Bayesian Potential Games to Model Cooperation for Cognitive Radios with Incomplete Information

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Abstract—This paper presents a Bayesian Potential game to model distributed joint power and channel allocation for cognitive radios with incomplete information. We propose a cooperative approach where secondary users (SUs) devote part of their transmission power in licensed channels to relaying primary users’ (PUs) messages. In addition, we consider incomplete information in the decision making process, so as to avoid the need for a Common Control Channel (CCC), where users share information. This hypothesis improves the robustness and feasibility of the cognitive radio network supported by the proposed approach. Simulation results show that cooperation benefits both PUs and SUs and that the lack of complete information in the decision process slightly reduces performances in terms of Signal to Interference and Noise Ratio (SINR) and outage probability.

I. INTRODUCTION

Cognitive Radio is a new paradigm in wireless communications to enhance utilization of limited spectrum resources. It is defined as a radio able to utilize available side information, in a decentralized fashion, in order to efficiently use the radio spectrum left unused by licensed systems. The basic idea is that a secondary (unlicensed) user (SU) is able to properly sense the spectrum conditions and, to increase efficiency in spectrum utilization, it seeks to underlay, overlay or interweave its signals with those of the primary (licensed) users (PUs), without impacting their transmission [1]. As a result, a hierarchical model is defined where PUs are entities characterized by a higher priority in a given frequency band (e.g. cell phones, TV stations, emergency service, etc.) than SUs. The PUs are not cognitive radio aware, that is, they are not able to facilitate signalling information to SUs to allow the access to their frequencies. On the other hand, SUs are characterized by cognitive radio capabilities, that is, in order to access certain frequencies, a SU has to continually monitor the radio spectrum to identify spectrum opportunities, to reliably detect the presence of PUs, and to evaluate the interference the SUs’ transmitters may cause on the PUs’ receivers. All the challenges related to the so called cognitive radio cycle [2] can highly benefit from cooperation, not only among SUs [3], but also between SUs and PUs. In this paper we focus on a cognitive radio scenario where concurrent primary and secondary communications are allowed by exploiting spatial reuse, as long as the SUs cooperate with the PUs active in the same frequency channels. SUs cooperate by relaying the primary messages to their final destination. The objective is to improve the performances of the PUs and to compensate them for the increased interference at their receivers.

In a cognitive radio scenario, the individual decisions of SUs strongly depend on the decisions made by the other SUs, since the PUs’ performances are limited by the aggregated interference generated by all the SUs simultaneously transmitting in their band. For this reason, we propose that a game theoretic framework, already proven good at analyzing interactions in decision making processes, will model this scenario. Specifically, we define a potential game to capture the aggregated interference the SUs cause to PUs receivers, the interference SUs generate to each other, as well as the throughput of the SUs. Game theory has often been considered in literature to model spectrum sharing problems for cognitive radios [4][5]. However, inherent in nearly all previous efforts has been the hypothesis of complete channel state information among SUs, that is, the wireless channel gains are assumed to be common knowledge across all SUs. This hypothesis implies the implementation of a common control channel (CCC) where the distributed SUs can share the information about their wireless channel gains. In literature, the hypothesis of such a fixed control channel in a cognitive radio context has often been rejected [6], since it requires a static assignment of licensed spectrum before deployment, which is basically against the same philosophy of cognitive radio. Additionally, this solution increases cost and complexity, limits scalability in terms of device and traffic density and is not robust to e.g. jamming attacks. As a result, in an effort to model a more reliable and realistic self-organized cognitive radio system we include uncertainty in the considered scenario and we do not rely on the existence of a preassigned CCC.

In this paper we propose a Bayesian Potential game (BPG) to model decentralized joint power and channel allocation for cooperative SUs with incomplete information. Simulation results will show that cooperation among SUs significantly improves performances of both PUs and SUs and that the more realistic hypothesis of incomplete information only slightly reduces performances of PUs and SUs with respect to the case of complete information. The outline of the paper is organized as follows. Section II describes the system model. Section III presents the game theoretic model and discusses the existence of a Bayesian Nash equilibrium. Section IV presents the simulation scenario where the proposed paradigm...
is evaluated. Section V is devoted to the discussion of representative simulation results. Finally, Section VI summarizes the conclusions of the paper.

