Classification of rotated and scaled textures using HMHV spectrum estimation and the Fourier-Mellin Transform

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Abstract

In the framework of image modeling for texture analysis, we propose the combination of the new parametric 2-D spectrum estimation method called HMHV (Harmonic Mean Horizontal Vertical) and the Fourier-Mellin transform. This latter technique allows the calculation of a set of texture descriptors from a 2-D spectrum estimate which is invariant under rotation and scaling. A comparison of the HMHV and the “standard” parametric HM (Harmonic Mean) methods on a synthetic and natural stochastic textures shows that HMHV method presents almost no spurious peaks and is quite isotropic. By performing the classification of a set of 60 images divided in 12 texture classes, descriptors computed with the HMHV method provide better results than those computed with the HM method.

1. Introduction

This paper addresses the problem of finding a robust statistical description of the textures which is invariant under any rotation and any scaling. A model-based solution has been proposed by Cohen & al. in [1]. This approach uses a 2-D Gauss-Markov Random Field which is a non-causal model and has the following major drawback: the estimation of the model parameters requires the optimization of a highly non-linear likelihood function. Recently, optimal and fast parameter estimation algorithms for 2-D AR model with causal supports have been derived [2][3]. Based on these developments, we have proposed a new parametric 2-D spectrum estimation called HMHV [4]. Furthermore, Ghorbel [5] has derived a complete set of invariant features for gray-level images by using the Analytical Fourier-Mellin Transform (AFMT) which can be used in pattern recognition [6]. In this paper, we propose to use a simplified set of such invariant features computed from the discrete FMT of the 2-D HMHV spectrum estimate as rotation and scale robust texture descriptors. In such a context, the HMHV method will be compared to the standard parametric HM method [7][8].

In Section 2, we will recall both HM and HMHV methods. In Section 3, a comparison of spectrum estimates of synthetic and natural textures is provided. In Sections 4 and 5, the classification procedure and experimental results will be given.

2. HM and HMHV methods

Let an image \( Y \) be a realization of a random field within a finite rectangular window of \((L \times T)\) discrete sites and \( y(i,j) \) is the gray level pixel intensity at site \((i,j)\) (Fig. 1). \( \{ y(i,j) \} \) is a 2-D zero-mean homogeneous stochastic field. The 2-D AR model (1), the linear prediction filter (2) and the prediction error \( \epsilon(i,j) \) (3) are given respectively by the following relationships:

\[
y(i, j) = \sum_{(n,m) \in D} a_{n,m} y(i-n, j-m) + \eta(i,j)
\]

\[
\hat{y}(i, j) = \sum_{(n,m) \in D} a_{n,m} y(i-n, j-m)
\]

\[
\epsilon(i, j) = y(i, j) - \hat{y}(i, j)
\]

where \( D \) is the prediction support. \( \{ \eta(i,j) \} \) is assumed to be a zero-mean white noise. \( \{ a_{n,m} \} \) is the set of transversal AR coefficients.

Let us define the causal \( D_{QP1} \) and \( D_{QP2} \) supports:

\[
D_{QP1} = \{(n,m) / 0 \leq n \leq N, 0 \leq m \leq M, (n,m) \neq (0,0)\}
\]

\[
D_{QP2} = \{(n,m) / -N \leq n \leq 0, 0 \leq m \leq M, (n,m) \neq (0,0)\}
\]

We consider now the vectorial approach (Fig. 1) [2][3] of 2-D linear prediction used to derive transversal 2-DFRLS (Fast Recursive Least Squares) and lattice 2-DFLRLS (Fast Lattice Recursive Least Squares) algorithms for the estimation of 2-D AR first Quarter Plane (QP1)
parameters. The forward prediction vector contains the data acquired within the $D_{QP1}$ support when it is shifted from $s$ to $s+1$ (Fig. 1); $s = jxL + i$ is the image linear scanning index.

$$y_{(i,j)} = y(s)$$

Pixels belonging to the $D_{QP1}$ support
Forward prediction vector ($y_{(i,j)}$ current pixel)

**Fig. 1. $D_{QP1}$ support and prediction vectors.**

Hence, this vectorial approach induces new 2-D linear prediction models according to their prediction supports (Fig. 2) that are different from $D_{QP1}$ and $D_{QP2}$ supports.

$$S_{QP1}(\omega_1, \omega_2) = \frac{\sigma_i^2}{H_{QP1}(\omega_1, \omega_2)^2}$$
with $H_{QP1}(\omega_1, \omega_2) = 1 + \sum_{[n,m] \in D_{QP1}} a_{n,m} e^{-j\omega_1 n} e^{-j\omega_2 m}$.

