Soft Output Detector for Convolutionally Encoded Parity Bit Selected Multicarrier Direct-Sequence Spread Spectrum System

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Abstract—In this paper we investigate the performance of the Parity Bit selected Multicarrier Direct Sequence Spread Spectrum (PB-MC-DS-SS) system using Log Likelihood Ratio (LLR). In this system information bits are convolutionally encoded prior to being used in the parity bit selected MC-DS-SS system. Reliability of the detected bits in the receiver is calculated in the form of LLRs and then used as the input of a soft input Viterbi decoder. The performance of the proposed system is compared to conventional coded multicarrier spread spectrum systems where no parity bit selected spreading sequence is used.

Keywords- Multicarrier spread spectrum communication; error control coding; log likelihood ratio

I. INTRODUCTION

In [1], a novel direct sequence spread spectrum (DS-SS) communication system employing parity bit selected spreading sequences is proposed. In this system, a block of \(k\) data bits is given to the parity bit calculator of a systematic \((n,k)\) linear block encoder. In contrast to the conventional systematic block codes where the \((n-k)\) parity bits are appended to the block of information bits; these parity bits are used to choose one of the mutually orthogonal spreading sequences. This work is extended in [2] where the authors introduce a soft output detector based on log likelihood ratios for convolutionally encoded systems.

In multicarrier DS-SS systems, the available spectrum is divided into \(N_c\) distinct sub-bands. If the bandwidth of each sub-band is \(W\), then the overall available bandwidth is \(WN_c\). Using direct sequence spreading, the data to be transmitted is spread to the bandwidth of a sub-band and then used to modulate one or more orthogonal carriers. Frequency diversity is obtained by transmitting the same data on multiple carriers. This comes at the expense of spectral efficiency. For optimal spectral efficiency, distinct information is carried by each carrier. In [3] and [4], the data is transmitted on every carrier, to maximize the frequency diversity. In these works, the fading on each sub-band is assumed to be Rayleigh and independent of the fading on other sub-bands. In [4], each user is assigned a unique set of mutually orthogonal spreading waveforms and then each sub-band is spread with a different spreading waveform. In [5], the data to be transmitted is serial to parallel converted and then used to modulate a subset of the orthogonal carriers.

In [6] a multicarrier DS-SS system employing parity bit selected spreading waveforms is presented. In this system parity bits are used to choose the spreading code that spreads the information bits on each sub-carrier. The ability of the receiver to exploit the frequency diversity in the process of spreading waveform determination, improves the bit error rate (BER) performance of this system compared to the conventional MC-DS-SS systems.

In the proposed receiver structure of [6], at first the spreading sequence used by the transmitter is determined by inputting the received signal from each sub-carrier to a bank of matched filters, each matched to a different spreading waveform. After determining the spreading sequence, the receiver has additional knowledge about the received data which can be used in detection. In other words, the receiver can make use of the fact that on each signaling interval a particular set of data bits is carried by a specific spreading sequence on each sub-carrier. It is shown in [6] that once the spreading code is identified in the first stage, the probability of error in determining the correct block of information bits is very low. In other words the probability of error is dominated by the errors caused by incorrect spreading sequence determination.

In this paper a new detection strategy for this system is proposed where the soft decision variables in terms of log likelihood ratios (LLRs) are calculated from the output of the bank of matched filters. The calculated LLR determines the reliability of each detected bit on each sub-carrier and by making use of these soft variables as the input to an outer soft-input decoder, a considerable improvement in the performance of the system in terms of BER is achieved.

The rest of the paper is organized as follows: in section 2, the parity bit selected multicarrier DS-SS system is explained. The proposed receiver structure is presented in section 3 where we describe the technique of calculating LLRs for each detected bit. In section 4 the simulation results for the proposed system are presented and section 5 deals with the conclusion drawn based on the results presented in this paper.
II. PARITY BIT SELECTED MULTICARRIER DIRECT-SEQUENCE SPREAD SPECTRUM SYSTEM

Fig. 1 shows a block diagram of the transmitter. The information stream is first convolutionally encoded and after passing through an interleaver, is converted into \( N_c \) parallel streams. On the \( i \)th signalling interval of \( T \), the output of the serial to parallel converter is:

\[
m^{(i)} = [m_0^{(i)}, m_1^{(i)}, \ldots, m_{N_c-1}^{(i)}]
\]

