A Novel Transmitter-Based Selective-Precoding Technique for DS/CDMA systems

C. Masouros and E. Alsusa
School of Electrical & Electronic Engineering
The University of Manchester
Manchester, UK
email: Chris.Masouros@postgrad.manchester.ac.uk, E.Alsusa@manchester.ac.uk

Abstract—In this paper a new transmitter precoding technique is presented that outperforms conventional precoding by making use of a portion of the interference between the users in a CDMA system downlink. The proposed technique selectively pre-decorrelates users that are experiencing destructive interference while allowing interference to other users when it is expected to contribute to their signal. The existence and exploitation of constructive interference effectively spreads the signal constellation and enhances the SNR at the receiver. The SNR improvement happens by making use of energy that is already in the system so the performance improvement is attained with no additional power-per-user investment. This however comes with the trade-off of some extra processing at the transmitter for the measurement of the expected interference. The proposed technique applies to the downlink of cellular CDMA systems employing PSK modulation. Theoretical analysis supported by comparative simulations of this and other precoding methods are presented and discussed.

Keywords—Adaptive signal processing, code division multi-access, interference multiuser channels, suppression

I. INTRODUCTION

The capacity of a code-division multiple-access (CDMA) system is limited by multiple access interference (MAI) from other users as well as intersymbol interference (ISI) between the symbols of the user of interest due to the frequency selectivity of the transmission medium. Multiuser Detection (MUD) techniques are traditionally used to mitigate these effects and improve the performance and capacity of CDMA systems. In order for complexity reduction at the Mobile Units (MUs) of a CDMA communication system, the current trend is towards transferring the computational burden to the Base Station (BS) by use of precoding techniques for the downlink transmission. Various methods have been proposed towards this end following an initial idea introduced in [1] applicable to general pulse amplitude modulation (PAM) systems. In [2] the authors propose transferring the Rake processing to the BS which yields the Pre-Rake technique. This technique’s main advantage is that matched filtering (MF) is applied for detection which alleviates the need for channel estimation and removes a significant burden from the MU. However, this is primarily a single user detection (SUD) technique so performance is poor in a multiuser scenario. The authors in [3] propose a system similar to the conventional receiver-based decorrelator-detector where the decorrelation procedure happens at the BS prior to transmission. The orthogonalization of the users comes with an increase in transmitted energy which calls for either scaling of the signal to be transmitted or applying constrained optimization in order for the power limitation to be maintained. Both these techniques are investigated in [3]. An improvement is attained by applying the decorrelating procedure in [4]. This optimization leads to the use of a decorrelation scheme that also employs Pre-Rake processing. This method offers both the benefits of pre-decorrelation as well as the advantages of Pre-Rake over the Rake technique as explained in [2]. The decorrelating methods introduced in [3,4] are blockwise, which results in a high computational complexity. In aim of mitigation of this defect, the authors in [5] propose a zero forcing bitwise decorrelating technique that has comparable performance when the ISI is limited by use of guard intervals. When severe multipath is introduced, however, the performance rapidly deteriorates. An improved bitwise technique using minimum mean square error (MMSE) optimization is presented in [6] that achieves performance comparable to [3,4] while maintaining reasonable computational complexity.

In this paper an improvement of the Transmitter Precoding (TP) and Joint Transmission (JT) techniques in [3,4] respectively is suggested which takes advantage of the constructive interference concept applicable in PSK modulation that will be analyzed below. The system analysis follows the one presented in [3]. In contrast to the TP and JT techniques where the users are fully orthogonalized, we propose a partial orthogonalization by means of pre-decorrelation, orientated to the users that are expected to suffer from destructive MAI. This reduces complexity and yields an increased signal to noise ratio (SNR) at the receiver. It should be noted that this technique is applicable to both the TP and JT methods irrespective of whether scaling or constrained optimization are used, assuming phase shift keying (PSK) modulation where the constructive MAI concept stands. In this paper unconstrained optimization followed by scaling is investigated.

