Localization using Radial Basis Function Networks and Signal Strength Fingerprints in WLAN

C. Laoudias*, P. Kemppi†, C. G. Panayiotou*

*KIOS Research Center for Intelligent Systems and Networks
Department of Electrical and Computer Engineering, University of Cyprus
Kallipoleos 75, P.O. Box 20537, 1678, Nicosia, Cyprus
Email: {laoudias, christosp}@ucy.ac.cy
†VTT Technical Research Centre of Finland
Vuorimiehentie 3, P.O. Box 1000, FIN-02044, Espoo, Finland
Email: Paul.Kemppi@vtt.fi

Abstract—Fingerprinting localization techniques provide reliable location estimates and enable the development of location aware applications especially for indoor environments, where satellite based positioning is infeasible. In our approach we utilize Received Signal Strength (RSS) fingerprints collected in known locations and employ a Radial Basis Function (RBF) neural network to approximate the function that maps fingerprints to location coordinates. We present a clustering scheme to reduce the size and computational complexity of the RBF architecture and demonstrate the applicability of this approach in a real-world WLAN setup. Experimental results indicate that the RBF based method is an efficient approach to the location determination problem that outperforms existing techniques in terms of the positioning error.

I. INTRODUCTION

Localization techniques enable the provision of location information regarding people, mobile devices and equipment. Estimating location accurately is a challenge especially inside buildings, where satellite-based positioning is not applicable due to the severe attenuation or blockage of satellite signals. Positioning accuracy is the key issue to effectively support advanced indoor location aware services. Indicative applications include in-building guidance, asset tracking in hospitals or warehouses and autonomous robot navigation.

A wide variety of localization techniques based on radio signal propagation models have been studied [1] and several types of measurements that relate the position of a mobile device to the known positions of fixed transmitters can be utilized. These measurements include Angle of Arrival (AOA), Time of Arrival (TOA), Time Difference of Arrival (TDOA) and Received Signal Strength (RSS). Even though relatively accurate propagation models are available for open areas, the presence of non line-of-sight paths between the receiver and the transmitter introduces additional measurement noise. Especially indoors, multipath conditions are common and lead to further accuracy degradation. An overview of technologies for wireless indoor location systems is detailed in [2].

Fingerprinting techniques address the issue of multipath propagation by utilizing fingerprints collected a priory in the entire area of interest and stored in a database. These fingerprints are associated with a set of predefined reference points and contain location related information, such as RSS measurements or Power Delay Profile (PDP) parameters. The unknown location is estimated by using the currently measured fingerprint to find the best match in the database. Matching is based on a distance measure between the current and collected fingerprints or on probability distributions.

Artificial Neural Networks (ANN) [3] provide an alternative solution to the location determination problem. ANNs comprise a number of non-linear transformation units, i.e. neurons and a sufficiently large set of free parameters, i.e. interconnection weights. In this context, localization can be viewed as a function approximation problem. The ANN exploits the fingerprint database in order to approximate the function that maps the fingerprints from the high dimensional signal space to coordinates in the plane by interpolating the collected data. Our approach is based on a special class of ANNs, called Radial Basis Function (RBF) networks and we utilize location fingerprints that contain RSS measurements from several Access Points (AP) available in WLAN. The contribution of this work is the presentation of an efficient localization method based on RBF networks. We apply a clustering scheme to reduce the network size and computational overhead during the localization process. Experimental results indicate that the proposed method provides more accurate location estimates compared to other approaches. Moreover, the underlying RBF architecture is scalable and can be easily applied to different WLAN setups, in which variable number of APs, reference points or fingerprints may be available.

The rest of the paper is structured as follows. Previous work related to indoor localization using RSS fingerprints is discussed in Section II. The proposed method based on RBF networks is detailed in Section III. In Section IV we present the WLAN experimental setup used for our performance evaluation, followed by the results regarding the positioning accuracy. Finally, Section V provides the conclusions and discusses some ideas for future work.