Each information block of \( m^{(i)} \) is input to the parity bit calculator of a systematic \((n,k)\) linear block encoder where \( k = N_c \). The parity bits produced by this operation, \( p^{(i)} = [p_0^{(i)}, p_1^{(i)}, \ldots, p_{(n-k)-1}^{(i)}] \), are used to select one spreading code from a set of \( 2^{(n-k)} \) mutually orthogonal spreading codes. The minimum length of these codes is also \( 2^{(n-k)} \). Therefore, if \( p_0^{(i)} = p_0 = [0,0,\ldots,0] \), then the spreading waveform selector will spread the parallel signals to be transmitted on the \( i \)th signalling interval with spreading waveform \( c_0(t-iT) \). Similarly, if \( p_0^{(i)} = p_n \), then spreading waveform \( c_n(t-iT) \) is employed on the \( i \)th signalling interval. This process is repeated every signalling interval.

Each parallel information bit is modulated using binary phase shift keying (BPSK) and then all of the modulated signals are multiplied by the spreading code. Each parallel spread signal is upconverted by a unique orthogonal carrier and multiplexed with other subcarrier signals. We assume that the carriers are sufficiently spaced so that the fading encountered by each can be assumed to be independent. We also assume that the data rates of each parallel data stream and spreading factor are low enough that the fading encountered by each carrier is frequency nonselective.

On the \( i \)th signalling interval, the transmitted signal is:

\[
s(t) = A_c \sum_{v=0}^{N_c-1} b_v^{(i)} c_v(t-iT) \cos(2\pi f_c t)
\]

where \( b_v^{(i)} = 2m_v^{(i)} - 1 \), \( A_c \) and \( f_c \) are the carrier amplitude and frequency of the \( v \)th subcarrier respectively and \( c_v(t-iT) \) is the spreading code selected by the parity bits.

III. PROPOSED RECEIVER STRUCTURE

The general structure of the receiver for the proposed system is given in Fig. 2. The detector for the \( v \)th carrier is depicted in Fig. 3 and the detector for the \( i \)th spreading waveform is simply a waveform correlator.

We assume the channel to be a slowly-varying, frequency selective, Rayleigh channel, where the DS-SS signals in each frequency band is fading nonselectively and independently. Thus, the transfer function of the \( v \)th frequency band in the \( i \)th signalling interval is given by \( h_v(t) = a_v^{(i)} e^{j\phi_v^{(i)}} \), where \( a_v^{(i)} \) and \( \phi_v^{(i)} \) are, respectively, an independently, identically distribution (i.i.d) Rayleigh random variable with unit second moment and i.i.d uniform random variable over \([0,2\pi]\). \([3]\)

The received signal on the \( i \)th signalling interval is then given by:

\[
r(t) = A_c \sum_{v=0}^{N_c-1} a_v^{(i)} b_v^{(i)} c_v(t-iT) \cos(2\pi f_c t + \phi_v^{(i)}) + n(t)
\]

where \( n(t) \) is additive white Gaussian noise with double sided noise spectral density \( N_0/2 \). The receiver computes the following decision variables:

\[
U_{v,c} = \text{Re}\left\{ U_{v,c}^{(i)} \right\} + j \text{Im}\left\{ U_{v,c}^{(i)} \right\}
\]

where

\[
\text{Re}\left\{ U_{v,c}^{(i)} \right\} = \int_{t-(i-1)T}^{iT} 2r(t)c_v(t-iT) \cos(2\pi f_c t) dt \quad (4)
\]

and

\[
\text{Im}\left\{ U_{v,c}^{(i)} \right\} = -\int_{t-(i-1)T}^{iT} 2r(t)c_v(t-iT) \sin(2\pi f_c t) dt \quad (5)
\]

Assuming that \( c_v(t-iT) \) is the spreading waveform employed on the \( i \)th signalling interval, then
channel state information (all sub-bands) then compensates the phase shift of the detectors’ outputs for words whose

We assume that the receiver is able to perfectly estimate the channel gains in all sub-bands.

