A Distributed Fixed-Step Power Control Algorithm for Time-Varying Systems

Huanshui Zhang†, Chung Shue Chen*, and Wing Shing Wong*

*Department of Information Engineering, The Chinese University of Hong Kong, Shatin, Hong Kong
†Information and Control Research Center, Shenzhen Graduate School of Harbin Institute of Technology,
HIT Campus, Shenzhen University Town, Xili, Shenzhen, 518055, P. R. China
Email: h_s_zhang@hit.edu.cn, {cschen9, wswong}@ie.cuhk.edu.hk

Abstract—This paper deals with a class of power control problems where the system link gains are assumed to be time varying and SIR estimates are allowed to be corrupted with bounded noises. A simple distributed algorithm of fixed-step power control is devised and the feedback requires only local information. As a generalization of the power control algorithm proposed by Sung and Wong, we have obtained a more robust solution which can handle time varying link gains and measurement noises. Convergence of the new algorithm is analyzed and numerical studies show that it is effective.

I. INTRODUCTION

Few people would dispute that power control is an important component of resource management in cellular wireless communications. Transmit power is adjusted to optimize the system signal-to-interference ratio (SIR) and to satisfy required quality-of-service (QoS). Various classes of power control problems can be formulated depending on the optimization objective. In the literature [1]–[3], many problems and solutions have been reported including centralized and distributed algorithms [4]–[9]. In most of the earlier works, transmit power is assumed to be continuous. Fixed-step power control algorithms [10], [11] have been proposed for digital wireless systems of discrete power levels. It is worth noting that most power control algorithms proposed in the literature are focused on fixed link gain model and assume the adjustment process is much faster than the rate of channel change. Power control for time varying gains in the case of fast fading remains to be investigated. Some discussions can be found in [12]–[15].

In this paper, we focus on studying power control problem in time varying systems. The gains are assumed to change freely in one region with known upper and lower bounds. A new distributed fixed-step algorithm is presented and the SIR of each user is guaranteed to converge to the target region as long as a feasible solution for the system exists. The size of the region depends, among other things, on the variances of link gains and measurement errors. As shown in [16], measurements errors could seriously affect the system performance and stability. It is well known that the convergence of Foschini and Miljanic algorithm [5] to the optimal solution requires that the SIR measurements are noise-free. The proposed work reported here extends the Foschini and Miljanic algorithm to allow fading and measurement errors and is completely independent of stochastic approximation techniques.

The rest of the paper is organized as follows. The system model is stated in Section II. The proposed algorithm is described in Section III. Section IV gives the numerical studies. The last section contains the conclusion.

II. SYSTEM MODEL

A cellular radio system with $M$ base station (BS) is considered. Each BS has at least one active mobile station (MS) homing onto it. To each communication link, we allocate a pair of orthogonal channels for the uplink and downlink. Since there is no interference between uplink and downlink, here we consider power control for uplink only. The result can be applied to downlink as well. We focus on the set of cells where a particular channel is used at a particular time and ignore the effect of adjacent channel interference. Terminals which use other channels have no interference to our considered set.

Let $P_i$ be the power transmitted by the $i$-th MS. Propagation loss and multipath effects are captured by an $M$ by $M$ gain matrix, $G(t) = \{G_{ij}(t)\}$, where $G_{ij}(t)$ is the link gain from MS $j$ to the BS home onto by the $i$-th BS. The SIR of transmission from MS $i$ to its BS at time $t$, $\Gamma_i(t)$, is given by

$$\Gamma_i(t) = \frac{G_{ii}(t)P_i}{\sum_{j\neq i} G_{ij}(t)P_j + \eta_i^0}$$

(1)

where $\eta_i^0$ is the receiver noise at BS $i$. For simplicity in discussion, we denote

$$Z_{ij}(t) \triangleq \frac{G_{ij}(t)}{G_{ii}(t)}$$

(2)

Thus, we can express (1) as

$$\Gamma_i(t) = \frac{P_i}{\sum_{j\neq i} Z_{ij}(t)P_j + \eta_i(t)}$$

(3)

where $\eta_i(t) = \eta_i^0/G_{ii}(t)$.

