Analysis of a Random Channel Access Scheme with Multi-Packet Reception

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Abstract—A key advantage of viewing communications in wireless networks as multiple access rather than a plurality of point-to-point transmissions, is its robustness towards multiple access interference. Concurrent packet transmissions are allowed to coexist thus deviating from the traditional view of enforcing collision-footprints around the transmitter-receiver pairs. What are the performance gains of employing channel access strategy based on a multiple access channel in a multihop wireless network? We consider a wireless multihop network, where nodes have a joint decoding capability to resolve up to $K$ multiple concurrent packet transmissions from other nodes in their range. The basic assumptions are that the packet transmissions are asynchronous, i.e., nodes are completely uncoordinated, and that the packet transmission at each node is based on a probabilistic model. In this paper, we show that a simple random access strategy for communication over such channels offers significant gains in throughput while reducing latency in congested wireless networks. More precisely, we characterize the throughput performance gains through an exact analysis for the case of $K = 2$ and also offer tight approximations for arbitrary $K$. Furthermore, we study the asymptotic throughput behavior and prove asymptotic optimality of random channel access over multiple access channel.

I. INTRODUCTION

Ad hoc wireless networking is poised to be the next revolution in data networks with plenty of applications ranging from surveillance and tracking to data mining and distributed computing. In such networks, transmission from a source to its destination is over multiple hops through other nodes that assist in relaying information. Communication among nodes is established without any infrastructure support in an uncoordinated fashion. Since packet transmissions from nodes are asynchronous and random over a common channel, efficient transmission protocols that achieve higher network performances are a necessity. Several communication protocols for random, uncoordinated channel access have ushered over the past decades. Examples of basic random access protocols include ALOHA, carrier sense multiple access (CSMA), token, and tree-based protocols.

A fundamental assumption in the design of these protocols is that a node can decode single-user transmission successfully while concurrent transmissions interfere with each other resulting in decoding failure of overlapping packets. Efficiency of such access protocols varies from $18\%$ offered by ALOHA [1] to $100\%$ for TDMA or centralized scheduling methods$^1$.

Throughput, delay and stability characteristics of various single-user transmission protocols have been thoroughly addressed in the literature [1].

Recent research on the code division multiple access (CDMA) communication has led to a broad variety of user cooperation strategies that differ in their performance and complexity. Joint-decoding and layering of CDMA signals provides a number of advantages such as resistance to near-far effect, and robustness to multiple access interference. Several novel joint detection methods are capacity achieving in many scenarios [2], [3]. With joint decoding it is possible to resolve multiple concurrent overlapping packet transmissions in wireless network environments. Packet collisions occur if the number of packets transmitted over the channel exceed the joint decoding capability at the receiver [4]. These attributes have inspired the use of joint decoding principles to various random packet transmission systems [5], [6]. Most of the related works have focused on characterizing the network performance offered by various decoding methodologies such as minimum mean-squared error, matched filtering, decorrelator, etc. [7] (and references therein) and a few others on understanding the throughput and stability properties of slotted multiple packet reception systems (MPR) [4], [8], [9].

In this work, we provide an analytical framework to study the behavior of a simple random access protocol with MPR capability. More precisely, we analyze the performance characteristics of a simple ALOHA-based random access protocol assuming an infinite population of users. First, we provide an exact calculation of throughput of a system capable of resolving at most two concurrent overlapping transmissions. Next, we provide an approximation for throughput of the system with arbitrary MPR capability. Simulations are also presented to show the tightness and validity of the approximation. Finally, an asymptotic throughput performance, i.e., for large MPR capabilities, is derived. We show that the simple ALOHA-based random access scheme offers significant improvement in throughput in congested wireless networks. Furthermore, the asymptotic analysis proves that ALOHA approaches the throughput upper bound of MPR system for packet arrival rates less than the MPR capability.

The rest of the paper is organized as follows. In Section II, we provide a system model for uncoordinated random channel access in wireless networks. For the system model, exact analysis and approximations to the system throughput are derived and compared with simulation results in Section III.

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$^1$Although the achievable optimal throughput with TDMA is appealing, distributed protocols are viable and scalable from an operational view.
This section also provides the asymptotic throughput characteristics of the random channel access with MPR capability. The conclusions follow in Section IV.

