eccentricity, with ripple amplitude quite proportional to the fault percentage.

4. When a dynamic eccentricity is added to the static one, an f/2 frequency component rises with the 2f component. Their relative amplitudes appear close to the relative gravity of static and dynamic faults in the 15kVA machine. Since real 2p-pole machines are usually affected by mixed-type faults, both the f/p and 2f frequency signals might be generally detectable. The relative evaluation of ripple amplitudes in the field current of SGs should permit one to obtain selectivity about the kind of fault (static or dynamic).
TABLE V
MEASURED PHASE VOLTAGE HARMONICS

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>6% of static eccentricity</th>
<th>25% of static eccentricity</th>
<th>50% of static eccentricity</th>
<th>6% of static eccentricity</th>
<th>25% of static eccentricity</th>
<th>50% of static eccentricity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% rated (2A)</td>
<td>% rated (0.5A)</td>
<td>% rated (2A)</td>
<td>% rated (0.5A)</td>
<td>% rated (2A)</td>
<td>% rated (0.5A)</td>
</tr>
<tr>
<td></td>
<td>dB V</td>
<td>dB V</td>
<td>dB V</td>
<td>dB V</td>
<td>dB V</td>
<td>dB V</td>
</tr>
<tr>
<td>1</td>
<td>46.5 220/40 22.1 33.2 23 45.5 7.8 7.4 45.8 79 38 38.6 94</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5.6 98 6 2 2 7 2 2 3 11 1 1 1 6 5 1 8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>14 5.0 15 5 6 6.6 4.5 7 6 2.7 7 2.2 7 2.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>-7 1.5 5 0.4 5 7 4.5 10 0.5 3 10 8 10 7 4.5 0.7 4.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-11 1.29 10 3.2 2 9 7 3.6 16 1 16 13 2.2 7 2.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>-20 0.1 16 0.2 2 12 0.4 17 14 16 0.1 14 19 14 16 0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>-22 0.7 10 3.2 2 6 1 5 0 0 9 9 12 6 28</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>-10 0.3 11 9 3.5 7 4.5 20 0 1 16 0.2 9 9 3.5 0.3 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>-15 0.1 14 0.1 2 12 0.2 5 0 0 19 0.1 17 14 14 0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>-20 0.1 16 0.1 2 16 0.2 3 0 0 15 0.1 17 15 18 0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>-24 0.7 11 11 0.1 13 1 11 0.1 1 0 24 0.7 15 17 0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>-25 0.1 19 0.1 19 0.1 2 11 0.1 2 0 25 0.1 17 17 0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>-20 0.1 19 0.1 19 0.1 2 19 0.1 1 0 20 0.1 17 18 0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>-21 0.1 19 0.1 19 0.1 2 19 0.1 1 0 21 0.1 17 18 0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>-23 0.1 19 0.1 19 0.1 2 19 0.1 1 0 22 0.1 17 18 0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>-25 0.1 19 0.1 19 0.1 2 19 0.1 1 0 25 0.1 17 18 0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(*) intrinsic static eccentricity on the non regulated side
(**) with added 25% of dynamic eccentricity

VII. CONCLUSION

The laboratory tests proved that a 2f frequency ripple rises in the SG rotor current as a consequence of static eccentricity. If ripple rms value is known for the healthy machine (with a given natural eccentricity), a direct comparison should permit to obtain a diagnosis. A dynamic eccentricity can be diagnosed too, when superimposed to the static one, thanks to an additional f/p frequency ripple. These indicators feature good fault selectivity in case of mixed-type faults. Both theory and experiments also proved that specific fault signatures rise in the terminal machine voltages, whose frequencies can be previewed, based on the winding structure. These frequencies can be different from machine to machine.

In this work, the method has been proved on a four-pole machine, with geometry in the middle between turbogenerators (2-4 poles) and hydrogenerators with many poles. Method sensitivity can vary in these two extreme cases. The 2f frequency ripple might show lower fault-sensitivity in many-pole machines, due to decaying $P_{c1}$ and $P_{c2}$ terms in (8). The f/p frequency ripple instead depends only on first-order eccentricity amplitudes in (D15), so it should not be adversely affected by the pole number. Fault sensitivity analysis for SGs with different pole pairs will be object of future works.

The action of a field regulator has been neglected here, assuming that in many practical cases the speed of the voltage feed-back regulation loop is usually too low to effectively dump the 2f frequency ripple in the field current. Obviously, in case a fast current loop was embedded in the voltage controller, the 2f frequency component should be searched in the field voltage, and no new harmonics due to (10), (11) should be expected in the terminal EMFs.

