A Cooperative Uplink Transmission Technique for the Single- and Multi-User Case

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Abstract—A new technique for the uplink transmission of a cooperative system consisting of single-antenna source and relay nodes and a multi-antenna destination node is presented. The proposed technique is based on the singular value decomposition of the channel matrix and, for the same rate, it exhibits higher diversity gain as compared to the existing ones. Theoretical analysis of the technique is carried out when the Decode-and-Forward protocol is employed. The analysis reveals the way the decoding errors occurring at the relays degrade the performance of the system. Furthermore, the proposed technique is extended to a multi-user environment so as to exploit multi-user diversity. A novel criterion is suggested for selecting the best transmitting user at each time slot. The performance of the system in the multi-user case is theoretically studied and the number of users needed so as the system to achieve its maximum performance is computed. The derived theoretical results are verified via typical simulations.

I. INTRODUCTION

Cooperative communications [1] have emerged, over the past few years, as a possible means to overcome the implementation problems of Multiple Input - Multiple Output (MIMO) systems. The main concept is to establish cooperation between single-antenna wireless systems in order to achieve some of the benefits of MIMO systems. In a generic scenario, the transmitter (source) transmits its data to adjacent nodes (relays) and then the relays forward the data to the receiver (destination). Thus, cooperation provides independent fading paths via which the data are transmitted to the receiver. Initially, the majority of the cooperative schemes presented in literature aimed to achieve the highest possible diversity gain. Note that when these schemes are applied in a half-duplex cooperative system, the transmission of a symbol needs double the time compared with a typical MIMO system, since relays need extra time slots to receive the data from the source prior forwarding them to the destination. Thus, diversity gain comes at a cost of bandwidth efficiency. Some recent works deal with the problem of extracting multiplexing gain from a cooperative system [2] [3]. In [4] a new technique which fits perfectly to an uplink transmission scenario of a cellular network was proposed. This technique is based on a proper use of Singular Value Decomposition (SVD) beamforming which has been a well-studied transmission scheme in MIMO systems [5]. The technique exploits the properties of channel matrix SVD to create multiple independent channels (singular channels) through which symbols are transmitted simultaneously to the transmitter. It turns out that the technique in [4] achieves superior performance in terms of probability of error as compared to existing transmission schemes. The technique also offers increased multiplexing gain as it needs only \( L + 1 \) time slots to transmit \( L \) symbols to the destination, where \( L \) is the transmission block size. The technique was developed for both the Amplify-and-Forward (AF) and Decode-and-Forward (DF) cooperation protocols while the performance of the technique for the DF protocol was examined under the assumption that the involved relays decode correctly every received signal.

In the present work we extend the technique of [4] so as to cope with the more realistic scenario which allows decoding errors to occur at the relays. The influence of the decoding errors in system’s performance is studied analytically. Moreover, in order to further improve performance we extend the new technique to a multi-user environment. Due to the existence of multiple users that are available for transmission, the system must employ a suitable criterion for user scheduling. In other words, at each time slot, the best user, according to a specific criterion, transmits its data to the relays and the base station. We suggest a criterion which is suitable for the problem at hand, and we show that if it is adopted for users’ scheduling the system achieves improved performance. We further compute the number of users needed so as the maximum available diversity gain to be achieved. Note that the suggested user selection scheme could be combined with the so-called relay selection techniques [6] to improve even further the overall performance.

In Section 2 we describe the new technique for the single-user case while in Section 3 we present the performance analysis of the proposed technique, called for convenience SURBUT (Single-User Relay-Based Uplink Transmission) technique. In Section 4 the SURBUT technique is extended to the multi-user case and the performance of the resulting Multi-User Relay-Based Uplink Transmission (MURBUT) technique is studied in Section 5. Section 6 includes a discussion on the combination of the MURBUT technique with relay selection schemes. In Section 7 we provide some typical simulations results which verify the preceding analysis. Finally, Section 8 concludes the work.