II. MODEL AND PRELIMINARIES

The system model consists of $M$ nodes each in the transmission-reception range of others. Each node is capable of receiving at most $K$ packets at any given time. In other words, if a receiving node sees more than $K$ packet transmissions over the channel at any given time over a packet length a decoding failure occurs at the receiver and all the overlapping packets are lost. We assume that all events happen at discrete time $t = 0, 1, \ldots$, whose interval equals that of a chip. All packet transmissions are of length $L$ and happen randomly, i.e., without synchronization among nodes.

We consider probabilistic-based packet transmission. Each node transmits a packet with probability $p$ over a packet time interval. With $M$ nodes in the network, the number of packets transmitted over the channel in the interval $[t, t + L]$ is a discrete binomial random variable $X(t)$. Assuming infinite node population, we can approximate the traffic (number of packets/packet interval) over the channel by a Poisson distribution:

$$Pr(X = i) = \frac{\lambda^i}{i!} e^{-\lambda},$$

where $\lambda = \lim M \to \infty Mp$ is the mean of the random variable $X$ that counts the number of packets per packet interval.

A. Upper Bound on Throughput of an MPR system

Throughput is defined as the time average of the number of packets received at a given node, i.e.,

$$T := \lim_{t \to \infty} \frac{N(t)}{t},$$

where $N(t)$ is the number of packets received at a node over time $t$. Assuming that there is a certain traffic rate $\lambda$ over the channel, the upper bound on $T$ is given by $\min(\lambda, K)$. This is illustrated in Fig. 1. The fundamental limitation is imposed by the MPR capability $K$. If the traffic rate is below the MPR capability, we see a linear increase in throughput. On the other hand if $\lambda$ is above $K$, best feasible throughput is limited by $K$ and hence we see a saturation. The region of interest is the shaded region. In this region, a bounded delay can be ensured, while in the saturation regime the system encounters an unbounded delay.

B. Main Result

Denoting $p_{\text{suc}}$ as the probability of a packet success, the average number of packets correctly received is denoted by $T = \lambda p_{\text{suc}}$. We now state the main result.

**Theorem 1:** For an asynchronous random access MPR system $K \to \infty$, and $\lambda \leq K - o(K)$, we have

$$p_{\text{suc}} \to 1 \text{ and } T_{\text{ALOHA}} \to \lambda.$$

2Since the probabilities are independent of $t$ and for case of notation we represent $X(t)$ by $X$.

III. PERFORMANCE ANALYSIS AND EVALUATION

First, we provide an exact analysis for the case $K = 2$ and then an accurate approximation to calculate the throughput for arbitrary $K$. We characterize all the error events that may lead to the decoding failure of a packet. Let $\tau_n$ be the time of $n$-th packet occurrence in the Poisson process. A collision occurs if the number of overlapping packets in the time window $[\tau_n, \tau_n + L - 1]$ is greater than $K$. For $K = 2$, this is illustrated in Fig. 2. The arrival of more than 2 packets in the window of the $n$-th packet leads to a collision of the packet. There are several such scenarios that lead to a collision. Further, note that only those packets that arrive during the interval $[\tau_n - L, \tau_n + L - 1]$ can cause interference to the decoding of the $n$-th packet. We are interested in the probability $P_K(n)$ of collision event for the $n$-th packet. The exact calculation of the collision probability for the $n$-th packet requires keeping track of error events that may occur during the interval $[\tau_n - L, \tau_n + L - 1]$. We divide the collision events into four mutually exclusive events and analyze the collision probabilities. An exact analytical expression for $P_2(n)$ is provided in Lemma 2.

**Lemma 2:** For $K = 2$,

$$P_2(n) = \sum_{a=0}^{L-1} \sum_{b=0}^{L-1-a} p_a \sum_{c=0}^{L-1-a} p_b \sum_{d=0}^{L-1-c} p_c,$$

$$+ \sum_{a=0}^{L-1} \sum_{b=L-a}^{L-1} p_a \sum_{c=L-a}^{L-1} p_b \sum_{d=0}^{L-1-c} p_c,$$

$$+ \sum_{a=L}^{L-1} \sum_{b=0}^{L-1-a} p_a \sum_{c=0}^{L-1-b} p_b \sum_{d=0}^{L-1-c} p_c,$$

$$+ \sum_{a=L}^{\infty} \sum_{b=0}^{L-1} p_a \sum_{c=0}^{L-1-b} p_b \sum_{d=0}^{L-1-c} p_c,$$

where $p_i = \frac{L}{\lambda} e^{-i\lambda/L}$, $i = 0, 1, \ldots$ is the distribution of the inter arrival times.