APPENDIX A: FILTERING ACTION OF STATOR WINDINGS

Let consider a single group of series connected conductors in the stator (a pole winding, or a whole phase for series-connected poles), with generic turn function as:

$$n_i = \sum n_k cos(k \theta_i + \xi_k), \quad k \in \mathbb{N}. \quad (A1)$$

The rotor flux linked with the group is [24]:

$$\psi = \rho_m l \pi B_g n_i \cos(\Omega t - \theta_i), \quad (A2)$$

where a generic flux density traveling wave is supposed:

$$B_g = B_s \cos(\Omega t - q \theta), \quad q \in \mathbb{N}. \quad (A3)$$

In (A2), only the term for k=q survives the integration:

$$\psi = \rho_m l \pi B_s n_i \cos(\Omega t + \xi), \quad (A4)$$

which only exists if the harmonic k=q exists in (A1). The MMF developed by the group of conductors has the same harmonics of (A1), so an EMF from (A4) arises only when (A3) matches some MMF space harmonic. If the phase contains 2p parallel poles, q must be also multiple of p to have voltages externally measurable. This additional condition is naturally verified in (A1) for series-connected poles (i.e., k is multiple of p), so the analysis can be always restricted to the series connection.

APPENDIX B: GENERATOR PARAMETERS AND WINDING SCHEME

Rated parameter of the experimental 15kVA SG are listed in Table VI. Fig. 30 shows the stator winding scheme.

TABLE VI
15kVA GENERATOR RATED PARAMETERS

<table>
<thead>
<tr>
<th>Stator part</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>apparent power</td>
<td>15kVA</td>
<td>real power</td>
<td>12kW</td>
<td></td>
<td></td>
</tr>
<tr>
<td>voltage</td>
<td>230/400V</td>
<td>current</td>
<td>22A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>frequency</td>
<td>50Hz</td>
<td>cosφ</td>
<td>0.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>phases</td>
<td>3</td>
<td>slots</td>
<td>30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>phase resistance</td>
<td>0.6/0.2Ω</td>
<td>turns/phase/pole</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rotor part</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>voltage</td>
<td>22V</td>
<td>current</td>
<td>2A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>poles</td>
<td>4</td>
<td>speed</td>
<td>1500rpm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>winding resistance</td>
<td>10.9Ω</td>
<td>turns/pole</td>
<td>320</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geometry/Inertia</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>stack length</td>
<td>200mm</td>
<td>air gap radius</td>
<td>100mm (mean)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>minimum gap</td>
<td>0.75mm</td>
<td>rotor inertia</td>
<td>0.1046kgm²</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix C: Permeance Harmonics for Salient-Pole Machines

Air gap permeance and air gap length functions share the same space-harmonics in a healthy (symmetric) machine:

$$\sum_{\mu=0,2p,4p\ldots} P_{\mu}^{(h)} \cos \mu \theta_s = \sum_{\mu=0,2p,4p\ldots} \frac{\mu_0}{g_{\mu}} \cos \mu \theta_s \cdot \frac{g_{\mu}}{g_{\mu}^{(h)}} \cos \mu \theta_s \cdot (C1)$$

A generic mixed-type eccentricity adds a first-order term to the gap in (C1), namely:

$$g_{1}^{(e)}(\theta) \cos (\theta + \gamma_{1}^{(e)}(\theta)) = \Delta g_{1} \cos (\theta + \delta_{1}) + \Delta g_{2} \cos (\theta + \delta_{2})$$

so that air gap permeance of the faulty machine is:

$$P_{\mu}^{(faulty)} = \sum_{\mu=0,2p,4p\ldots} \frac{\mu_0}{g_{\mu}} \cos \mu \theta_s + g_{1}^{(e)}(\theta) \cos (\theta + \gamma_{1}^{(e)}(\theta)) \cdot (C3)$$

Space harmonics of $P_{\mu}^{(faulty)}$ must be carried out by performing an harmonic balance with the second member of (C3). The solution can be always written as:

$$P_{\mu}^{(faulty)} = P_{\mu}^{(h)} + P_{\mu}^{(e)}$$

which is a generalization of (7). Terms $P_{\mu}^{(e)}$ in (C4) mainly depend on term $g_{1}^{(e)}(\theta)$ in (C3), since they are weakly linked to terms $g_{\mu}^{(h)}$ in the harmonic balance. With good approximation, for the eccentric-rotor-salient-pole machine it can be written:

$$P_{\mu}^{(e)} = \frac{\mu_0}{g_{0}^{(h)} + g_{1}^{(e)}(\theta) \cos (\theta + \gamma_{1}^{(e)}(\theta))} - P_{0}^{(h)}$$

which is exactly verified only for double-cylinder machines.