II. SYSTEM MODEL

The cognitive radio network we consider consists of \( M \) transmitting-receiving PUs pairs, and \( N \) transmitting-receiving SUs pairs. In this paper we will indicate the transmission power levels of the PUs’ transmitters as \( p_{ij}^P, i = 1, \ldots, M \), and the transmission power levels of the SUs’ transmitters as \( p_{ij}^S, j = 1, \ldots, N \). PUs and SUs, both transmitters and receivers, are randomly and uniformly distributed in a circular coverage region of a primary network with radius \( R_{max} \). Primary communications can be characterized by a long distance between the transmitting and the receiving device, whereas secondary communications are in general characterized by short range. The nodes are either fixed, or are moving slowly (slower than the convergence of the proposed algorithm). The system architecture considered in this paper is represented in Fig. 1, where PU\(_i\), PU\(_r\), SU\(_t\) and SU\(_r\) indicate, respectively, the PU’s transmitter, the PU’s receiver, the SU’s transmitter and the SU’s receiver. According to this architecture, a SU distributively selects the frequency channel, among \( l \) available, and the transmission power level, among \( m \) available, in order to maximize its throughput while at the same time not causing harmful interference to the PUs. Besides, the SUs have to compensate the PUs for using their frequency band; they do so by devoting part of their transmission power to relaying the primary transmission. As a result, the SU’s transmission power level is split in two parts, 1) a power level \( p_{ij}^{SP} \), \( j = 1, \ldots, N \) for its own transmission, and 2) a cooperation power level \( p_{ij}^{SS} \), \( j = 1, \ldots, N \) for relaying the PU’s message on the selected band, where \( p_{ij}^S = p_{ij}^{SP} + p_{ij}^{SS} \). The cooperative scheme used by the SUs is shown in Fig. 2. We assume that the PU transmission is divided into slots, and each slot further into T micro-slots (ms). The SU listens to the primary for one micro-slot and then starts transmitting cooperatively in a decode and forward mode. If the SU is able to operate in full-duplex mode, then it keeps on relaying the PU signal. If it operates in half-duplex mode (as is often the case in practice), then it alternates between listening and transmitting microslots. We also assume that such a transmission scheme can be decoded by the PU (for example using an encoding scheme such as the one described in [7]).

We shall analyze the network performance in terms of Signal to Interference and Noise ratio (SINR) of both PUs and SUs and outage probability. As for the notation, we indicate with \( h_{ij}^{PP} \) the link gain between a PU’s transmitter \( i \) and a PU’s receiver \( j \); with \( h_{ij}^{PS} \) the link gain between a PU’s transmitter \( i \) and a SU’s receiver \( j \); with \( h_{ij}^{SP} \) the link gain between a SU’s transmitter \( i \) and a PU’s receiver \( j \) and with \( h_{ij}^{SS} \) the link gain between a SU’s transmitter \( i \) and a SU’s receiver \( j \). Finally, \( \sigma^2 \) is the noise power (assumed to be the same in each channel).

A. Signal to Interference and Noise Ratio (SINR)

The SINR \( \gamma_i \) for a pair \( i \) of PUs in a frequency channel \( c_i \) is given by:

\[
\gamma_i = \frac{p_{ij}^P h_{ij}^{PP} + \sum_{j=1}^{N} p_{ij}^S h_{ij}^{SP} f(c_j, c_i)}{\sum_{j=1}^{N} p_{ij}^S h_{ij}^{SP} f(c_j, c_i) + \sigma^2}, \quad i = 1, \ldots, M \tag{1}
\]

where \( f \) is defined as:

\[
f(c_i, c_j) = \begin{cases} 
1 & \text{if } c_i = c_j \\
0 & \text{if } c_i \neq c_j
\end{cases} \tag{2}
\]