The following assumption is made throughout the paper.
Assumption 1: The gain $Z_{ij}(t)$ is a stochastic time-varying process with lower and upper bounds $\bar{Z}_{ij}$ and $\hat{Z}_{ij}$ respectively, such that

$$Z_{ij} \leq Z_{ij}(t) \leq \hat{Z}_{ij}. \tag{4}$$

The receiver thermal noise $\eta_i(t)$ has lower and upper bounds $\bar{\eta}_i$ and $\hat{\eta}_i$ respectively, such that

$$\bar{\eta}_i \leq \eta_i(t) \leq \hat{\eta}_i. \tag{5}$$

By (1) and (4), we have

$$\frac{P_i}{\sum_{j \neq i} Z_{ij}P_j + \bar{\eta}_i} \leq \frac{P_i}{\sum_{j \neq i} Z_{ij}(t)P_j + \eta_i(t)} \leq \frac{P_i}{\sum_{j \neq i} Z_{ij}P_j + \hat{\eta}_i}. \tag{6}$$

We denote

$$\bar{\Gamma}_i \triangleq \frac{P_i}{\sum_{j \neq i} Z_{ij}P_j + \bar{\eta}_i} \tag{7}$$

where $\bar{\Gamma}_i$ is the lower bound SIR of the time-varying $\Gamma_i(t)$ for a given power control vector.

In addition, we define

$$\alpha_{ij} \triangleq \frac{Z_{ij}}{\hat{Z}_{ij}}, \tag{8}$$

$$\alpha_i \triangleq \min_j \{\alpha_{ij}\}. \tag{9}$$

It is obvious that $\alpha_i \leq \alpha_{ij} \leq 1$.

In practice, it is impossible to measure the SIR, $\bar{\Gamma}_i(t)$, exactly since measurement errors cannot be completely avoided. We assume that the measured or estimated SIR, $y_i(t)$, is related to $\Gamma_i(t)$ via the equation

$$y_i(t) = \bar{\Gamma}_i(t) + v_i(t) \tag{10}$$

where $v_i(t)$ is a random noise process. It is assumed that the measurement error has a known lower bound, $\underline{v}_i$, and a known upper bound, $\overline{v}_i$, such that

$$\underline{v}_i \leq v_i(t) \leq \overline{v}_i. \tag{11}$$

It is worth pointing out that there is no requirement the measurement noise has a zero mean, unlike the stochastic approach. In practice, the error can be quite small [17].

III. POWER CONTROL FOR TIME VARYING SYSTEMS

Before stating the proposed algorithm, it is helpful to recall some background results.

A. Algorithm for Fixed Link Gains

In [10], the SIR is assumed to be known exactly and the link gains are time invariant. Each MS adjusts its transmit power $P_i^{(n+1)}$ at the $(n+1)$-th iteration based on the following rules:

$$P_i^{(n+1)} = \begin{cases} 
\delta P_i^{(n)}, & \text{if } \Gamma_i^{(n)} < \delta^{-1}\gamma_i \\
\delta^{-1}P_i^{(n)}, & \text{if } \Gamma_i^{(n)} > \gamma_i \\
P_i^{(n)}, & \text{otherwise}
\end{cases} \tag{12}$$

where $\gamma_i$ is the SIR threshold and the step size $\delta > 1$.

It can be seen from the above description that an SIR target region $[\delta^{-1}\gamma_i, \delta\gamma_i]$ is defined for each MS. If the SIR is below the region, the BS will instruct the MS to raise its transmit power to the next higher level. If the SIR is above the region, the power will be adjusted downwards by one level. The upper threshold is $2\delta$ higher than the lower threshold. The sign $x^{(dB)}$ is used to denote the decibel value of $x$, i.e., $x^{(dB)} = 10\log_{10} x$.

It is proved that under some suitable conditions, the algorithm given by (12) converges to a fixed point $P_i$, which is the optimal solution for the power control. In the following subsection we are concerned with power control for systems with time-varying link gains.

B. Algorithm for Time-Varying Systems

For time varying link gains SIR with measurement errors, we modify (12) to define an updating algorithm as follows:

$$P_i^{(n+1)} = \begin{cases} 
\delta P_i^{(n)}, & \text{if } y_i^{(n)} < \delta^{-1}\gamma_i + \underline{v}_i \\
\delta^{-1}P_i^{(n)}, & \text{if } y_i^{(n)} > \gamma_i + \overline{v}_i \\
P_i^{(n)}, & \text{otherwise}
\end{cases} \tag{13}$$

where $\delta > 1$, $\gamma_i^0 = \alpha_i^{-1}\gamma_i$ and $\alpha_i$ is defined as in (9).