This estimate can be improved by combining multiple spectrum estimate using different regions of support. HM (Harmonic Mean) spectrum [7] is computed by using 2-D spectrums estimated with the 2-D AR QP1 and QP2 parameters:

$$\frac{1}{S_{HM}(\omega_1, \omega_2)} = \frac{1}{2} \left( \frac{1}{S_{QP1}(\omega_1, \omega_2)} + \frac{1}{S_{QP2}(\omega_1, \omega_2)} \right)$$

Considering the vectorial approach, each filter of the forward prediction vector provides a 2-D spectrum estimate ($1 \leq \mu \leq M$):

$$S_{QP1}^\mu(\omega_1, \omega_2) = \frac{\sigma_i^2}{H_{QP1}^\mu(\omega_1, \omega_2)^2}$$
with $H_{QP1}^\mu(\omega_1, \omega_2) = 1 + \sum_{[n,m] \in D_{QP1}} a_{n,m} e^{-j\omega_1 (n+\mu)} e^{-j\omega_2 (m-\mu)}$.

So, the MHM (Multichannel Harmonic Mean) spectrum estimate can be performed [4]:

$$\frac{1}{S_{MHM}(\omega_1, \omega_2)} = \frac{1}{M+1} \left( \frac{1}{S_{QP1}(\omega_1, \omega_2)} + \sum_{\mu=1}^{M} \frac{1}{S_{QP1}^\mu(\omega_1, \omega_2)} \right)$$

The harmonic mean of two MHM estimates (i.e. for an horizontal scanning with QP1 (MHM1h) and a vertical scanning with QP2 (MHM2v)) provides the HMHV (Horizontal Vertical) estimate:

$$\frac{1}{S_{MHHV}(\omega_1, \omega_2)} = \frac{1}{2} \left( \frac{1}{S_{MHM1h}(\omega_1, \omega_2)} + \frac{1}{S_{MHM2v}(\omega_1, \omega_2)} \right)$$

**Fig. 2. Prediction supports involved in the vectorial approach.**

It is possible to write for each linear prediction model the equations of the associated predicted values (4) and the linear prediction errors (5) ($1 \leq \mu \leq M$ where $\mu$ is a vectorial index):

$$\hat{y}_\mu(i, j) = - \sum_{(n,m) \in D_{QP1}} a_{n,m} y(i-(n+1), j+\mu-m)$$

$$e^\mu = y(i, j) - \hat{y}_\mu(i, j)$$

The computation of $\{a_{n,m}\}$ and $\{e^\mu\}(1 \leq \mu \leq M)$ parameters is directly performed by using the transversal 2-DFRLS algorithm proposed in [2]. These parameters can also be obtained from the 2-D reflection coefficients by using the 2-D Fast Lattice RLS (2-D FLRLS) algorithm [3].

The 2-D spectrum (or Power Spectral Density function - PSD) estimated with the 2-D AR QP1 parameters is expressed as follows:
3. Comparison of the spectrum estimates

In order to compare the HM and HMHV estimates, we have generated a synthetic 2-D random field (Fig. 3.a). Its periodogram is provided in Fig. 3.b. The (10,10) order HM and HMHV spectrum estimates have been calculated (Fig. 3c & d) in order to reveal the eventual spurious peaks. The log spectrum obtained with HMHV method presents almost no spurious peak in comparison to HM method.