**Proof:** See Appendix.

The throughput performance of the MPR system for $K = 2$ is plotted against traffic rate $\lambda$ in Fig. 3. For the simulation, a packet length of 1460 bytes is considered. The packet

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transmissions are asynchronous and random. From this figure, we can observe that the throughput of the ALOHA-based random access protocol is enhanced with MPR capability ($K = 2$). However, the performance is much lower than the upper bound on the throughput of the MPR system. This is due to the random channel access that leads to packet collisions. Moreover, we can see that the simulation results coincide with the analysis.

Fig. 2. Collision event for the $n$-th packet.

We are interested in calculating the collision event defined as: $\exists i \in [0, L-1] \mid I_{i+\tau_n} > K$. The probability of collision event is given by

$$P_K(n) = P (\exists i \in [0, L-1] \text{ s.t. } I_{i+\tau_n} > K). \quad (3)$$

An instance of the random process $I_{\tau_n}$ is shown in Fig. 4. In this figure the random process $I_{i+\tau_n}$ indicates the level of interference. The interference level at time instants $\tau_n, \tau_n + 1, \ldots, \tau_n + L - 1$ is given by $I_{\tau_n}, I_{\tau_n+1}, \ldots, I_{\tau_n+L-1}$ respectively. Notice that if the process $I_{i+\tau_n}$ is below the MPR capability $K$ for the entire duration of the $n$-th packet, leads to the success of the packet. On the other hand, if the process exceeds $K$ at any instant in the duration of the packet leads to a collision. Lemma 3 states the asymptotic result. An accurate approximation is provided in the proof (see Appendix).

Fig. 4. A random walk denoting the level of interference for the $n$-th packet.

The throughput performance of the MPR system with $K = 10$ is plotted for different values of traffic rate $\lambda$ in Fig. 5. From this figure, we can notice that the approximation provided in Lemma 3 is quite tight in comparison with simulations. In addition, Lemma 3 provides an asymptotic throughput analysis and implies Theorem 1. It shows that random access ALOHA achieves the upper bound on the throughput of an MPR system.

Fig. 5. A comparison of simulation and approximation to throughput of a MPR system with $K = 10$

Extending the analysis provided in Lemma 2 for larger values of $K$ is non-trivial. We therefore, provide a general approach for arbitrary $K$ and $L$. We characterize all the error events that may lead to the decoding failure of a packet. We are interested in understanding the behavior of the interference process, which is a random process. Let the random variables $X_{\tau_n-L}, X_{\tau_n-L+1}, \ldots, X_{\tau_n+L-1}$ denote the number of packet transmissions at time instants $\tau_n-L, \tau_n-L+1, \ldots, \tau_n+L-1$ respectively. The random variables are i.i.d. Poisson with mean $\lambda/L$.

The number of overlapping packet transmissions in the duration of the $n$-th packet indicate the interference level and can be calculated as follows. Let $I_{i+\tau_n}$ denote the interference level at time $i + \tau_n$, $i = 0, \ldots, L-1$, i.e., the number of overlapping packets at time $i + \tau_n$. The interference level for the $n$-th packet $I_{i+\tau_n}$ can be obtained as follows

$$I_{i+\tau_n} = \sum_{j=1}^{L} X_{\tau_n+i-L+j}, \quad i = 0 \ldots L - 1 \quad (2)$$

**Lemma 3:** For $\lambda = K - C_1 K^{3/4}$, and some constant $C_1$, the probability of packet success $p_{\text{succ}}$ approaches 1 as $K \to \infty$.
Proof: See Appendix.

The implication of Lemma 3 is well illustrated in Fig. 6. In this figure, the throughput of an MPR system with $K = 10$, $K = 500$ and the corresponding upper bounds are plotted. We can see that when $K$ gets large, and the arrival rate $\lambda$ approaches $K$, the probability of success gets close to one and the throughput approaches $\lambda$. This shows that random access ALOHA asymptotically achieves the upper bound on the throughput of the MPR system.

As an immediate extension to Lemma 3 we can obtain asymptotic performance of a synchronous, slotted system, whose slot length equals the length of the packet duration. In our model, this corresponds to the case $L = 1$.