This algorithm is a generalization of the previous work (12) for time invariant systems, which is the special case when $\alpha_i^{-1} = 1$. The bound value $\alpha_i$ as defined in (9) can be seen as an indicator of the amplitude of link gain fluctuations. The width $[\delta^{-1}\gamma_i + \underline{v}_i, \delta\gamma_i + \overline{v}_i]$ increases with the value of $\alpha_i^{-1}$ since $\gamma_i^0 = \alpha_i^{-1}\gamma_i$.

1) Quantization of Power Level: The following basic assumption is made throughout the paper.

Assumption 2: For any $i$, there exists a non-negative vector $P^*$ such that

$$\bar{\Gamma}_i(P^*) \equiv \frac{P_i^*}{\sum_{j \neq i} Z_{ij}P_j^* + \overline{v}_i} = \gamma_i \tag{14}$$

for $i = 1, 2, \cdots, N$.

We say the set of SIR targets $\gamma_i$ is lower bound feasible if there is a nonnegative finite vector $P^*$ that satisfies (14). In this model, the power level is quantized and the step size is equal to $\delta^{(dB)}$. The SIR of each user is required to converge to a target region instead of a target value.

Lemma 1: If there exists a power vector $P^*$ such that (14) holds for all $i$, then there exists a quantized power vector $P$ such that

$$\delta^{-1}\gamma_i \leq \Gamma_i(t, \hat{P}) \leq \delta\gamma_i^0 \tag{15}$$

for all $i$, where $\gamma_i^0 = \alpha_i^{-1}\gamma_i$.

Proof: Given $P^*$, we can always find one and only one discrete power level $\hat{P}_i$ such that

$$\delta^{-1/2}P^*_i \leq \hat{P}_i < \delta^{1/2}P^*_i. \tag{16}$$
Let $\hat{P}$ be the quantized power vector of $P^*$. Then,

$$\Gamma_i(t, \hat{P}) = \frac{\hat{P}_i}{\sum_{j \neq i} Z_{ij}(t) \hat{P}_j + \eta_i(t)} \geq \frac{\hat{P}_i}{\sum_{j \neq i} Z_{ij} \hat{P}_j + \eta_i} \geq \frac{\hat{P}_i}{\sum_{j \neq i} Z_{ij} \hat{P}_j^* \delta^{-1/2} + \eta_i} \geq \delta^{-1} \frac{\sum_{j \neq i} Z_{ij} \hat{P}_j^* + \eta_i}{\sum_{j \neq i} Z_{ij} \hat{P}_j + \eta_i} = \delta^{-1} \gamma_i.$$ (17)

On the other hand,

$$\Gamma_i(t, \hat{P}) \leq \frac{\hat{P}_i}{\sum_{j \neq i} Z_{ij}(t) \hat{P}_j + \eta_i} \leq \frac{\hat{P}_i}{\sum_{j \neq i} Z_{ij} \hat{P}_j^* \delta^{-1/2} + \eta_i} \leq \delta \frac{\sum_{j \neq i} Z_{ij} \hat{P}_j^* + \eta_i}{\sum_{j \neq i} Z_{ij} \hat{P}_j + \eta_i}. \quad (18)$$

From (9), $Z_{ij} = Z_{ij} \alpha_{ij} \geq Z_{ij} \alpha_i$ and $\eta_i \geq \eta_i \alpha_i$. Thus,

$$\Gamma_i(t, \hat{P}) \leq \delta \frac{\sum_{j \neq i} \alpha_{ij} Z_{ij} \hat{P}_j^* + \eta_i}{\sum_{j \neq i} \alpha_{ij} \hat{P}_j + \eta_i} \leq \delta \frac{\alpha_i \sum_{j \neq i} Z_{ij} \hat{P}_j^* + \eta_i}{\sum_{j \neq i} \hat{P}_j + \eta_i} = \delta \gamma_i^n. \quad (19)$$

Hence, Lemma 1 holds.

One can see from the above Lemma that a feasible solution exists if for all user $i$ the target region is $[\delta^{-1} \gamma_i, \delta \gamma_i^n]$.

2) Convergence Property: Results on the convergence of proposed algorithm are stated below.

**Theorem 1**: If there exists a power vector $P^*$ such that (14) holds for all $i$, then by algorithm (13), the power vector, $P(n)$, at any iteration $n$ has an upper bound and a lower bound which depend on the initial power vector.