\[ N_{v,c}^{(i)} \] is the response of the \( v \)th detector to the input noise on \( i \)th signalling interval. Also,

\[ U_{v,c}^{(i)} = \text{Re}[U_{v,c}^{(i)}, \text{Im}[U_{v,c}^{(i)}] \]

\[ \begin{align*}
U_{v,x}^{(i)} &= A_c b_v^{(i)} T \alpha_v^{(i)} \cos(\phi_v^{(i)}) + j A_c b_v^{(i)} T \alpha_v^{(i)} \cos(\phi_v^{(i)}) + N_{v,x}^{(i)} \\
&\quad \text{for } c \neq x
\end{align*} \]

Figure 2. Receiver of the proposed system

\[ W_{v,c}^{(i)} = U_{v,c}^{(i)} e^{-j \phi_v^{(i)}} = \begin{cases} 
A_c b_v^{(i)} T \alpha_v^{(i)} + \eta_v^{(i)}, & c = x \\
\eta_v^{(i)}, & c \neq x
\end{cases} \] (8)

where \( \eta_v^{(i)} = h_v^{(i)} e^{-j \phi_v^{(i)}} \). It can be shown that \( \eta_v^{(i)} \) is a Gaussian random variable with 0 mean and variance \( N_o T \).

Log-Likelihood Ratio Calculation

Let \( \Lambda(b_v^{(i)}) \) indicate the LLR of the information bit transmitted on the \( v \)th subcarrier on the \( i \)th signalling interval defined as

\[ \Lambda(b_v^{(i)}) = \ln \frac{\Pr \left( b_v^{(i)} = +1 \right) \left| \mathbf{W}^{(i)} \right| \left| \xi^{(i)} \right|}{\Pr \left( b_v^{(i)} = -1 \right) \left| \mathbf{W}^{(i)} \right| \left| \xi^{(i)} \right|} \] (9)

where \( \mathbf{W}^{(i)} = [W_{v,0}^{(i)}, W_{v,1}^{(i)}, \ldots, W_{v,2(n-k)-1}^{(i)}, W_{1,0}^{(i)}, \ldots, W_{1,2(n-k)-1}^{(i)}, \ldots, W_{N_k-1,2(n-k)-1}^{(i)}] \) is the vector of all detectors’ outputs after phase compensation and \( \xi^{(i)} = [\xi_0^{(i)}, \xi_1^{(i)}, \ldots, \xi_{N_k-1}^{(i)}] \) is the vector of channel gains in all sub-bands.

We define \( \mathcal{M}(b_v = +1) \) as a set of indices of all message words whose \( v \)th bit is +1:

\[ \mathcal{M}(b_v = +1) = \{ q : b_q = [\ldots, b_v = +1, \ldots] \} \] (10)

and similarly for \( \mathcal{M}(b_v = -1) \):

\[ \mathcal{M}(b_v = -1) = \{ q : b_q = [\ldots, b_v = -1, \ldots] \} \] (11)

Then we have

\[ \Pr (b_v^{(i)} = +1 | \mathbf{W}^{(i)}, \xi^{(i)}) = \sum_{q \in \mathcal{M}(b_v = +1)} \Pr (b_q | \mathbf{W}^{(i)}, \xi^{(i)}) \] (12)

\[ \Pr (b_v^{(i)} = -1 | \mathbf{W}^{(i)}, \xi^{(i)}) = \sum_{q \in \mathcal{M}(b_v = -1)} \Pr (b_q | \mathbf{W}^{(i)}, \xi^{(i)}) \] (13)

Figure 3. Structure of \( v \)th carrier detector

We assume that the receiver is able to perfectly estimate the channel state information \( \xi_v^{(i)} \) for all sub-bands. The receiver then compensates the phase shift of the detectors’ outputs for all sub-bands:
As the transmitted message words are equiprobable we can rewrite (9) as

\[
\Lambda \left( h_q \right) = \ln \sum_{q \in M(h_q = +1)} \Pr \left( W_q | b_q, \xi_q \right) = \ln \sum_{q \in M(h_q = +1)} \Pr \left( W_q \mid b_q, \xi_q \right)
\]  

(14)

Considering \( W_q \) derived in (8), as \( W_q \) is a Gaussian random variable, the probability of \( W_q \) given the transmitted vector of \( b_q \) and the channel gain vector \( \xi_q \) is calculated by

\[
\Pr \left( W_q | b_q, \xi_q \right) = \frac{1}{\pi N_0 T} \exp \left( -\frac{d_q^2}{N_0 T} \right)
\]

(15)

where \( d_q \) is the Euclidian distance between the received signal (detectors outputs after phase compensation) and the transmitted signal when the \( q \)th information signal is sent. This distance is calculated by

\[
d_q = \sqrt{\sum_{p=0}^{N_c-1} (W_{p,q} - A_p T \alpha_p b_{p,q})^2 + \sum_{i=0}^{2^{(n-1)}-1} \sum_{j=0}^{N_c-1} (W_{ji})^2}
\]

(16)
in which \( W_{p,q} \) is the output of the \( p \)th detector on the \( q \)th sub-band after phase compensation and \( p \) is the parity vector produced by word \( q \). In this equation, \( b_{p,q} \) indicates the \( v \)th bit of the \( q \)th information signal.