Corollary 4: For a slotted MPR system, whose slot length equals the length of the packet, the probability of collision $P_K(n)$ has the following behavior

$$\lim_{K \to \infty} P_K(n) = \begin{cases} 0 & 0 < \lambda < K \\ 1/2 & \lambda = K \\ 1 & \lambda > K. \end{cases}$$

Proof: See Appendix.

IV. CONCLUSION

In this paper, we have shown that a simple random access strategy over multiple access channels offers significant gains in throughput in congested wireless networks. We characterized the throughput performance gains through an exact analysis for case of two packet MPR and also offered tight approximations for the general case. Furthermore, we studied the asymptotic throughput behavior and showed the optimality of the simple ALOHA-based, random channel access over multiple access channel. Similar conclusions for slotted random access system follows as a corollary to the main result.

REFERENCES


APPENDIX

Proof of Lemma 2 The $n$th packet arrival moment $\tau_n$ is a sum of inter-arrival times $\xi_i$

$$\tau_n = \sum_{i=1}^{n} \xi_i, \quad n \geq 1,$$

where the inter-arrival times $\{\xi_i \geq 0\}$ are i.i.d. random variables with exponential distribution $p(\xi_i = k) = p_k$.

We say that a time instant $i$ is covered by the $n$-th packet if $0 \leq i - \tau_n \leq L - 1$. By definition a collision event for the $n$-th packet occurs if there exists a time instant $i$ such that $0 \leq i - \tau_n \leq L - 1$, and $i$ is also covered by any other $\geq 2$ packets. We are interested in the probability $P_2(n)$ of the collision event for the $n$th packet. We have

$$P_2(n) = \Pr \{\tau_n - \tau_{n-2} \leq L - 1\} \quad + \sum_{i=0}^{L-1} \Pr \{\tau_n - \tau_{n-2} \geq L; \tau_n - \tau_{n-1} = i; \text{collision}\} \quad + \Pr \{\tau_n - \tau_{n-1} \geq L; \tau_{n+2} - \tau_n \leq L - 1\}.$$ (4)

From the definition of the inter arrival times $\xi_n$ it follows that

$$\Pr \{\tau_n - \tau_{n-2} \leq L - 1\} = \Pr \{\xi_{n-1} + \xi_n \leq L - 1\} \quad (5)$$
and

$$\Pr \{\tau_n - \tau_{n-1} \geq L; \tau_{n+2} - \tau_n \leq L - 1\} = \Pr \{\xi_{n+1} + \xi_{n+2} \leq L - 1\}.$$ (6)

Also, for $1 \leq i \leq L - 1$ we have

$$\Pr \{\tau_n - \tau_{n-2} \geq L; \tau_n - \tau_{n-1} = i; \text{collision}\} = \Pr \{\xi_{n-1} + \xi_n \geq L; \xi_n = i; \text{collision}\} = \Pr \{\xi_{n-1} \geq L - i; \xi_{n+1} \leq L - 1 - i\} + \Pr \{\xi_{n+1} \geq L - i; \xi_n = i; \xi_{n+1} \geq L - i; \xi_{n+1} + \xi_{n+2} \leq L - 1\} = \Pr \{\xi_{n-1} \geq L - i\} \Pr \{\xi_n = i\} \Pr \{\xi_{n+1} \leq L - 1 - i\} + \Pr \{\xi_{n+1} \geq L - i\} \Pr \{\xi_n = i\} \Pr \{\xi_{n+1} \geq L - i; \xi_{n+1} + \xi_{n+2} \leq L - 1\},$$

and

$$\Pr \{\xi_{n+1} \geq L - i; \xi_{n+1} + \xi_{n+2} \leq L - 1\} = \sum_{j=L-i}^{L-1} \Pr \{\xi_{n+1} = j\} \Pr \{\xi_{n+2} \leq L - j - 1\}. \quad (7)$$

Similarly for $i = 0$, we can write

$$\Pr \{\tau_n - \tau_{n-2} \geq L; \tau_n - \tau_{n-1} = 0; \text{collision}\} = \Pr \{\xi_{n-1} \geq L; \xi_n = 0; \text{collision}\} = \Pr \{\xi_{n-1} \geq L; \xi_n = 0; \xi_{n+1} \leq L - 1\} = \Pr \{\xi_{n-1} \geq L\} \Pr \{\xi_n = 0\} \Pr \{\xi_{n+1} \leq L - 1\}. \quad (8)$$