II. RELATED WORK

Several solutions to the location determination problem using RSS fingerprints have been studied in the literature. These
approaches differ in the underlying localization algorithm, however they all rely on a RSS map that covers the entire area of interest. During the measurement campaign we use a set of predefined reference points \( \ell_i = (x_i, y_i), \) \( i = 1, \cdots, L \) to collect RSS values from \( n \) neighboring APs. A reference fingerprint \( s = [s_1, \cdots, s_n]^T \) is a vector of RSS samples and \( s_j \) denotes the RSS value related to the \( j \)-th AP. A series of reference fingerprints \( s(\ell_i, m), \) \( i = 1, \cdots, L \) and \( m = 1, \cdots, M, \) is collected at each reference point and stored in a database followed by the physical coordinates \((x_i, y_i)\). In the localization step we exploit the reference data in order to obtain a location estimate \( \hat{\ell} \) given a new fingerprint \( s' = [s'_1, \cdots, s'_n]^T \) measured at the unknown location \( \ell \).

In the deterministic techniques location is estimated by minimizing an error function, e.g. the Euclidean distance between \( s' \) and the reference fingerprints in the database. Localization time can be greatly reduced by preprocessing the reference fingerprints collected at \( \ell_i \) to obtain the mean of the combined fingerprints \( \bar{s}(\ell_i) = \frac{1}{M} \sum_{m=1}^{M} s(\ell_i, m) \). The Nearest Neighbor algorithm proposed in [4], [5] is based on this approach. The nearest neighbor is essentially the location with the shortest distance from \( s' \) in the \( n \)-dimensional signal space. The \( K \) Nearest Neighbors (KNN) variant estimates location as the centroid of \( K \) locations with the shortest distances. The Database Correlation Method (DCM) introduces an additional term in the error function to penalize missing RSS values in the fingerprints [6].

On the other hand, probabilistic techniques estimate the unknown location by calculating the conditional probabilities \( p(\ell_i | s') \). Applying Bayes rule the problem reduces to estimating \( p(s' | \ell_i) \). Assuming that RSS measurements from neighboring APs are independent we get \( p(s' | \ell_i) = \prod_{j=1}^{n} p(s'_j | \ell_i) \). The Kernel method introduced in [7] and the Histogram method presented in [7], [8] may be used to estimate \( p(s'_j | \ell_i) \) from the reference data. Probabilistic techniques are reported to achieve higher positioning accuracy at the expense of increased computational complexity.

In the case of ANNs, the objective is to approximate the function that maps RSS fingerprints to locations. The reference fingerprints and corresponding coordinates \((x_i, y_i)\) are employed to train the network and adjust the weights accordingly. Subsequently, a location estimate is obtained when a new fingerprint \( s' \) is presented to the network inputs. Recently, RBF networks have been discussed for indoor localization in Wireless Sensor Networks (WSN) by utilizing distance measurements [9] or series of successive RSS fingerprints [10]. Positioning techniques based on ANNs have also been applied to areas where WLAN infrastructure is available. Authors in [11] propose a Multi Layer Perceptron (MLP) network to perform localization using RSS measurements. A Generalized Regression Neural Network (GRNN) architecture, which is a RBF-type network with slightly different output layer, is evaluated in [12] using RSS values from three transmitters.

Our main contribution is that we provide well defined ways for setting the RBF parameters. The resulting RBF is small enough, easy to train, has good localization performance and avoids over-fitting. In [9], a small scale (3 \times 3m) test bed with low noise and line-of-sight propagation conditions is used, and the standard RBF (sRBF) is reported to perform better than other algorithms. However, sRBF is prone to over-fitting and we show that in a realistic WLAN environment with noisy RSS measurements the proposed clustered RBF outperforms sRBF.

In [10] the size of the RBF network is decided experimentally and even worse, it ends up being excessive compared to our approach. The MLP network [11] is significantly more difficult to train and must be retrained if new data become available. In addition, the size of the network needs to be determined experimentally. Finally, the GRNN method [12] uses a predefined way for setting the network weights, but loses some of the flexibility that other approaches have and thus it has lower performance.