An other comparison has been realized by estimating the PSD of texture D68 of the Brodatz album [7] at two different orientations (Fig. 4: left column for original texture; right column for 60° rotated texture). We compute for each image, the logarithm of the periodogram (Fig. 4c & d), the (3,3) HM spectrum (Fig. 4e & f) and the (3,3) HMHV spectrum (Fig. 4g & h). In comparison to periodograms, the HM method leads to some distortions of the corresponding PSDs, while the HMHV method gives well-conditioned and identically shaped PSDs. This estimate thus seems to be the most appropriate for texture classification.

4. Classification method

A set of invariant parameters with respect to rotation and scaling can be calculated from the AFMT of the PSD of an image [5]:

\[
M_s(k,s) = \int_{\rho=0}^{0.5} \int_{\theta=0}^{2\pi} \rho^{-s} e^{-ik\theta} \hat{S}(\rho,\theta) d\theta \frac{d\rho}{\rho}
\]

(11)

with \( k \in \mathbb{Z}, s = -\sigma_o + i\nu, (\sigma_o, \nu) \in \mathbb{R}^+ \times \mathbb{R} \).

The following set is invariant under a rotation or a scaling transformation of the DSP:

\[
I_s(k,s) = \left| M_s(k,s) M_s(0,1)^t \right|
\]

(12)

A classification scheme can thus be derived.

In this paper, we have computed the invariant features of 60 images (128x128) belonging to 12 different textures (Fig. 5), 10 from the Brodatz album [9] and 2 synthetic 2-D random fields which are a sum of a deterministic field, a stochastic field and an evanescent field [10]. For each texture, the original image (512x512) has been rotated and scaled 4 times with different rotation angles and scale factors. So, 5 (128x128) images have been obtained one from the original and 4 from the transform images.
The parametric HM and HMHV spectrum estimates have been computed with a (5,5) order prediction support. This size of the support was a compromise between the approximation of the exact PSD of the different textures and the apparition of spurious peaks using HM method.

We used an Euclidean distance in order to compute:

1. $\tau$, the ratio between inter-class and intra-class average distances using the set of invariant features of original (128×128) texture images:

$$
\tau = \frac{2}{Nc(Nc-1)} \sum_{i=1}^{Nc} \sum_{j=1}^{Nc} d(O_i, O_j) - \frac{1}{Nc} \sum_{i=1}^{Nc} \sum_{j=1}^{Nc} d(O_i, O'_j)
$$

where $Nc$ is the number of texture classes ($Nc=12$), $Np$ the number of images per class ($Np=5$), $O_i, i \in [1,Nc]$, the sets of invariant features of the (128×128) original images, and $O'_j, j \in [1,Np-1]$, the sets of invariant features of (128×128) transform images of the texture $i$. This ratio gives an estimation of the discrimination performance of the method. We obtained 2.66 with HM spectrum and 4.43 with HMHV spectrum.

2. the nearest class to each image by considering the 1st, 2nd, 3rd and 4th nearest neighbors belonging to a class. Tab. 1 provides the percentage of classification errors.

<table>
<thead>
<tr>
<th></th>
<th>HM</th>
<th>HMHV</th>
</tr>
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<tbody>
<tr>
<td>1st</td>
<td>8.33%</td>
<td>5%</td>
</tr>
<tr>
<td>2nd</td>
<td>21.67%</td>
<td>6.67%</td>
</tr>
<tr>
<td>3rd</td>
<td>30%</td>
<td>13.33%</td>
</tr>
<tr>
<td>4th</td>
<td>40%</td>
<td>28.33%</td>
</tr>
</tbody>
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Tab. 1. percentage of classification errors.

We considered that 5 images per textures did not allow us to implement unsupervised classification scheme based on fuzzy logic or Bayesian approach. Nevertheless, these results show the improvement in classification by using HMHV spectrum estimation against HM method. Most of the classification errors are due to D9, D29 and D92 textures for both HM and HMHV methods, and D38, D68 and W1 textures for the HM method.

5. Conclusion

Using a synthetic random field and a natural stochastic texture we showed that HMHV spectrum estimate generates less spurious peaks and is quite isotropic in comparison to HM one. The better results in classification of rotated and scaled textures are mainly due to the better estimation properties provided by the HMHV method. We can conclude that the combination of the two recent techniques, the HMHV spectrum estimate and the AFMT, leads to the computation of a robust set of texture invariants.

References