II. SINGLE-USER CASE: SYSTEM DESCRIPTION

Let us consider a system with a single-antenna source and a multi-antenna base station (destination), and let us also assume that other single-antenna users are available and willing to participate in a cooperative transmission scenario. A system
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The received vector at the destination is post-coded with the corresponding left singular vector sub-matrix $U_{1:M,1:L'}$, resulting in

$$y'_D = U_{1:M,1:L'}^H y_D (L + 1) = U_{1:M,1:L'}^H \{ H_{RD} V_{1:N,1:L'} x (L + 1) + w_2 (L + 1) \} = \Sigma x (L + 1) + w' (L + 1),$$

where $\Sigma$ is a diagonal matrix containing the singular values of the decomposition and $w'(L + 1) \in C^{L' \times 1}$ due to the unitary property of singular vectors. Thus, we have an equivalent system in which the $L'$ symbols are transmitted via the respective independent parallel singular channels. Note that the $i$-th channel tap is equal to the $i$-th singular value.

Since the singular values are independent of the $h_{SD}$ channel taps we can use a combination of (1) and (5) to extract higher diversity gain. More precisely, for each symbol that was detected correctly from the relays, we have $M$ transmission paths from (1) and one path from (5). So, Maximum Ratio Combining (MRC) is employed on the equivalent SIMO system

$$y'_D = h_{eq}^i x_i + w'_i, \quad 1 \leq i \leq L$$

where $y'_{D_i} = [y'_{D1}(i), y'_{D2}(i), \ldots, y'_{DM}(i)]^T$, $y'_{D2}(i) = (L + 1)$ is the $i$-th element of $y'_{D1}$, $h'_{eq} = [h_{SD}^T(i), \sigma_i]^T$, $\sigma_i$ is the $i$-th singular value, $w'_i = [w^T(i), w'_i(L + 1)]$ and $w'_i(L + 1)$ is the $i$-th element of $w'(L + 1)$ so as to extract diversity gain from the independent transmission paths.

We should note that if a symbol is not correctly detected in one of the relays, then this relay cannot be used for transmission during period T2. This is because, if one or more relays forward erroneous symbols, matrix $\Sigma$ of equation (5) will no longer be diagonal and hence the system will have inferior performance.

As it is shown in Section 3, the performance of the system is degraded by these detection errors at the relays and we must employ multi-user communication or relay selection methods in order to achieve the maximum performance that the system can provide.

III. SINGLE-USER CASE: PERFORMANCE ANALYSIS

In this section we present a performance analysis of the proposed technique. Note that in [4] an initial version of the SURBUT was presented and the resulting Diversity-Multiplexing Tradeoff (DMT) was studied under the assumption that no detection errors occur at the relays. For that case it was proved that the system achieves diversity gain equal to $(M - L + 1)(N - L + 1) + M$.

In a real world system, the assumption that the transmission to the relays is "error-free", is of course not correct and the performance analysis carried out at [4] is actually an upper bound for the performance of the system. To compute the
performance of the system more accurately we must see how fast the outage probability tends to zero as the value of the Signal-to-Noise Ratio (SNR) tends to infinity assuming that decoding errors do occur at the relays.

Let us begin by simply considering the case where an outage event occurs in one relay. According to the system description the relay in which the outage event appeared cannot be used to forward data since it will not be able to decode successfully the data received from the source. Thus, the symbols during time period T2 can be forwarded to the destination through the remaining $N - 1$ relays. Therefore, now the outage of the overall system is, the outage of the equivalent SIMO system (see eq. (6)) constructed of the M direct source-destination channels plus the singular channel of the $(N - 1) \times M$ virtual MIMO system. In a similar manner, if an outage event occurs in K of the available relays, then the L symbols are forwarded from the remaining $N - K$ relays, provided that the inequality $N - K \geq L$ is satisfied. In case the aforementioned inequality is not true, then, as it was already mentioned, the number of available relays is not sufficient in order to forward all the L symbols that the relays have received during period T1. In such a case, some symbols are not forwarded at all from the relays and for their detection we can rely only on the direct transmission paths between the source and the destination antennas. Thus, in the latter case the system’s performance is dominated from the performance of this SIMO system. Therefore we deduce that, depending on the number of the relays that suffer an outage probability event we have a different equivalent system with a different outage probability. In order to see the actual outage probability of the overall system we must study the performance of the constituent systems and see whose performance dominates the performance of the overall system. The following theorem refers to the performance of the overall system in the DF case.