III. RADIAL BASIS FUNCTION NETWORKS

A. Function Approximation with RBF

In general, RBF networks are ANNs that have an input layer, a single hidden layer with non-linear radial, i.e. distance based, basis functions and an output layer. The architecture of a fully connected RBF network is depicted in Fig. 1. The input vector \( x \) is provided as input to all radial basis functions and the output \( f(x) \) is given by

\[
f(x) = \sum_{i=1}^{C} w_i \varphi(||x - c_i||) \tag{1}
\]

where \( ||x - c_i|| \) is the Euclidean distance between \( x \) and the \( n \)-dimensional basis function center \( c_i \). The number of basis functions is \( C \) and \( w_i \) are the network weights. Usually the Gaussian radial basis function is used, i.e. \( \varphi(||x - c||) = \exp(-\beta||x - c||^2) \). We can use RBF networks to approximate any continuous function by fitting the values of the function \( f(x_i) = b_i, \ i = 1, \cdots, C \) at known points \( x_i \). We set the centers of the basis functions equal to \( x_i \) in (1) and then determine the weights by using \( b_i \) and solving the system of linear equations. A RBF approximation example in one dimension is illustrated in Fig. 1.

B. Localization Method

In the context of indoor localization using fingerprints the RBF has \( n \) inputs, corresponding to the RSS measurements.

![Fig. 1. Architecture of a Radial Basis Function network.](image-url)
the RBF network may be expressed as a weighted sum of normalized basis functions, since they provide improved sensitivity to be adjusted. Decreasing \( \beta \) leads to wider basis functions such that there exists more overlap among them. Therefore, when a new fingerprint \( s' \) is present several basis functions will give fairly large outputs, leading to a more accurate location estimate. On the other hand, when \( \beta \) is larger reference data are fitted more sharply. In this case, \( C' \) is larger reference data are fitted more sharply. In this case, \( C' \) may be obtained during localization. The appropriate value must be selected in case additional reference data are collected, e.g. when new reference points are used to cover more rooms. In the cRBF method a heuristic is used to set the width \( \beta \) according to

\[
\beta = \frac{1}{2d_{\text{max}}}
\]

where \( d_{\text{max}} = \max ||c_i - c_j|| \) for \( i, j = 1, \ldots, L \). The intuition is that when the distance among centers in the \( n \)-dimensional signal space increases, the value of \( \beta \) is reduced to ensure that the basis functions still overlap enough to produce accurate location estimates. With this scheme the value of \( \beta \) can be easily adjusted to provide high level of accuracy when a variable number of reference points, reference fingerprints or APs is used; see the experimental results in Section IV-B.}

**C. ANN Implementation Comparison**

The cRBF method is more efficient compared to other ANN based methods. The reduced network size ensures that the weights \( w_k \) are obtained faster compared to the sRBF design. Moreover, the memory overhead for storing the basis function centers and weights, as well as the number of operations required for estimating a location, are reduced by a factor from all \( n \) available APs and two outputs representing the 2-dimensional coordinates. Given a RSS fingerprint \( s = [s_1, \ldots, s_n]^T \) measured at location \( \ell = (x, y) \) the output of the RBF network may be expressed as a weighted sum of normalized Gaussian basis functions

\[
\ell(s) = \sum_{k=1}^{C} w_k u(||s - c_k||) = \sum_{k=1}^{C} w_k \frac{\varphi(||s - c_k||)}{\sum_{j=1}^{C} \varphi(||s - c_j||)} \tag{2}
\]

where \( w_k \) are 2-dimensional weights. Note that we have used normalized basis functions, since they provide improved performance compared to unnormalized functions, especially as the input dimensionality increases. The RBF network is used in order to approximate the function that maps RSS fingerprints to locations in physical space. The parameters \( c_k, \beta \) and \( w_k \) may be determined to obtain a good approximation by optimizing the fit between (2) and the reference data. Thus, we form the following set of equations

\[
\ell_i = \sum_{k=1}^{C} w_k u(||s(\ell_i, m) - c_k||), \quad i = 1, \ldots, L, \quad m = 1, \ldots, M \tag{3}
\]