In case of pure static eccentricity, (C5) must be a stationary permeance distribution in the stator frame:

$$P_{\mu}^{(e)} = \frac{\mu_0}{g_{0}^{(h)} + \Delta g_{1} \cos \theta_s} - P_{0}^{(h)} = \sum_{\mu=0,2p\ldots} P_{\mu}^{(e)} \cos \mu \theta_s \cdot (C6)$$

with constant $P_{\mu}^{(e)}$ coefficients ($\delta_{1}=0$ has been supposed).

Appendix D: Inductance pulsations for an eccentric rotor

A. Rotor Inductance Carried Out by Using WFA

The rotor inductance in case of generic rotor eccentricity from the WFA theory is:

$$L_r = \rho_m l \sum_{k=0,3,5,p\ldots} \sum_{n=0} n_i \langle P_{k}^{(h)} + P_{k}^{(e)} \rangle \cdot (D1)$$

where a series-connected 2p-pole rotor winding has been assumed, with turn function and winding function as:

$$n_i = \sum_{k=0} n_i \cos (k \theta_s) \cdot (D2)$$

$$N_r = \sum_{k=0} \langle n_i (P_{k}^{(h)} + P_{k}^{(e)}) \rangle \cdot (D3)$$

By using permeances (C4) in (D1), the latter furnishes:

$$L_r = L_{r}^{(h)} + L_{r}^{(e)}(\theta)$$

$$= \rho_m l \frac{\pi}{2} \sum_{k=0} \sum_{n=0} n_i n_k \left( P_{k}^{(h)} + P_{k}^{(h)} + P_{k}^{(h)} + P_{0}^{(h)} + N_r^{(h)} \right)$$

$$= \rho_m l \frac{\pi}{2} \sum_{k=0} \sum_{n=0} n_i n_k$$

$$P_{k}^{(h)} + P_{h}^{(h)} + P_{h}^{(h)} - \frac{P_{h}^{(h)}}{P_{0}^{(h)} + P_{0}^{(h)}}$$

where $P_{k}^{(h)} = P_{k}^{(h)}(\theta) \cos \phi_{k}^{(h)}(\theta)$ is posed for brevity. Note $L_{r}^{(h)}$ is a constant (rotor inductance for healthy machine).

For $k = h = p$ in (D4), three essential terms appear in the inductance due to the first harmonic of rotor winding:

$$L_r = \rho_m l \frac{\pi}{2} \sum_{n=0} \left( 2 \left(P_{0}^{(h)} + N_r^{(h)} \right) + P_{2}^{(h)} + P_{2}^{(h)} - \frac{P_{p}^{(h)}}{P_{0}^{(h)} + P_{0}^{(h)}} \right)$$

The first term is due to the permeance mean value, the second one to saliences, and the last appears in case of eccentircities.

B. Case of Static Eccentricity

Permeance (C6) with $\theta_s = \theta_r + \theta$ is:

$$P_{\mu}^{(e)} = \sum_{\mu=0,1,2\ldots} \frac{\mu_0}{g_{\mu}} \cos (\mu \theta_s + \mu \theta)$$

where the generic phase is $\phi_{\mu}^{(e)} = \mu \theta$. From (D4), (D6), and with $\theta = \omega t/p$, the rotor inductance descends as:

$$L_r = L_{r}^{(h)} + \rho_m l \frac{\pi}{2} \sum_{k=0} \sum_{n=0} n_i n_k$$

$$\left( P_{k}^{(h)} + P_{h}^{(h)} \right) \frac{1}{2} \cos \left( \frac{k+h}{p} \right) o t$$

$$+ \left( P_{k}^{(h)} + P_{h}^{(h)} \right) \frac{1}{2} \cos \left( \frac{k-h}{p} \right) o t$$

where it is easily recognized that $(k-h)/p$ is multiple of two.