**Theorem 1.** The diversity order of a cooperative system with $N$ relays and $M$ destination antennas employing the SURBUT technique under the DF protocol is $N + M - L + 1$.

**Proof:** Let us first provide some notations. With $P_{\text{out}}^{R_{k}(N-K)}$ we denote the probability that an outage event occurs in K relays, with $P_{\text{out}}^{\text{eq}_{R_{k}(N-K)}}$ we denote the outage probability of the overall equivalent system (of eq. (6)) when $N - K$ relays do not exhibit an outage event, with $P_{\text{out}}^{\text{SIMO}}$ we denote the outage probability of the SIMO system consisted only of the direct paths between the source and the destination antennas, and with $P_{\text{out}}^{\text{SISO}}$ we denote the outage probability of a SISO system (as the one between the source and a relay). We assume that every channel tap can be modeled as an i.i.d. Rayleigh variable. So, the overall system performance can be written as

$$
P_{\text{out}} = \left(\begin{array}{c}
P_{\text{out}}^{R_{1}(N-1)} \\
\vdots \\
P_{\text{out}}^{R_{k}(N-K)}
\end{array}\right) \left(\begin{array}{c}
1 - P_{\text{out}}^{R_{k}(N-K)}
\end{array}\right) P_{\text{out}}^{\text{eq}_{R_{k}(N-K)}} +
$$

for $N - K < L$

$$
\vdots
$$

for $N - K \geq L$

$$
\left(\begin{array}{c}
P_{\text{out}}^{R_{N}(N)}
\end{array}\right) P_{\text{out}}^{\text{eq}_{R_{N}(N)}} +
$$

We can easily see that the term $P_{\text{out}}^{R_{k}(N-K)} (1 - P_{\text{out}}^{R_{k}(N-K)})$ can be written as $(P_{\text{out}}^{\text{SIMO}})^{K} (1 - P_{\text{out}}^{\text{SIMO}})^{N-K}$, where $P_{\text{out}}^{\text{SIMO}} \sim 1/\text{SNR}$ as $\text{SNR} \to \infty$ (e.g. see [7]). We can also verify that $P_{\text{out}}^{\text{eq}_{R_{k}(N-K)}} \sim 1/\text{SNR}^{M}$ for M receiver antennas and $P_{\text{out}}^{\text{eq}_{R_{N}(N)}} \sim 1/\text{SNR}^{M} (M-L+1)(N-L+1)+M$ for N relays, $M$ receiver antennas and L symbols block size, as $\text{SNR} \to \infty$ according to [7] and [4] respectively. Then equation (7) becomes

$$
P_{\text{out}} = \left(\begin{array}{c}
N \\
\vdots \\
K
\end{array}\right) P_{\text{out}}^{\text{SIMO}} \left(\begin{array}{c}
1 - P_{\text{out}}^{\text{SIMO}}
\end{array}\right) P_{\text{out}}^{\text{eq}_{R_{k}(N-K)}} +
$$

for $N - K < L$

$$
\vdots
$$

for $N - K \geq L$

$$
\left(\begin{array}{c}
N \\
\vdots \\
K
\end{array}\right) P_{\text{out}}^{\text{SIMO}} \left(\begin{array}{c}
1 - P_{\text{out}}^{\text{SIMO}}
\end{array}\right) P_{\text{out}}^{\text{eq}_{R_{N}(N)}} +
$$

where $\Phi(K) = (M-L+1)(N-L+1)+M$. From equation (8) we can see that the term that dominates the decaying speed of the outage probability of the system, as $\text{SNR} \to \infty$, is the $\left(\begin{array}{c}
K \\
\vdots \\
N
\end{array}\right) P_{\text{out}}^{\text{SIMO}} \left(\begin{array}{c}
1 - P_{\text{out}}^{\text{SIMO}}
\end{array}\right) P_{\text{out}}^{\text{SIMO}}$ one for

$$
\sim 1/\text{SNR}^{M} (N-L+1)+M
$$

for $N - K < L$

and the diversity order achieved by the proposed scheme under the DF protocol is $M + N - L + 1$.

By inspecting equation (8) we observe that the term $(1 - P_{\text{out}}^{\text{eq}_{R(N)}}) P_{\text{out}}^{\text{eq}_{R(N)}}$ is an upper bound for the system performance in case that no decoding errors occur at the relays.