Notice that, part of the SU power contributes to improving the SINR by increasing the useful signal power at the receiver (cooperation power). Moreover, the assumption that all secondary transmitters are able to correctly decode the primary signal was made. Regarding the SUs, it is reasonable to assume that all secondary receivers will be able to detect the primary signal. In this case, all interference corresponding to the primary signal can be removed following a successive interference cancellation approach. Under that assumption, the only remaining interference term for SUs is the term corresponding to the interference caused by other secondary transmissions. Consequently, the SINR for the SUs is given.
by:
\[
\gamma_i = \frac{p_i^S h_{ij}^{SS}}{\sum_{j=1, j \neq i}^{N} p_j^S h_{ij}^{SS} f(c_j, c_i) + \sigma^2}, \quad i = 1, \ldots, N \quad (3)
\]

B. Outage Probability

The outage probability is the probability that a user \( i \) perceives a SINR \( \gamma_i < \gamma \) dB, where the threshold is set according to the primary receiver sensitivity.

III. GAME FORMULATION AND BAYESIAN NASH EQUILIBRIUM EXISTENCE

Game theory constitutes a set of mathematical tools to analyze interactions in decision making processes. In this paper we model joint channel and transmission power selection for cognitive radios with incomplete information as the output of a Bayesian Potential game. In particular, the considered game of incomplete information is defined as: \( \Gamma = \{N, \{S_i\}_{i \in N}, \{\eta_i\}_{i \in N}, \{f_H(\eta_i)\}_{i \in N}, \{u_i\}_{i \in N} \) where:

i) \( N \) is the finite set of players, i.e. the SUs. Additionally, \( N^+ \) is a finite set with \( N^+ \supseteq N \) and \( N^+ \setminus N \) is the set of outside players (i.e. the PUs).

ii) For every \( i \in N, S_i \) is the set of strategies of player \( i \). More in particular they are:

- a power level \( p_i^S \) in the set of power levels \( P^S = \{p_1^S, \ldots, p_m^S\} \);
- the power level \( p_i^{SP} \) that the player devotes to its own transmissions, in the set of power levels \( P^{SP} = \{p_1^{SP}, \ldots, p_q^{SP}\} \), where \( q \) is the order of set \( P^{SP} \);
- the cooperative power level \( p_i^{SS} \) that the player devotes to relaying a PU transmission and which is computed as \( p_i^{SS} = p_i^S - p_i^{SP} \). The set of these power levels, \( P^{SS} \), is the same as \( P^{SP} \);
- a channel \( c_i \) in the set of channels \( C = \{c_1, \ldots, c_l\} \).

These strategies can be combined into a composite strategy \( s_i = (p_i^S, p_i^{SP}, p_i^{SS}, c_i) \in S_i \). We define \( S = \times S_i, i \in N \) as the strategy space.

iii) A game of incomplete information, with respect to a game of complete information, is characterized by the player’s type, which embodies any information that is not common knowledge to all players and is relevant to the players’ decision making. This may include the player’s utility function, his belief about other player’s utility functions, etc. For every \( i \in N^+ \), \( H_i \) is the finite set of possible types of player \( i \), \( \eta_i = (h_{i1}^{SS}, \ldots, h_{i1}^{SS}, h_{i1}^{SS}, h_{i1}^{SS}, \ldots, h_{iN}^{SS}) \in H_i \), which includes the wireless channel gains of player \( i \). Each player is assumed to observe perfectly its type, but is unable to observe the types of its neighbors.

iv) \( f_H(\eta_i) \) is a probability distribution on \( H = \times H_i, i = 1, \ldots, N \), with the a priori probability density function (PDF) on \( H \) defining the wireless channel gain PDF.

v) For every \( i \in N, u_i : S \times H \rightarrow \mathbb{R} \) is the utility function of player \( i \).