II. CONSTRUCTIVE MAI DERIVATION FOR PSK MODULATION

A. Downlink Signal Model and Constructive MAI Definition

Consider the downlink transmission in a discrete-time synchronous frequency selective CDMA system of $K$ users, where the channels’ path delays are assumed to be an integer
number of chip periods. All codes and channels are assumed to have normalized energy of one and length of \(L\) and \(P\) chips respectively. The data frame is \(N\) symbols long. \(T_b\) and \(T_c\) are the symbol and chip periods respectively. The received signal at the \(u\)-th MU can be expressed as:

\[
r_u(t) = \sum_{i=1}^{N} \sum_{k=1}^{K} \sum_{p=1}^{P} a_{k} x_{i}(i) c_4(t - iT_b - pT_c) h_{pu}(i) + n_u(t)
\]

where \(x_{i}(i)\), \(a_{k}\), \(c_4\) are the \(k\)-th user’s PSK modulated data symbol for the \(i\)-th symbol period, amplitude and code, \(h_{pu}(i)\) and \(n_u(t)\) are the \(u\)-th MU’s channel \(p\)-th tap coefficient and additive white Gaussian noise (AWGN) corrupting the signal of interest. The output of a Rake receiver of the \(u\)-th user can be expressed as:

\[
d_u(i) = \sum_{p=1}^{P} \int \frac{h_{pu}(i) r_u(t) c_4(t - iT_b - pT_c)}{dt}
\]

\[
= a_{u} x_{iu} + ICI_{iu} + ISI_{iu} + MAI_{iu} + \eta_{iu}
\]

Here \(x_{iu}\) is a compact representation of the desired \((u\)-th\) user’s signal for the \(i\)-th period of interest, \(ICl_{iu}\) is the InterChip Interference between adjacent chips, \(ISI_{iu}\) is the InterSymbol Interference caused by adjacent symbols, \(MAI_{iu}\) is the Multiple Access Interference caused by the other \(K\)-1 users and \(\eta_{iu}\) is the noise component at the Rake output. If a discrete time representation is adopted and \(T_b\), \(T_c\) can be omitted the above quantities are defined as:

\[
ICl_{iu} = a_{u} x_{iu} \sum_{l=1}^{L+P-1} s_{iu}[l] s_{iu}^*[l] - a_{u} x_{iu}^2
\]

\[
ISI_{iu} = a_{u} \sum_{n=-1}^{N-1} x_{iu}[n] \sum_{l=1}^{L+P-1} s_{iu}[l] s_{iu}^*[l+nL]
\]

\[
MAI_{iu} = \sum_{n=-1}^{N-1} \sum_{k=1}^{K} a_{k} x_{iu}[k] \sum_{l=1}^{L+P-1} s_{ku}[l] s_{ku}^*[l+nL]
\]

\[
= \sum_{n=-1}^{N-1} \sum_{k=1}^{K} a_{k} x_{iu}[k] \rho_{ku}
\]

\[
s_{ku} = \sum_{p=1}^{P} c_{kp} h_{pu}
\]

respectively, where \(c_{kp}\) is the \(k\)-th user’s delayed (by \(p\) chips) version of the signature waveform and \(\rho_{ku}\) is the crosscorrelation of the users’ multipath corrupted signature waveforms (\(s_{iu}\)) in (6).

The MAI is constructive when it adds to the desired user’s signal energy, thus improving the effective SNR. When MAI is constructive, \(|\eta_{iu}| > |a_{u} x_{iu}| + |MAI_{iu}|\) is required for an error, while when MAI is destructive \(|\eta_{iu}| > |a_{u} x_{iu}| - |MAI_{iu}|\) is enough to lead to erroneous decision. Having the above in mind, the SNR (for PSK modulation) instead of:

\[
SNR = \frac{S}{MAI + N}
\]

can effectively be written as:

\[
SNR_e = \frac{S + MAI_{constructive}}{MAI_{destructive} + N}
\]

In the following, constructive MAI is derived for binary PSK (BPSK) and generalized to \(M\)-ary PSK (MPSK) modulation.