\footnote{The decaying speed of the factors in the terms of (8) are indicated just below the corresponding horizontal braces.}
IV. Multi-User Case: System Description

As expected and shown analytically in the previous Section, the performance of the SURBUT technique is degraded by the decoding errors that occur at the relays. A possible means to cope with this problem is to employ a proper relay selection technique, since, as shown in literature, such techniques may improve considerably the performance of cooperative schemes. Here, instead of resorting to a relay selection technique, we investigate an alternative approach to improve performance via exploitation of multi-user diversity. The existence of multiple user’s in a system gives the opportunity to select the “best” one at each transmission slot, according to a proper criterion, and thus improve the performance of the system. The more users are available, the greater the possibility is for one of them to have relatively good channels thus increasing the performance of the overall system. At this point, it should be noted that, unlike the relay selection techniques which require specific setup of the wireless network, the above multi-user based strategy seems to be more appropriate for a real world wireless network in which an excessive number of users compete to gain access to the wireless medium. Moreover, the performance of the multi-user system can be further enhanced by employing relay selection techniques. A discussion with respect to the latter issue is provided in Section 7.

Let us consider that there are $W$ users willing to transmit at each time slot. We assume that we have a fixed set of relays that will help the users to transmit their data to the multi-antenna destination. We can consider the relays either as available users or as cheap single antenna base stations. In the latter case, we can use cells that cover large areas with only one multiple antenna base station, that has a relative high cost, and then place in specific locations the single antennas base stations in order to lower the cost of the network construction. In such a case, special care should be taken so that the relays - destination channels have a relatively good condition.

In either case, the "weak" parts of the system are the source-relays channels, since they are the only SISO ones. Recall that for a SISO channel the probability of outage decays very slowly, as $1/SNR$. In order to deal with this problem we will exploit the opportunistic nature of multi-user systems. In case a TDMA system is assumed, then at each time slot the user that maximizes the performance of the system, under a criterion, is scheduled to transmit. In other words, the relays’ network is provided to the user with the best user-relays channel.

Before proceeding, we should note that the optimal strategy would be to select the user with the best user-relays and user-destination channels. We can easily verify that $P_{out}^{SISO} << P_{out}^{SMO}$ as the number of the destination antennas $M$ increases. Thus, for simplicity and given that all channel taps are assumed to follow the same distribution, the users are scheduled according to the quality of the worst user-relay channel. In other words, at each time slot, the transmitting user is the one whose worst user-relay channel has the best quality.

More formally, the criterion for selecting the user $S^*$ to transmit out of $W$ users is given by

$$S^* = \arg \max_i \left\{ \min_j \left\{ |h_{S,R_j}|^2 \right\} \right\}, \quad (10)$$

for $1 \leq i \leq W$ and $1 \leq j \leq N$.

The suitability of the above suggested criterion is investigated in Section V where a performance analysis of the technique is carried out in a multi-user environment.

V. Multi-User Case: Performance Analysis

The following theorem refers to a performance measure of the multi-user relay-based uplink transmission (MURBUT) technique of the previous section.

**Theorem 2**: The diversity order of a cooperative system with $N$ relays and $M$ destination antennas employing the MURBUT technique under the DF protocol in a multi-user environment with $W$ users is equal to $W(N - L + 1) + M$.

**Proof**: Adopting the user selection criterion of equation (10) implies that an outage event in a source-relay channel will be upper bounded by the probability of the worst channel of the scheduled user. Since the users are scheduled according to the quality of their worst channel, it is easy now to see that $P_{out} \{ R^{(1)} \} \leq P_{out}^{bw}$, where $P_{out}^{bw}$ denotes the outage probability of the worst source-relay channel of the best user. It can be proved that $P_{out}^{bw} \sim 1/SNR^W$ if the number of potential users equals $W$. Replacing this in (8) we deduce that

$$P_{out} \sim \frac{1}{SNR^{W(N - L + 1) + M}}$$

hence the diversity order achieved by the proposed scheme under the multi-user DF case is equal to $W(N - L + 1) + M$.