We calculate \( w_k \) by solving the system of linear equations based on (3) using the reference fingerprints in the database and the corresponding coordinates. Subsequently, the weights \( w_k \) are used during localization to obtain a location estimate \( \hat{\ell} \) given a new fingerprint \( s' \) according to

\[
\hat{\ell}(s') = \sum_{k=1}^{C} w_k u(||s' - c_k||). \tag{4}
\]

Selecting the proper value for \( C \) and choosing the centers \( c_k \), is not trivial and affects the performance of the RBF network. In the standard RBF (sRBF) architecture each reference fingerprint defines a center \( c_k \) so that the total number of basis functions is \( C = L \cdot M \). In this case, the linear system has a unique solution and the sRBF design guarantees exact fitting for all reference data. However, this architecture has high memory requirements since all reference fingerprints, used as centers for the basis functions, are required to perform localization. Moreover, computational complexity is high both for the calculation of \( w_k \) and location estimation.

Several center selection schemes may be applied to reduce \( C \) and design more efficient RBF architectures. For instance, we may sample \( C < L \cdot M \) centers randomly among the reference fingerprints or select the desired number of centers by using the Orthogonal Least Squares algorithm. An alternative solution is to obtain a good value for \( C \) experimentally, as in [10]. However, these approaches may be time consuming and/or do not always ensure the best data fitting. We adopt a clustering method to dramatically reduce \( C' \), while maintaining adequate localization performance. In the proposed clustered RBF (cRBF) architecture the centers are set equal to the mean value fingerprints \( \bar{s}(\ell_i), \quad i = 1, \ldots, L \). In this case, \( C = L \) and the weights \( w_k \) are calculated in a least squares sense by solving the overdetermined system of equations based on (3).

The \( \beta \) parameter is also important in the cRBF approach since it determines the accuracy of the fit between the function approximation in (2) and the reference data. The value of \( \beta \) specifies the width of the Gaussian basis functions and allows their sensitivity to be adjusted. Decreasing \( \beta \) leads to wider basis functions such that there exists more overlap among them. Therefore, when a new fingerprint \( s' \) is present several basis functions will give fairly large outputs, leading to a more accurate location estimate. On the other hand, when \( \beta \) is larger reference data are fitted more sharply. In this case, the errors between the cRBF model and the reference data are minimized, however highly inaccurate location estimates may be obtained during localization. The appropriate \( \beta \) value is usually selected experimentally based on the reference data and can be further fine-tuned if testing data are available. However, this approach can be time consuming and a new value must be selected in case additional reference data are collected, e.g. when new reference points are used to cover more rooms. In the cRBF method a heuristic is used to set the width \( \beta \) according to

\[
\beta = \frac{1}{2d_{\text{max}}}
\]
of $M$. Therefore, the proposed method based on cRBF is attractive when localization is performed on a mobile device with limited memory and processing power.

The GRNN design is similar to sRBF and each reference fingerprint defines a basis function center in the architecture. However, contrary to sRBF, the weights are set equal to the coordinates of the respective reference points in (4). GRNN calculates the distance between the fingerprint $s'$ and the centers, thus interpolating the location estimate $\hat{\ell}$ according to the fingerprint database. In this approach no time is spent to obtain $w_k$, but the memory requirements and computational complexity during localization are the same with sRBF.

The MLP network uses the sigmoidal function $f(x) = 1/(1 + \exp(-x))$ in the hidden layer. This function can have non-zero outputs over a large region of the input space, as opposed to Gaussian radial basis functions that respond to relatively small regions. Therefore, only a small set of weights is required to fit the reference data and memory overhead as well as localization time are significantly reduced. The major drawbacks in this approach is that the size of the MLP network can only be decided experimentally and long training time is needed to calculate the weights with the back propagation gradient descent learning algorithm. MLP training suffers from local minima, while in RBF networks linearity ensures that optimum weight values are easily found. Another disadvantage of the MLP is that it must be retrained in case additional reference fingerprints are collected, as opposed to RBF networks where retraining time can be greatly reduced by using appropriate matrix operations.