B. Constructive MAI Derivation for BPSK Modulation

For BPSK modulation the desired user’s signal \(x_{iu} \in \{-1, 1\}\), so constructive is the \(MAI_{iu}\) that has the same sign as \(x_{iu}\), as depicted in the shadowed part of the BPSK constellation diagram in Fig. 1a. When the multipath delay spread is not large and MAI from previous and next symbols can be neglected, this, using (5), leads to:

\[
a_{u} x_{iu} \sum_{k=1}^{K} a_{k} x_{ia} \rho_{ku} > 0
\]

The above equation can be expressed using matrices representation as:

\[
\sum(M_{i} - \text{diag}(M_{i})) > 0
\]

where \(M_{i}\) is the crosscorrelation matrix of the codes in (6), modulated by the data, defined as:

\[
M_{i} = \text{diag}(x_{i}) R \text{diag}(x_{i})
\]

In (10), \(A=\text{diag}(a_{k})\), \(k=1...K\) is the diagonal matrix of amplitudes, \(x_{i} = [x_{i1}, x_{i2}, ..., x_{iK}]\) is the data matrix for the \(i\)-th symbol period and \(R\) is the \(K\times K\) crosscorrelation matrix of the codes in (6), defined as:

\[
R = \begin{pmatrix}
1 & \rho_{12} & \cdots & \rho_{1K} \\
\rho_{21} & 1 & \cdots & \rho_{2K} \\
\vdots & \ddots & \ddots & \vdots \\
\rho_{K1} & \rho_{K2} & \cdots & 1
\end{pmatrix}
\]
Therefore, $M_i$ is the matrix:

$$
M_i = \begin{pmatrix}
    a_i^2 & a_i x_{i1} x_{i2} & \cdots & a_i x_{i1} x_{iK} \\
    a_i x_{i1} x_{i2} & a_i^2 & \cdots & a_i x_{i2} x_{iK} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_i x_{i1} x_{iK} & a_i x_{i2} x_{iK} & \cdots & a_i^2
\end{pmatrix}
$$

(12)

Each element of $M_i$ provides information on the interference between two users. By taking the sign of $\text{sum}(M_i - \text{diag}(M_i))$ for each column, one can derive whether the cumulative MAI component is constructive or destructive for each user, for the $i$-th symbol period of interest. In a multipath scenario where the delay spread is considerable and severe ISI is present the previous and next symbols need be taken into account for accurate MAI derivation. The expansion to a blockwise investigation using $NK \times NK$ crosscorrelation matrices expression as in [3] for this case is straightforward.

C. Constructive MAI Derivation for MPSK Modulation

From the above analysis and by observing the constellation diagrams in Fig. 1, it can be derived that constructive is the MAI that causes the noiseless part of the received signal to fall in the part of the constellation diagram where the distance from the decision thresholds is increased. Applying this rule to QPSK and 8PSK yields the shadowed part in Fig 1.b,c. The generalization to MPSK is simple.

III. SELECTIVE PRECODING METHOD ANALYSIS

In the following analysis BPSK modulation is investigated for simplicity and the spreading sequences are assumed normalized to unit energy. As shown in [3,4] when precoding is applied the transmitted signal is given as:

$$
g = fxATC \quad \text{for TP} \\
g = fxATCH^T \quad \text{for JT}
$$

(13)

where $A=\text{diag}([a_1, a_2, \ldots, a_K])$, is the diagonal matrix of amplitudes, $x=[x_1, x_2, \ldots, x_K]$ is the data matrix, $T$ is the precoding matrix, $C=[c_1, c_2, \ldots, c_K]^T$ is the matrix containing the users’ codes and $H=[H_1, H_2, \ldots, H_K]^T$ is the channel matrix. $f$ is the scaling factor that ensures that the average transmitted power per user is equal to the one without precoding and is given as in [3] by:

$$
f = \sqrt{\frac{\sum_{i=1}^{N} a_i^2}{\sum_{i=1}^{N} a_i^2 T_{k,k}}}
$$

(14)

where $N$ is the number of symbols of data being decorrelated. $N$ is one for bitwise and equal to the framength for blockwise techniques.