By using the reduced expression (D5), we have:
where the permeance terms $P_{2p}^{(e)}$ and $P_{2p}^{(e)}$ represent 'virtual' saliences due to static eccentricity. These stationary virtual saliences can be thought on the stator-side of the gap, since they are distinct from the real (rotating) saliences on the rotor. The virtual saliences interact with rotor windings rising a 2f frequency fluctuation in rotor inductance and current. This effect is generally valid for any rotor pole connection (series or parallel).

C. Case of Mixed Eccentricity

In the general case of mixed-type eccentricity, (C5) must be solved by an harmonic balance such as in (complex form):

$$\sum_{k=-\infty}^{\infty} \bar{g}_k \bar{P}_{k-1} = \bar{\mu}_k, \; k=0,1,2,...$$ (D9)

with quantities defined in Table VII. Since the spectrum of space frequencies is bounded for permanence in (C5), i.e. $P_e \rightarrow 0$ for $k \rightarrow \infty$, system (D9) is solved by imposing $P_{2e}=0$. A backward substitution leads, for the second equation of (D9), to:

$$\bar{P}_{1}=G_{1}\bar{P}_{0} = \frac{-\bar{g}_1}{\bar{g}_0 + \bar{g}_1^* G_{k-1}}$$ (D10)

The coefficient $G_k$ is recursively defined in (D10). The convergence implies:

$$\lim_{k \rightarrow \infty} G_k - G_{k-1} = 0$$ (D11)

and $G_k$ is computed by solving (D10) with $G_k = G_{k-1} = G_k$. Substitution of (D10) in the first equation of (D9) furnishes:

$$\bar{P}_0 = \frac{\mu_0}{\sqrt{\bar{g}_0^2 - 4 \bar{g}_1 \bar{g}_1^*}}$$ (D12)

Finally, all other components can be obtained in cascade:

$$\bar{P}_{1} = \frac{-2 \bar{g}_1}{\bar{g}_0 + \sqrt{\bar{g}_0^2 - 4 \bar{g}_1 \bar{g}_1^*}} \cdot \frac{\mu_0}{\sqrt{\bar{g}_0^2 - 4 \bar{g}_1 \bar{g}_1^*}}$$ (D13)

$$\bar{P}_{k} = \frac{-\bar{g}_0 \bar{P}_{k-1} - \bar{g}_1 \bar{P}_{k-2}}{\bar{g}_1^*}, \; k=2,3,4...$$ (D14)

By using (C2) with $\delta_0=\delta_0=0$ and $\theta = \omega t/p$, (D12) becomes (D15), which finally proves that a mixed-type eccentricity induces time oscillations with rotor rotation frequency in the term $(P_0^{(e)}+P_0^{(p)})$ of (D5), and in rotor inductance and current. Permeance terms (D13), (D14) are also generally affected by oscillations with frequencies 2f and f/p, for mixed-type faults.

**ACKNOWLEDGMENT**

The authors thank Angelo Cori, Pietro Volpi, Luigi Salvatori, and Vittorio Di Rosa from Forestal, Rome, Italy, for the valuable and precise manufacture of the eccentric flange.

**REFERENCES**


Claudio Bruzzese (S'05-M'08) received the M.Sc. and Ph.D. degrees from the University of Rome “La Sapienza,” Rome, Italy, in 2002 and 2008, respectively.

In 2002, he was with the National Transmission Network Management Company. Since September 2002, he has been with the Department of Electrical Engineering, University of Rome “La Sapienza,” as Assistant Researcher. Currently, he is promoter of research projects funded by the Italian Ministry of Defense. His interests cover diagnostics of power induction and synchronous machines, finite element analysis, railway and naval power systems, and electromechanical design and advanced modeling.

He has authored or coauthored about 40 technical papers, and holds one patent.

Dr. Bruzzese is a Registered Professional Engineer in Italy. He is member of the IEEE Industrial Electronics Society.

Gojko Joksimovic (M’98) received his B.Sc.(Hons.) (1991), M.Sc.(1995) and Ph.D. degrees (2000), all in electrical engineering, from the University of Montenegro, Podgorica, Montenegro.

He is an Associate Professor at the Department of Electrical Engineering at the same University. His main research areas include analysis of electrical machines, condition monitoring of electrical machines, power electronics and control. During 1997/98 he was visiting research fellow at the Department of Engineering, University of Aberdeen, Scotland, UK. During 2001/02 he was research fellow of the Alexander von Humboldt Foundation, at the Institute of Electrical Energy Conversion, Darmstadt University of Technology, Darmstadt, Germany. He is author of a few books and several papers published in leading international journals.

Prof. Joksimovic is a IEEE member since 1998.