IV. PERFORMANCE EVALUATION

A. Measurement Setup

The localization trial was carried out in a typical office environment at the premises of VTT Technical Research Centre. The size of the 3-storey building is roughly 110m×45m. Each floor consists of 8 wings containing office rooms, open plan offices and meeting rooms connected with corridors. The measurement campaign was conducted in the second floor using 107 distinct reference points. These points are located 2-3 meters apart from each other and form a grid that covers all the public spaces and meeting rooms. There are 10 Cisco Aironet APs installed in this floor that use the IEEE802.11b/g standard. The reference points are depicted in Fig. 3 and the color scheme denotes the number of hearable APs at each reference point. We used a Fujitsu-Siemens Pocket Loox smart phone to collect RSS measurements with 1dBm resolution due to hardware limitations. Typical RSS values range from -101dBm to -34dBm in close proximity to an AP. We used a small constant (-110dBm) to handle the missing RSS values in the fingerprints. A total of 3210 reference fingerprints were collected, corresponding to 30 fingerprints per reference point and stored in the database with the physical coordinates. Fingerprints were also collected by following a predefined route that consists of 192 locations. One fingerprint is recorded at each location and the same route is sampled 3 times.

B. Experimental Results

We evaluate the performance of the cRBF architecture in terms of the positioning error, defined as the Euclidean distance between the actual and estimated location. All fingerprints collected by sampling the same route multiple times are utilized to obtain accuracy results. Unless otherwise noted, fingerprints contain RSS values from $n = 10$ WLAN APs and all reference fingerprints, i.e. $L = 107$ and $M = 30$, are stored in the database and used by the localization methods.

We have implemented the MLP, GRNN, sRBF and cRBF architectures, as well as the deterministic DCM algorithm and the probabilistic method (PROB). For DCM we used the 2 Nearest Neighbor approach, while PROB is based on the Kernel method described in [7]. Accuracy results are summarized in Table I and cRBF has the best localization performance according to the mean and median error. The standard deviation (Std) of the error is also low. This is followed by PROB, GRNN and DCM. The PROB method slightly improves accuracy in the given setup compared to DCM and does not justify the computational overhead. MLP and sRBF provide less accurate location estimates. These results indicate that cRBF is a promising localization method that performs better than other ANN approaches.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>POSITIONING ERROR [M]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MLP</td>
</tr>
<tr>
<td>Min</td>
<td>0.0</td>
</tr>
<tr>
<td>Max</td>
<td>23.7</td>
</tr>
<tr>
<td>Mean</td>
<td>4.6</td>
</tr>
<tr>
<td>Median</td>
<td>3.8</td>
</tr>
<tr>
<td>Std</td>
<td>3.7</td>
</tr>
</tbody>
</table>

\(^1\)The MLP network has ten inputs, two outputs and the implementation that gave the best results in the given setup consists of a single hidden layer with 30 sigmoidal functions.
The positioning error in the cRBF location estimates with respect to the value of $\beta$ is plotted in Fig. 4. If $\beta$ is large the basis functions are very narrow and reference data are fitted more sharply. The resulting implementation minimizes the error between the cRBF approximation and the reference data, however when a new fingerprint is presented only few basis functions respond and the estimates can be inaccurate. On the other hand, if $\beta$ is small the basis functions are wide and the reference data are fitted very smoothly. In this case, each basis function is essentially responding in the same, large region of the input space and the cRBF performance during localization is degraded. The curve in Fig. 4 indicates that a proper value may be selected in order to minimize the mean error in the location estimates. The heuristic introduced in Section III-B, based on the maximum distance among centers in the input space, ensures that the basis functions have the appropriate width. This is used in our experiments to set the value of $\beta$ accordingly.