**Proof**: The proof of this theorem is based on Lemma 1 and is modelled after the approach in [10], [11].

Let $P^{(0)}$ be the initial power vector. According to Lemma 1, a quantized vector $\hat{P}$ that satisfies (15) exists. $P^{(0)}$ differs from $\hat{P}$ by a multiple of $\delta$, i.e., $P^{(0)} = \hat{P} \delta^{a(i,0)}$ where $a(i,0)$ is an integer. In general, we define $a(i,n)$ by

$$P^{(n)}_i = \hat{P}_i \delta^{a(i,n)}. \quad (20)$$

Note that $a(i,n)$ is an integer and $|a(i, n + 1) - a(i, n)| \leq 1$. Denote that $K(n) = \min_i \{a(i,n), 0\}. \quad (21)$

For user $i$ where $a(i, n) = K(n)$, it follows that

$$\Gamma^{(n)}_i(t_n) = \frac{P^{(n)}_i}{\sum_{j \neq i} Z_{ij}(t_n) P^{(n)}_j + \eta_i(t_n)} \leq \frac{\hat{P}_i \delta^{a(i,n)}}{\sum_{j \neq i} Z_{ij}(t_n) \hat{P}_j \delta^{a(j,n)} + \eta_i(t_n)} \leq \frac{\hat{P}_i \delta^{K(n)}}{\sum_{j \neq i} Z_{ij}(t_n) \hat{P}_j \delta^{K(n)} + \eta_i(t_n)} \leq \Gamma_i(t_n, \hat{P}) \leq \delta^{\gamma_i^n}. \quad (22)$$

Therefore, those users which achieve $K(n)$ will not decrease its power at iteration $n + 1$. In other words, $a(i, n + 1) \geq a(i, n) = K(n)$.

For user $i$ where $a(i, n) > K(n)$, it follows

$$a(i, n + 1) \geq a(i, n) - 1 \geq K(n). \quad (23)$$

Hence, $K(n)$ is a non-decreasing sequence. So, for each $i$,

$$P^{(n)}_i \geq \hat{P}_i \delta^{K(0)}. \quad (24)$$

The existence of an upper bound can be shown in the same way to complete the proof of the Theorem.

Since the power level is quantized with fixed step size and by the result of Theorem 1, the power vector has a finite number of states. Assumed that link gains and SIR measurement errors are independent identically distributed (iid), the algorithm behaves as a homogeneous finite state Markov chain. Following Lemma 1, it is clear that an absorbing state of the power control exists. Moreover, an absorbing state corresponds to the case where all users do not change their power level. It can be proved that there is no recurrent state [18] in the system other than absorbing state. Therefore, algorithm (13) converges to a fixed point $\hat{P}$ such that (15) is satisfied.

IV. NUMERICAL STUDIES

Some numerical studies are conducted. The underlying system is assumed to have a standard hexagonal cellular layout with 16 cochannel cells [8] corresponding to a reuse pattern of 7. There is one active MS per cell. The location of the MS is distributed uniformly inside the cell. We assume the link gains vary in accordance with shadow fading only and the effect of multipath fading is averaged out and not considered.

Let $A_{ij}(t)$ be the dB attenuation due to shadow fading which is usually modeled as a Gaussian random variable with zero mean and standard deviation $\sigma$. The discrete time model for the link gain matrix is given below:

$$G_{ij}^{(n)} = \frac{10^{-A_{ij}^{(n)}/10}}{d_{ij}^\alpha} \quad (25)$$

where $d_{ij}$ is the distance between the $i$-th BS and the $j$-th MS, and $\alpha$ is the path loss exponent. We assume $d_{ij}$ is a constant and only consider the dynamics of shadow fading.
According to Gudmundson [19], $A_{ij}^{(n)}$ can be represented by the following model based on a Gauss-Markov process

$$A_{ij}^{(n+1)} = \rho A_{ij}^{(n)} + \sqrt{1 - \rho^2} W_{ij}^{(n)}$$  \hspace{1cm} (26)

where $\rho$ is the correlation coefficient and $W_{ij}^{(n)}$ has a normal distribution with mean zero and variance $\sigma^2$. For example, in a GSM system, $\rho$ is equal to 0.51 for a user traveling with a speed of 100 km/h. For a speed of 10 km/h, $\rho$ is equal to 0.94. The value of $\rho$ also depends on the length of iteration block. We are interested in the region $0.5 < \rho < 1$.