From the above analysis it can be seen that the system can benefit from the existence of constructive interference, so there is no need for it to be removed by fully pre-decorrelating. Using Channel State Information (CSI), knowledge of all users’ codes and data, readily available at the BS at downlink, and with the help of criteria (8) and rule (9), the interference to each user can be estimated at the BS prior to transmission to provide matrix $M_i$. By observation of the matrix $M_i$, the elements of the crosscorrelation matrix $R$ to be removed via decorrelation can be determined. Hence the following MMSE optimization could be applied:

$$
J = E_{x,a} \left\{ \|xAR_{\text{con}} - d\|^2 \right\} = E_{x,a} \left\{ \|xAR_{\text{con}} - (xATR + \eta)\|^2 \right\}
$$

(15)

where $T$ is the precoding matrix and $R_{\text{con}}$ is the constructive crosscorrelation matrix that contains the elements of $R$ that yield constructive interference according to the observation of $M_i$ at every symbol period. For TP and JT we would have $R_{\text{con}}=I$ which leads to full orthogonalization. It should be noted that $R=CH^T C^T$ for TP in [3] and $R=CH^T HC^T$ for JT in [4]. The solution to optimization (15) is:

$$
T = R_{\text{con}}^{-1}
$$

(16)

As will be shown, $R_{\text{con}}$ is always closer to $R$ than $I$ so moving from noiseless receiver output $\hat{x}A = xAR_{\text{con}}$ of a conventional system to the proposed $\hat{x}A = xAR_{\text{con}}$ needs less manipulation than moving to $\hat{x}A = xAI$ which is proposed in [3,4]. Hence, the matrix $T$ in (14) will have elements smaller than the elements of $R^{-1}$ which is the solution of (15) for TP and JT. Thus, the scalar $f$ will be larger and the useful energy will be more efficiently exploited.

Three different criteria can be followed for the formation of $R_{\text{con}}$, and therefore three different methods will be presented.

A. Selective Precoding Method a)

The simplest method would be fully orthogonalizing the users that experience destructive MAI and leaving the users that expect constructive MAI correlated. In mathematical terms for the $i$-th period of interest this could be expressed as:
If $\sum_{k=1}^{K} a_k x_{ik} \rho_{ij} < 0$ then $R_{\text{con}_{jk}} = 0$ for all $j \neq k$

Else $R_{\text{con}_{jk}} = \rho_{jk}$ for all $j \neq k$

It is clear, as mentioned, that $R_{\text{con}}$ is closer to $R$ than $I$ is.

B. Selective Precoding Method b)

An alternative to the above method would be to orthogonalize every user but only to the users that impose destructive interference to the useful signal at each symbol period. This would completely remove all destructive while allowing all constructive interference. This could be expressed as:

If $a_s a_k x_{ik} x_{jk} \rho_{is} < 0$ then $R_{\text{con}_{k \mu}} = 0$

Else $R_{\text{con}_{k \mu}} = \rho_{is}$

Obviously, this method aims at enhanced performance, but it requires a higher amount of decorrelation and hence increased transmitted power and scaling. The tradeoff to useful energy, though, is better than for method a) which yields improved performance. However, since decorrelation is fuller, the complexity is increased but still less than in JT and TP.