The utility function for player \( i \) is a function of its realized channel gains \( \eta_i \in H_i \) and its chosen strategy \( s_i \in S_i \), as well as the channel gains of the other SUs and PUs (i.e. \( \eta_{-i} \) and the strategies of other players \( s_{-i} \)).

\[
u(s_i, s_{-i}; \eta_i, \eta_{-i}) = -\sum_{j=1}^{M} p_j^S h_{ij}^{SP} f(c_i, c_j) - \sum_{j=1, j \neq i}^{N} p_j^S h_{ij}^{SS} f(c_j, c_i) - \sum_{j=1}^{M} p_j^{SS} h_{ij}^{SS} f(c_i, c_j)
+ b \log(1 + p_i^S h_{ii}^{SS}) + \sum_{j=1}^{M} p_i^{SS} h_{ij}^{SS} f(c_i, c_j) \quad (4)
\]

The expression presented in (4) consists of five terms. The first and the third terms account for the interference perceived by the PUs and by the other SUs in \( c_i \) from player \( i \), which only consists of the power the user \( i \) devotes to the secondary transmission (i.e. \( p_i^S \)). The second term accounts for the interference generated by the SUs active in channel \( c_i \) on player \( i \). The fourth term represents an incentive for the individual players to increase the power level devoted to their own communications. We weight this term by a coefficient \( b \) to give it more or less importance than the other terms of the utility function. Finally, the last term, is a positive contribution to the utility function and accounts for the benefit provided to the PUs by the relaying realized by the SUs. This term is positively defined to encourage SUs to cooperate with PUs in exchange for using their frequency channel.

It can be easily demonstrated that the game \( \Gamma \), with utility function defined in (4), is a Bayesian Potential game. Specifically, a Bayesian game is Potential if there exists a function \( Pot : S \rightarrow \mathbb{R} \) such that, for all \( i \in N, s_i, s_i' \in S_i \) and \( \eta_i, \eta_{-i} \in H \):

\[
Pot(s_i, s_{-i}; \eta_i, \eta_{-i}) - Pot(s_i', s_{-i}; \eta_i, \eta_{-i}) = u(s_i, s_{-i}; \eta_i, \eta_{-i}) - u(s_i', s_{-i}; \eta_i, \eta_{-i}) \quad (5)
\]

The function \( Pot \) is called exact potential function of the game \( \Gamma \) and it reflects the change in utility for any unilaterally deviating player. For the previously defined game we can define the potential function defined in (6). The demonstration can be found in Appendix A.

\[
Pot(s_i, s_{-i}; \eta_i, \eta_{-i}) = \sum_{i=1}^{N} \left( \sum_{j=1}^{N} (p_j^{SS} - p_i^{SS}) h_{ij}^{SS} f(c_i, c_j) \right) + \sum_{i=1}^{N} \left( -a \sum_{j=1, j \neq i}^{N} p_j^{SS} h_{ij}^{SS} f(c_j, c_i) - (1 - a) \sum_{j=1}^{M} p_j^{SS} h_{ij}^{SS} f(c_i, c_j) \right) + \sum_{i=1}^{N} b \log(1 + p_i^{SS} h_{ii}^{SS}) \quad (6)
\]

The players make decisions in a decentralized fashion, and independently, but they are influenced by the other players’ decisions. In this context, we are interested in searching an equilibrium point for the joint power and channel selection problem of the SUs from which no player has anything to gain by unilaterally deviating. This equilibrium point, in games
of complete information, is known as Nash equilibrium. We define a Bayesian Nash equilibrium as a Nash equilibrium of a Bayesian game. In particular, a strategy profile \( s^* = (s^*_1, \ldots, s^*_N) \) is a Bayesian Nash equilibrium if \( s^*_i(\eta_i) \) solves (7), assuming that types of different players are independent.

\[
s^*_i(\eta_i) \in \arg\max_{s_i \in S_i} \sum_{\eta_{-i}} f_H(\eta_{-i})u_i(s_i, s_{-i}; \eta_i, \eta_{-i}) \tag{7}
\]

As it is proven in [8], the existence of a Bayesian Nash equilibrium is an immediate consequence of the Nash existence theorem. As a result, considering that the potential games have shown to always converge to a Nash Equilibrium when a best response adaptive strategy is applied, it can be derived that for the Bayesian Potential game \( \Gamma \) there exists a Bayesian Nash equilibrium, which maximizes the expected utility function defined in (7).