In the following analysis, we assume the path loss exponent $\alpha$ is 4 and the standard deviation $\sigma$ in shadow fading is equal to 4. The receiver thermal noise is $10^{-12}$W at all the BS. The samples of estimation error $v_i$ on SIR are independently drawn from a uniform distribution such that $v_i \in (-1.26, 1.26)$. Each MS generates an initial power uniformly between 0.0001 and 1W. For acceptable link quality, we assume the threshold $\gamma_i$ is 16 dB for all $i$.

A typical plot on the maximum and minimum SIR among all users is shown in Fig. 1. The correlation coefficient $\rho$ is equal to 0.94. All the SIR values plotted are computed based on the current adjusted transmit power attenuated with respect to the gain matrix in the next iteration instant. This gives a stricter setup for measuring the algorithm performance than assuming the SIR’s are measured instantaneously. Fig. 1 shows the convergence of the proposed algorithm in the case where there exists a feasible solution for the system. The maximum and minimum SIR for all $i$ converge into the target window $[\delta^{-1}\gamma_i + \tau_i, \delta\gamma_i + \tau_i]$ as defined in (13). The upper and lower thresholds are set according to provided bounds on the link gains and measurement noises. The performance with a step size $\delta$ of 2 dB under the same link gain matrix and initial power vector is provided for comparison. In both cases, we observe the convergence. As expected, the convergence rate of a larger step size power control is faster. However, the target window in the case of (b) is 2 dB wider than that in (a) due to their difference in step size.

To investigate the performance of the proposed algorithm, we compare it with Foschini-Miljanic algorithm [5], which allows the power to take any positive value and can minimize the total transmit power. The algorithm is given below:

$$P_i^{(n+1)} = \frac{\gamma_i P_i^{(n)}}{\Gamma_i^{(n)}}.$$  \hspace{1cm} (27)

As shown in Fig. 2, generally the Foschini-Miljanic algorithm converges faster than the proposed one in the case both algorithms converge. This is not surprising since the Foschini-Miljanic algorithm uses a continuous power level and assumes perfect precision in each adjustment. In the proposed fixed-step algorithm, the power levels are discrete and only two bits are required to represent power increase, decrease, or no change for each adjustment. More importantly, the Foschini-Miljanic algorithm cannot always guarantee a convergence of the algorithm and maintain the minimum SIR to be above the requirement in a time varying model. This can be shown by Fig. 2 in which the minimum SIR by the Foschini-Miljanic algorithm drops below the lower threshold frequently. This may lead to call drops. Comparatively, the proposed algorithm is more stable in the sense that it guarantees all the SIR’s are always within the target window and above the lower threshold. The width of guaranteed target window is related to the variance of the link gain in the model. Due to quantized power level, we can only require the SIR to converge to a specified region rather than to an exact target value.

To further demonstrate the significant difference between the two algorithms for time-varying model, we decrease $\rho$ from 0.94 to 0.51. In this case, the link gain will vary with a lower correlation and consequently the channel gain fluctuation is higher. Fig. 3 shows a typical performance result. Here, we
can see the effectiveness of the proposed algorithm even under such a fluctuation. All the SIR’s are still within the guaranteed target window and above the SIR requirement. However, the Foschini-Miljanic algorithm cannot provide such a result. Compared with Fig. 2, the SIR outage will be more serious in a time varying model with a higher fluctuation.

Next, we investigate the effect of estimation error on the proposed algorithm. Stability of the algorithm under measurement noise is our main concern. To focus on the effect of estimation error, we set \( \rho \) to 0.94 such that its effect on channel fluctuation will be relatively small. Fig. 4 shows the observation under different estimation errors. Since the range of estimation error is different, the width of the target window is different. Only the target window in the case of (a), which is the narrowest one among the three cases, is shown. Results in Fig. 4 show that the proposed algorithm is not too sensitive to measurement noises. Convergence of the proposed algorithm in all the three cases is observed.

V. CONCLUSION

In this paper, we investigate the fixed-step power control problem for time-varying systems. A simple distributed algorithm is devised as a generalization of [10]. The link gains are assumed to be changed randomly in a bounded region with known upper and lower bounds which may be obtained from statistical results. The algorithm works well even when the estimates of SIR are corrupted by noises. The SIR is guaranteed to converge to a target region set above a specified threshold. Numerical result shows that the proposed power control algorithm is effective for time varying systems.

REFERENCES