C. Selective Precoding Method c)

Here an optimization between the complexity, the required scaling and the constructive interference held in the system is attempted. This is done by orthogonalizing the users experiencing destructive cumulative MAI only to the users that impose destructive MAI on them, while leaving the remaining users completely un-decorrelated. This criterion can be expressed as:

If $\sum_{k=1}^{K} a_k x_{ik} \rho_{ij} < 0$ then

\begin{align*}
\left\{ \begin{array}{ll}
\text{If } a_s a_k x_{ik} x_{jk} \rho_{is} < 0 & \text{then } R_{\text{con}_{k \mu}} = 0 \\
\text{Else } & R_{\text{con}_{k \mu}} = \rho_{is}
\end{array} \right.
\end{align*}

Else $R_{\text{con}_{jk}} = \rho_{jk}$ for all $j \neq k$

Evidently, this method requires the least decorrelating and scaling, leaving a larger portion of the useful signal to be exploited at the receiver. Due to the existence of destructive MAI in the system, however, performance is expected to be worse than for method b), but the advantage of this method is the reduced complexity compared to the previously proposed techniques. It can be proven that for all three precoding techniques proposed, the precoding matrix is easier to construct than in cases [3,4] since it is not a full orthogonalization matrix as in TP and JT. Moreover, the Coppersmith-Winogard [7] method using temporary variables can be used for matrix construction to reduce the complexity from $O(W^3)$ to $O(W^{0.379})$ where $WxW$ is the size of $T$, $W=K$ for bitwise and $W=N.K$ for blockwise methods.

Another point worth mentioning is that the performance enhancement is achieved with no additional power-per-user investment. The improvement is attained by simply allowing the existence of useful interference and energy that is already in the system, from which the users’ signals can benefit. With full orthogonalization this benefit is lost.

IV. Performance Analysis

A. MAI Variance and SNR Calculation

In this section performance analysis is done for a simple system in order to avoid tedious computations. Synchronous transmission is presumed, all users are supposed to have equal power. AWGN channel and BPSK modulation is considered. For a large $K$, interference between two users can be modeled as Gaussian and its variance, for random codes, can be calculated as in [8]:

$$\sigma_{\text{MAI}_{k \mu}}^2 = E_b / L$$ (17)

where $E_b$ is the energy per unmodulated bit. The AWGN variance is $\sigma^2 = N_0/2$. Using these values and (7) the instantaneous per symbol period SINR can be expressed as:

$$\gamma = \frac{2 f E_s + a (U_c f E_s) / L}{b (U_d f E_s) / L + N_0}$$ (18)

where $U_c$, $U_d$ are the users that experience constructive and destructive interference respectively at each symbol period. For conventional precoding $a=0$, $b=0$ since full orthogonalization is performed, which means that single user performance is achieved with a reduced SNR due to the factor $f$. In the case of method b) where only constructive MAI is held $a=1$, $b=0$. Hence the SNR is enhanced. For methods a),c), $a$, $b \in \{0, 1\}$ and it can be viewed that $b$ is the same for both, but $a$ is larger for method c) than for a) yielding better performance.

B. BER Performance

For BPSK modulation using Gaussian noise approximation with zero mean and variance $\sigma^2 = N_0/2$ the BER can be calculated using an average $\gamma$ of the SINR in (18) as:

$$\text{BER} = P\{d_{in} < 0 \mid x_{in} = +1\} = P\{\eta_{in} < a_s x_{in} + \text{MAI}_{in}\}$$

$$= Q\left(\sqrt{\gamma}\right) = Q\left(\frac{f \cdot E_b + \tilde{a} (\tilde{U}_c f \cdot E_b) / L}{b (\tilde{U}_d f \cdot E_b) / L + N_0 / 2}\right)$$ (19)

Here $\tilde{a}, \tilde{b}, \tilde{U}_c, \tilde{U}_d$ are statistical averages of the instantaneous values in (18) used for the approximate calculation of the average SNR. A perfectly accurate analytical expression of the BER performance is unattainable due to the complexity imposed by the fluctuations of the elements in $R_{\text{con}}$ and hence
a, b, \(U_a\), \(U_b\) at every symbol period. The above expression can be used as a qualitative measure for comparison.