IV. SIMULATION SCENARIO

The scenario considered to evaluate the proposed framework consists of a circular area with radius \( R_{\text{max}} = 150 \) mt. With respect to the strategy space, the set of power levels \( P^S = (p^S_1, \ldots, p^S_m) \) is defined as \( P^S = (0, 5, 10, 15, 20) \) dBm, i.e. \( m = 5 \). On the other hand, the SUs can be scheduled over \( l = 4 \) available frequency channels, so that the set of channels \( C = (c_1, \ldots, c_l) \) is defined as \( C = (1, 2, 3, 4) \). Each channel is assumed to have a bandwidth \( B_s = 200 \) KHz. We consider \( M = 4 \) PUs pairs, one pair for each frequency channel, and \( N \) SUs pairs, which at simulation start are randomly distributed over the \( l \) frequency channels. The PUs pairs are randomly located in the scenario. Specifically, the maximum distance between a PU transmitter and a PU receiver is randomly selected depending on their random position in the coverage area. On the other hand, the maximum distance between a SU transmitter and receiver is 20 mt. We consider a wireless channel gain of \( h_{ii} = \left( \frac{10}{\eta_{ii}} \right) \), where \( d_{ii} \) is the distance from transmitter \( i \) to receiver \( i \). The transmission power of a PU is 43 dBm. The minimum SINR for a user not to be in outage is \( \gamma = 3\text{dB} \). In order to define the PDF of the wireless channel gains, we proceed by simulations. We discretize the random variable \( R \) representing the distance between two channel nodes, and accordingly the possible values of wireless channel gains, into \( K \) equally spaced values. In this way we generate a path loss probability mass function (PMF) of the wireless channel gains, which is represented in Fig. 3.

V. DISCUSSION

In this section we compare simulation results in terms of outage probability and SINR, for SUs and PUs, when partial (Bayesian Potential game - BPG) and complete (Potential game - PG) information are considered. Notice that for the game with complete information, the hypothesis of a CCC where PUs and SUs share their communication information is made. Additionally, the cooperative approach is compared to the case when cooperation is not considered, that is, when \( p^S_i = 0 \) and \( p^S_i = p^S_i \). Fig. 4 compares the behavior of the BPG and the PG. It can be noticed how the lack of complete information slightly reduces performances in terms of SINR for both PUs and SUs. Performances can be also significantly improved for PUs by including cooperation in the proposed games, as it is shown in Fig. 5. PUs’ performances are particularly benefited by the cooperative scheme in those cases when the SU’s transmitter is close to the PU’s receiver, since the message relayed by the SU is received with a higher quality by the PU receiver. However, the cooperative approach reduces the SUs performances, which is the price to pay for being allowed to access primary channels. These simulation results have been obtained considering that the parameter \( b \) in (4), is 10. By reducing \( b \) we can discourage the SUs from increasing their transmission power, so reducing the harmful interference caused to the PUs. Let us consider two different values of \( b \) for which both the cooperative and non cooperative games provide the PUs with less than 3% of outage probability, (i.e. \( b = 10 \), for the cooperative BPG and \( b = 0.05 \) for the non cooperative BPG). It can be observed from Fig. 6 that even if the PUs’ results in terms of outage are comparable, the SUs’ performances are reduced, when considering a lower \( b \), due to their lower transmission power levels. This demonstrates that, under the condition of limited interference on the PUs, also the SUs are benefited by cooperation. In fact, they are allowed to transmit with higher power levels, as long as they devote a part of it to relaying primary communications, which results more favorable to them than not cooperating and reducing the \( b \) parameter of the game. Finally, it is worth noting that better results in all the cases could be obtained by considering a “no-talk” protected region around the PU transmitter, as already proposed in different papers available in literature [9][10]. However, for the sake of simplicity, this assumption is not made in this paper since it would not modify the behaviour of the players.
VI. CONCLUSION