By substituting (15) in (14), the exact formula for calculating the LLRs is derived:

\[
\Lambda \left( h_q \right) = \ln \frac{\sum_{q \in M(h_q = +1)} \exp \left( -\frac{d_q^2}{N_0 T} \right)}{\sum_{q \in M(h_q = -1)} \exp \left( -\frac{d_q^2}{N_0 T} \right)}
\]

(17)

This equation can be efficiently simplified by considering the following approximation [7]:

\[
\ln \left( e^x + e^y \right) \approx \max (x, y)
\]

(18)

By making use of this approximation, a simplified equation for computing the approximate values of LLRs is derived as follows:

\[
\Lambda \left( h_q \right) = \ln \left( \sum_{q \in M(h_q = +1)} \exp \left( -\frac{d_q^2}{N_0 T} \right) \right) - \ln \left( \sum_{q \in M(h_q = -1)} \exp \left( -\frac{d_q^2}{N_0 T} \right) \right)
\]

\[\approx \min_{q \in M(h_q = -1)} \{d_q\} - \min_{q \in M(h_q = +1)} \{d_q\}
\]

(19)

In other words, to calculate the LLR of each detected bit, first we divide all message vectors into two groups: one includes all message words where the desired bit is +1 and the other group is the set of all message words with -1 in the position of the desired bit. Then Euclidean distance for all vectors of each group is computed by (16). Finally by finding the minimum distance in each group and making use of (19), the LLR for each detected bit is calculated.

IV. SIMULATION RESULTS

The simulations are performed for the system based on linear (7,4) and (10,6) codes whose parity matrices are

\[
P_{7,4} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad P_{10,6} = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}
\]

In the transmitter, at first, a block of bits are encoded by 1/2 rate convolutional encoder with the generator matrix of \( G(D) = [1 + D^2 \ 1 + D + D^2] \). The coded bits are interleaved and serial-to-parallel converted such that \( N_c \) parallel coded bits (\( N_c \) is 4 and 6 for (7,4) and (10,6) codes respectively) are transmitted simultaneously on \( N_c \) orthogonal subcarriers.
block of \( N_c \)-bit is then used to choose one of the \( 2^{(n-k)} \) spreading sequences that spreads the signals in all parallel sub-bands. The cross-correlation between all spreading codes is assumed to be zero (orthogonal codes) and the channel is a slowly-varying, frequency selective, Rayleigh channel, where the DS-SS signals in each frequency band is fading nonselectively and independently.

The BER performance of the proposed algorithm for (7,4) and (10,6) based coded PB-MC-DS-SS systems are shown in figures 3 and 4 respectively. These figures show that the proposed coded MC-DS-SS system with parity bit selected spreading sequence, outperforms the conventional coded MC-DS-SS systems for \( E_b/N_0 \) above 3dB. At a BER of \( 10^{-4} \), the (7,4) based system has a 2.5dB gain over the conventional system while the (10,6) based system has a 3dB gain.

The improved BER in parity bit selected MC-DS-SS is achieved by creating dependency between the bits carried on different carriers. In other words, if the spreading code is correctly determined by the receiver, the information bits transmitted on the sub-bands encountering deep fades can still be recovered with high probability because of the relationship between the spreading waveform and the information it carries.

V. CONCLUSION

A soft output detector for parity bit selected multicarrier DS-SS system is presented in this paper. The exact and approximate expressions to calculate the LLRs for each detected bits are derived. The calculated LLRs are then used in the outer coding layer where the soft-input Viterbi decoder is employed. The simulation results show that the proposed system outperforms the conventional coded MC-DS-SS system at higher \( E_b/N_0 \). The improved BER performance can be traded off against additional users in a code division multiple access (CDMA) system.

REFERENCES