In the following we further investigate the performance of the proposed cRBF method compared to DCM. The actual and estimated locations, obtained with cRBF and DCM, are depicted in Fig. 5 for a single route. The cRBF location estimates are very accurate and reflect the actual traveled route. The Cumulative Distribution Function (CDF) of the positioning error is plotted in Fig. 6. Accuracy of less than 5m is achieved for 80% of the cRBF estimates, while the mean error is 3.4m for cRBF and 4.0m for DCM. Approximately 5% of the DCM estimates have positioning error higher than 10m and the maximum error is 13.1m and 21.4m for cRBF and DCM, respectively. The cRBF method can alleviate high positioning errors in the location estimates and outperforms DCM in terms of estimation accuracy.

Collecting reference data to build the RSS map is a cumbersome task and it is a major restriction in fingerprint based localization techniques. There is a trade off regarding the time spent on data collection and the positioning accuracy. We address this issue by using a variable number of reference points or fingerprints and investigate the performance of cRBF method. First, we increase the number of reference points $n$ and each fingerprint $\ell_i$, which are selected to cover the whole floor. This affects the size of the cRBF architecture and the localization performance is illustrated in Fig. 7. In all cases, cRBF achieves higher accuracy compared to DCM. For $L = 60$ the mean error in the cRBF estimates falls below 4m and when $L = 80$ the accuracy is approximately 3.5m. Increasing the value of $L$ further does not provide significant improvement.

The mean error achieved by DCM and cRBF, with respect to the number of reference fingerprints $M$, is depicted in Fig. 8. Even when only 5 or 10 fingerprints are collected at every reference point, the cRBF performs better and the mean error is 3.7m. By increasing the value of $M$ the improvement observed in the DCM accuracy is marginal. On the contrary, the performance of cRBF is improved by using more fingerprints and the error is about 3.4m if 25 fingerprints are used. This can be justified as follows. The number of reference fingerprints $M$ affects the center of each Gaussian basis function, which is set equal to the mean value fingerprint $\bar{\ell}(\ell_i)$. The cRBF method is more sensitive to the value of distance between the new fingerprint $s'$ and each $\bar{\ell}(\ell_i)$ compared to the DCM algorithm, due to the exponential basis function. Thus, when more fingerprints are available per reference point the centers are closer to their actual values and cRBF performs better.

Results in the previous experiments show that the proposed method can provide adequate level of accuracy without excessive data collection time requirements. The number of reference points can be reduced and few fingerprints can be collected without significant performance degradation. On the other hand, the number of APs in the area of interest affects the size of the input layer in the cRBF architecture. We use a variable number of APs $n$, which are selected to provide good coverage for the whole route; see Fig. 9. As expected, by increasing $n$ the positioning error is decreased since the fingerprints contain RSS measurements from more APs. By using 6-7 APs the positioning error is about 4m. The number of APs is related to the dimensionality of the signal space and the cRBF architecture can map RSS fingerprints to location coordinates more accurately.

V. CONCLUSIONS

The proposed cRBF algorithm is a promising fingerprint based technique that provides more accurate location estimates compared to other localization methods. Experimental results indicate that adequate level of accuracy is achieved even with limited number of reference points, collected fingerprints or installed APs. The proposed method is scalable and can be easily applied to other environments. The heuristic used to set the value of $\beta$ ensures that under different conditions the cRBF design interpolates the reference data smoothly and performs well during localization.

Our future work includes the application of the cRBF approach to other WLAN environments in order to verify the localization performance. We also plan to integrate this method into an indoor positioning platform. We emphasize that the proposed cRBF is a ANN design which is simple to train and is very efficient after training, thus it can be easily incorporated into real time applications. Both, network and user-plane positioning architectures can be used. In a network-based architecture, a mobile terminal sends the currently observed RSS fingerprint to the positioning server which responds with the location coordinates. However, we envision a system where
the previously collected fingerprints will be used to train the cRBF off-line and determine the weights. Once a terminal enters an area, it receives from a server this small set of weights and using the currently measured fingerprints it can localize and track itself independently thereafter.

ACKNOWLEDGMENT
This work is partly supported by the Cyprus Research Promotion Foundation under contract ENIΣΧ/0506/59 and the European Science Foundation (ESF) in the framework of the Middleware for Network Eccentric and Mobile Applications (MiNEMA) activity.

REFERENCES