In this paper we have introduced a Bayesian Potential game to model joint channel and power allocation for cooperative cognitive radios with incomplete information. In particular, we have proposed a cooperative scheme where SUs are allowed to use licensed channels as long as they provide compensation to PUs by means of cooperation. We have modeled this cooperative scheme through a Potential game, which is always characterized by a pure Nash equilibrium. In addition to this, in order to avoid the implementation of a CCC, we have considered a game with incomplete information, where SUs are unaware of the wireless channel gains of the other PUs and SUs. We have shown that the resulting Bayesian Potential game is characterized by a pure Bayesian equilibrium. Simulation results have shown that cooperation benefits both PUs and SUs and that the hypothesis of incomplete information only slightly reduces performance results with respect to the case of complete information.

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REFERENCES
APPENDIX A

We prove that the game with the utility function defined in (4) and the potential function $Pot(S, H)$ defined in (6) is an exact potential game. The proposed potential function consists of three contributions:

$$Pot(S, H) = W(S, H) + X(S, H) + Y(S, H)$$  \hspace{1cm} (8)

where:

$$W(S, H) = W(s_i, s_{-i}; \eta_i, \eta_{-i}) = \sum_{i=1}^{N} \left( \sum_{j=1}^{M} (p^{S'}_i - p^S_i) h^{SS}_{ij} f(c_i, c_j) \right)$$  \hspace{1cm} (9)

$$X(S, H) = X(s_i, s_{-i}; \eta_i, \eta_{-i}) = \sum_{i=1}^{N} \left( -a \sum_{j=1}^{M} p^{S'}_j h^{SS}_{ji} f(c_j, c_i) - (1-a) \sum_{j=1, j \neq i}^{N} p_i^{S'} h^{SS}_{ij} f(c_j, c_i) \right)$$  \hspace{1cm} (10)

$$Y(S, H) = Y(s_i, s_{-i}; \eta_i, \eta_{-i}) = \sum_{i=1}^{N} b \log(1 + p^{S'}_i h^{SS}_{ii})$$  \hspace{1cm} (11)

The first term $W(s_i, s_{-i}; \eta_i, \eta_{-i})$ can be rewritten in the following way:

$$W(s_i, s_{-i}; \eta_i, \eta_{-i}) = \sum_{j=1}^{M} (p^{S'}_i - p^S_i) h^{SP}_{ij} f(c_i, c_j)$$  \hspace{1cm} (12)

$$+ \sum_{k=1, k \neq i}^{N} \left( \sum_{j=1}^{M} (p^{S'}_k - p^S_k) h^{SK}_{kj} f(c_k, c_j) \right)$$

$$= \sum_{j=1}^{M} (p^{S'}_i - p^S_i) h^{SP}_{ij} f(c_i, c_j) + W(s_{-i}, \eta_i, \eta_{-i})$$

where

$$W(s_{-i}; \eta_i, \eta_{-i}) = \sum_{k=1, k \neq i}^{N} \left( \sum_{j=1}^{M} (p^{S'}_k - p^S_k) h^{SK}_{kj} f(c_k, c_j) \right)$$  \hspace{1cm} (13)

and it does not depend on the strategy of player $i$.

As for the second term, it can be rewritten as follows:

$$X(s_i, s_{-i}; \eta_i, \eta_{-i}) =$$

$$-a \sum_{j=1, j \neq i}^{N} p_j^{S'} h^{SS}_{ji} f(c_j, c_i) - (1-a) \sum_{j=1, j \neq i}^{N} p_i^{S'} h^{SS}_{ij} f(c_i, c_j)$$  \hspace{1cm} (14)

$$+ \sum_{k=1, k \neq i}^{N} \left( -a \sum_{j=1, j \neq k}^{N} p_j^{S'} h^{SS}_{kj} f(c_j, c_k) - (1-a) \sum_{j=1, j \neq k}^{N} p_k^{S'} h^{SS}_{kj} f(c_k, c_j) \right)$$