V. NUMERICAL AND SIMULATION RESULTS

Monte Carlo simulations have been performed for variable numbers of users and both flat and frequency selective fading decentralized AWGN channels whose estimation is assumed to be perfect. BPSK modulation has been employed and random codes of length \(L=16\) have been used to allow for the worst case. In all cases the average SNR=\(E_b/N_0\) per user is considered.

A. BER Performance Simulations

In Fig. 2 the BER vs SNR performance for the case of \(K=8\) is depicted in an AWGN channel for conventional TP and the three proposed selective precoding (STP) techniques. It should be noted that for the AWGN case TP and JT have identical performance since MF is applied and there is no distinction between Rake and Pre-Rake processing. It can be seen that selective precoding outperforms conventional precoding in all three cases, due to the benefit from the existence of constructive interference. Since the second method allows pure constructive MAI it offers the best performance with an SNR benefit that reaches 5dB. It should be reminded that this technique has the highest complexity of the three proposed techniques.

In Fig. 3 the BER vs \(K\) performance for SNR=7 is shown. All techniques rapidly saturate for \(K>L\). This is because orthogonalization is unattainable for that point, for the same reasons that no orthogonal codes exist for \(K>L\). In mathematical terms the crosscorrelation matrix \(R\) is always close to singular and there is no exact inverse which means that the MMSE optimization and the orthogonalization have no exact solution. Still the best performance is achieved by method b) as it benefits from the existence of pure constructive MAI. It can be seen that selective precoding yields significant capacity improvement for all techniques. For method b) the capacity is almost doubled if a BER of 10\(^{-2}\) is required.

For the multipath scenario it was shown in [4] that JT outperforms TP. For this reason, in Fig. 4 all three proposed techniques are compared to JT for a Rayleigh fading channel of \(P=3\) paths occupied by \(K=12\) users. The selective precoding yields an SNR gain up to 6dB for method b).

B. Noiseless Received Signal Constellation Simulations

In Fig. 5 the noiseless received signal constellation diagram is depicted for the Rayleigh fading channel case of \(P=11\) for \(K=8\) where no scaling or power constraint is applied to show the effect of the constructive MAI concept as presented theoretically in Fig. 1. For conventional JT the noiseless received symbols are identical to the symbols of the alphabet since complete orthogonalization is performed. For Selective Joint Transmission (SJT) the received symbols have moved further away from the decision threshold and the constellation has effectively been expanded since constructive interference is allowed in the system.

VI. CONCLUSIONS AND FUTURE WORK

In conventional precoding schemes energy is wasted in two ways. Firstly, this is due to the lack of exploitation of constructive MAI which is readily available in a communication system and would increase the received SNR without the need for increased transmitted energy per user. Secondly, additional energy is invested for the pre-orthogonalization of users that interfere constructively. The proposed method benefits from taking these facts into account and hence improved performance is attained as proven by mathematical analysis and numerical results.

The selective precoding method and the constructive MAI concept can be combined with all existent precoding techniques given that PSK modulation is employed. Further research can be carried out towards the combination of the proposed technique with the constrained optimization of [3] as well as the DPF and INVF techniques investigated in [4,5].

![Figure 2. BER vs SNR Performance of conventional TP and the proposed precoding methods in an AWGN channel of \(K=8\) users with \(L=16\).](image2.png)

![Figure 3. BER vs \(K\) Performance of conventional TP and the three proposed precoding methods in an AWGN channel for SNR=7dB, \(L=16\).](image3.png)
Figure 4. BER vs SNR Performance of conventional JT and the three proposed precoding methods in a Rayleigh fading channel of $P=3$ paths for $K=12, L=16$

Figure 5. Noiseless received signal constellation for conventional JT and the three proposed SJT in a Rayleigh fading channel of $P=11$ paths for $K=8, L=16$ with no power scaling

REFERENCES