Combining (5), (6), (7) and (8) leads to the expression in Lemma 2.
Proof of Lemma 3} We consider an approximation of the $L$-segmented interference process (2) as

$$
Y_0 = X_{r_n+1-L} + X_{r_n+2-L} + \ldots + X_{r_n-1} \\
Y_1 = Y_0 + \xi_1 \\
Y_2 = Y_1 + \xi_2 \\
\vdots \\
Y_{L-1} = Y_{L-2} + \xi_{L-1},
$$

where $X_i \sim \text{Po}(\lambda/L), i = 0, 1, \ldots, L-1$ are Poisson random variables and $\xi_i \sim \text{Skellam}(\lambda/L), i = 1, \ldots, L-1$ are Skellam distributed random variables. Note that $Y_0 \sim \text{Po}(\lambda)$.

We would like to show that throughput $\lambda$ asymptotically close to $K$ is achievable. Let us choose $\lambda = K - C_1K^{3/4}$ where $C_1$ is a constant and show that with this $\lambda$ probability of packet success goes to 1 as $K \to \infty$. Let us introduce a value $x' = K - C_2K^{3/4}$ such that $C_2 < C_1$. Then the following proposition holds.

**Proposition 5:** For random variable $Y_0 \sim \text{Po}(\lambda)$

$$
\Pr(Y_0 \geq x') \leq \frac{1}{(C_1 - C_2)^2K^{1/2}}.
$$

**Proof:** Let us consider Tchebychev inequality in the form

$$
\Pr(Y_0 - EY_0 \geq k\sqrt{\text{var}(Y_0)}) \leq \frac{1}{k^2} \Leftrightarrow \\
\Pr(Y_0 - \lambda \geq k\sqrt{\lambda}) \leq \frac{1}{k^2} \Leftrightarrow \\
\Pr(Y_0 \geq k\sqrt{\lambda} + \lambda) \leq \frac{1}{k^2}
$$

(13)

Let us choose the value of the parameter $k$ such that

$$
k\sqrt{\lambda} + \lambda = x' \\
\text{i.e. } k = (x' - \lambda)/\sqrt{\lambda}. \quad \text{(15)}
$$

Substituting (15) into (13) we obtain

$$
\Pr \left( Y_0 \geq x' \leq \frac{\lambda}{(x' - \lambda)^2} \right) = \frac{K - C_1K^{3/4}}{(C_1 - C_2)^2K^{6/4}} < \frac{1}{(C_1 - C_2)^2K^{1/2}}. \quad \text{(16)}
$$

The next proposition determines the probability of the packet success conditioned on $Y_0 < x'$.

**Proposition 6:** For $Y_0 < x'$ the probability that the random walk (9) newer exceeds $K$ goes to 1 as $K \to \infty$.

**Proof:** The proof is omitted due to space limitations.

Finally using Propositions 5 and 6 we complete the proof of Lemma 3. We can lower the probability of success as

$$
\rho_{\text{succ}} \geq \Pr(\text{succ} | Y_0 \leq x') = \Pr(\text{succ} | Y_0 \leq x') \Pr(Y_0 \leq x') \geq \Pr(\text{succ} | Y_0 \leq x') \left( 1 - \frac{1}{(C_1 - C_2)^2K^{1/2}} \right) \quad \text{(17)}
$$

where the last inequality comes from Proposition 5. On the other hand from Proposition 6 we know that

$$
\lim_{K \to \infty} \Pr(\text{succ} | Y_0 \leq x') \to 1. \quad \text{(18)}
$$

The lemma is proved.

**Proof of Corollary 4** We are interested in understanding the system behavior when $\lambda \to K$ and $L = 1$. For $X \sim \text{Po}(\lambda)$ it can be shown that $X \overset{d}{\to} N(\lambda, \lambda)$ as $\lambda \to \infty$. The success probability is given by $\Pr(X < K)$. Define $Z = (X - \lambda)/\sqrt{\lambda}$. The probability of success is given by

$$
\Pr \left( Z < \frac{K - \lambda}{\sqrt{\lambda}} \right) = 1 - Q \left( \frac{K - \lambda}{\sqrt{\lambda}} \right) \geq 1 - \frac{1}{2} e^{-\lambda^3/2(K-1)/2}
$$

Looking at the asymptotic behavior of the above expression for $K \to \infty$ leads to the statement of the Corollary 4. This result was also shown in [8].