$$= -a \sum_{j=1, j \neq i}^{N} p_j^{S'} h^{SS}_{ji} f(c_j, c_i) - (1-a) \sum_{j=1, j \neq i}^{N} p_i^{S'} h^{SS}_{ij} f(c_i, c_j)$$  \hspace{1cm} (15)

$$+ \sum_{k=1, k \neq i}^{N} \left( -a p_i^{S'} h^{SS}_{ik} f(c_k, c_i) - (1-a) p_k^{S'} h^{SS}_{ki} f(c_k, c_i) \right)$$

The last term does not depend on $s_i$, so that

$$X(s_i, s_{-i}; \eta_i, \eta_{-i}) =$$

$$-a \sum_{j=1, j \neq i}^{N} p_j^{S'} h^{SS}_{ji} f(c_j, c_i) - (1-a) \sum_{j=1, j \neq i}^{N} p_i^{S'} h^{SS}_{ij} f(c_i, c_j)$$  \hspace{1cm} (16)

$$+ \sum_{k=1, k \neq i}^{N} \left( -a p_i^{S'} h^{SS}_{ik} f(c_k, c_i) - (1-a) p_k^{S'} h^{SS}_{ki} f(c_k, c_i) \right)$$

Finally, $Y(s_i, s_{-i}; \eta_i, \eta_{-i})$ can be rewritten as:

$$Y(s_i, s_{-i}; \eta_i, \eta_{-i}) = b \log(1 + p_i^{S'} h^{SS}_{ii}) + Y(s_{-i}; \eta_i, \eta_{-i})$$  \hspace{1cm} (17)

where $Y(s_{-i}; \eta_i, \eta_{-i}) = \sum_{k=1, k \neq i}^{N} b \log(1 + p_k^{S'} h^{SS}_{kk})$, and it does not depend on $s_i$.

As a result,

$$Pot(s_i, s_{-i}; \eta_i, \eta_{-i}) =$$

$$\sum_{j=1}^{M} (p^{S'}_i - p^S_i) h^{SP}_{ij} f(c_i, c_j)$$  \hspace{1cm} (18)

$$- \sum_{j=1, j \neq i}^{N} p_j^{S'} h^{SS}_{ji} f(c_j, c_i) - \sum_{j=1, j \neq i}^{N} p_i^{S'} h^{SS}_{ij} f(c_i, c_j)$$

$$+ b \log(1 + p_i^{S'} h^{SS}_{ii}) + W(s_{-i}; \eta_i, \eta_{-i}) + X(s_{-i}; \eta_i, \eta_{-i})$$

$$+ Y(s_{-i}; \eta_i, \eta_{-i})$$

$$= \left( -a - 1 + a \right) \sum_{j=1, j \neq i}^{N} p_j^{S'} h^{SS}_{ji} f(c_j, c_i)$$  \hspace{1cm} (19)

$$- \left( 1 - a \right) \sum_{j=1, j \neq i}^{N} p_i^{S'} h^{SS}_{ij} f(c_i, c_j)$$

and it is a function that does not depend on the strategy of player $i$. As a result, if player $i$ changes its strategy from $s_i$ to $s'_i$, we then obtain that:

$$Pot(s_i', s_{-i}; \eta_i, \eta_{-i}) = u(s_i', s_{-i}; \eta_i, \eta_{-i}) + F(s_{-i}; \eta_i, \eta_{-i})$$  \hspace{1cm} (20)

$$+ Y(s_{-i}; \eta_i, \eta_{-i})$$

and consequently

$$Pot(s_i, s_{-i}; \eta_i, \eta_{-i}) - Pot(s'_i, s_{-i}; \eta_i, \eta_{-i}) = u(s_i, s_{-i}; \eta_i, \eta_{-i}) - u(s'_i, s_{-i}; \eta_i, \eta_{-i})$$  \hspace{1cm} (21)