Equations (8) and (11) imply that a proper exploitation of the multiple users leads to performance improvement as compared to the SURBUT case. The more users are available for transmission, the better the performance is. However, as we saw in Section 3 there is an upper bound in the performance of the overall system. In other words there is a maximum number of users over which any further increase of users offers no gain in system performance in terms of diversity order. We can compute this users’ number based on the following Corollary.

**Corollary 1**: The multi-user system attains its maximum performance if the number $W$ of potential users (i.e., users willing to transmit) is at least

$$W = (M - L + 1) \quad (12)$$

**Proof**: As it was already stated in Section 3 the term that determines the maximum diversity gain that the system can achieve is $\left\{ 1 - P_{out} \{ R^{(N)} \} \right\} P_{out}^{bw} \{ R^{(N)} \}$. Thus, in order to compute the number of users required to achieve this diversity gain we must equate the diversity gain of this term with the one of Theorem 2 that determines the performance of the system in the multi-user DF case. So, we have

$$W (N - L + 1) + M = (M - L + 1) (N - L + 1) + M \Rightarrow W = (M - L + 1)$$

$^2$We skip the proof due to space limitations.
which is the desired number of users.

VI. COMBINING THE MURBUT TECHNIQUE WITH RELAY SELECTION SCHEMES

In this Section the incorporation of a relay selection scheme to the MURBUT technique is briefly discussed. Up to this point, we have studied the way that multi-user diversity enables the suggested technique to combat the decoding errors at the relays and thus enhance its performance. A reasonable question is whether a relay selection scheme would further improve the performance of the technique. Let us consider that the SURBUT technique is to be applied in a system with $N$ relays, $M$ receiver antennas and $L$ symbols’ block size. Let us also assume that there are available $Q$ additional relays. It is clear that, a relay selection criterion needs to be adopted for selecting $N$ relays out of the $N+Q$ ones. A simple criterion is the selection of the best $N$ relays based on the quality of the corresponding user-relays channels. We note again that we emphasize only in the source relay-channels due to their low diversity as they are the only SISO ones in the overall system. Theoretical analysis and simulations (not presented here due to space limitations) have shown that this relay selection scheme (with $Q$ additional relays) attains the same performance with the MURBUT technique with $Q$ available users.

The incorporation of the above relay selection scheme to the MURBUT technique is now straightforward. For each available user (out of $W$) the best $N$ relays are selected and then the user with the best worst channel is scheduled to transmit. Again, theoretical analysis and simulations have shown that the performance enhancement is in this case proportional to $Q \times W$.

VII. SIMULATIONS

In Figure 2 the probability of outage is depicted for a system with 2 relays and 2 receiver antennas for 1, 2, 4 and 16 available users. In the same figure the performance of the system which is attained when no decoding errors occur at the relays is also depicted (max_per_sim). For clarity, the theoretical curves for the one user case (min_theor) and the ideal error-free case (max_theor) are also plotted in the figure. As we can see, in the single-user case the system achieves a diversity gain equal to $M-L+N+1$, thus confirming the result of Theorem 1. The existence of $W$ users enhances the performance, as stated in Theorem 2, leading to a system with diversity $W(N-L+1)+M$. According to Corollary 1, in the case of the examined system, 2 users are required in order for the system to achieve its maximum performance. As shown in Figure 2, the system with the 2 users does attain the maximum diversity order, though the outage curve is worse than the ideal case where no decoding errors occur at the relays.

Even though the maximum diversity order is achieved for the number of users specified by Corollary 1, the expression that gives the outage probability of the system still includes all the terms of (8). These terms are decaying with a rate that depends on the number of potential users. That is, as the number of users becomes larger the aforementioned terms are decaying faster with an increase in the number of the users, enabling the system to approach the performance of the ideal one given by the term $P_{out}^{eq} \{ R(N) \}$.

VIII. CONCLUSION

A new Single-User Relay-Based Uplink Transmission technique has been presented and its performance has been studied for the DF protocol case. The technique has been extended to the multi-user case and the performance of the resulting so-called MURBUT technique was studied theoretically. As it has been proved the technique exploits properly the multi-user diversity so as to regain the performance loss due to the decoding errors that occur to the relays. The theoretical analysis was verified by typical simulations.

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REFERENCES