Abstract—In this paper we propose network coding to perform multi-hop multicast wireless transmissions. More specifically, we address the problem of reducing redundancy in the completely distributed operation of network coding, so as to increase network throughput. In order to do so, we propose a distributed system where network nodes autonomously make decisions with respect to the packets to encode and forward, with the goal of maximizing the data detection probability and minimizing the overhead of transmitted packets. Our setup consists of a Slotted Aloha network where nodes inject packets at a rate which depends on how useful these packets are to other nodes. We model the selection of transmission probabilities by means of a game theoretic approach, in which nodes reach the desired transmission probabilities as an equilibrium solution to the game. Simulation results show that the proposed scheme outperforms a Slotted Aloha system with optimal uniform retransmission probability.

I. INTRODUCTION

Wireless networks involving multiple wireless hops, such as ad-hoc, mesh, vehicular, or relay networks, have recently been the object of considerable research activity. The main motivation behind such interest is that multi-hop wireless transmission is inefficient due to channel errors on one hand and to poor medium access efficiency on the other. Distributed coding techniques have proven useful in increasing the throughput of multihop transmission. At the physical layer, distributed space-time codes have been proposed to improve the efficiency of wireless relaying. At upper layers, network coding has proved useful as well, achieving capacity in multicast transmissions [1],[2],[3]. Moreover, network coding adds redundancy in a completely distributed fashion, allowing end nodes to recover the transmitted message in the presence of channel erasures [4]. Some proof of concept implementations of wireless network coding have also been described [5].

Practical network coding schemes have been proposed in [6], consisting in inserting the encoding vector at each transmitted packet, with a small loss in efficiency. However, while network coding increases the robustness of the transmitted message by adding redundancy, it also may decrease the actual data throughput. The distributed nature of the encoding scheme makes it difficult to control the amount of redundancy added and requires the network nodes to make decisions about which packets are worth retransmitting, and which have to be discarded. The decision criteria are based on the amount of redundancy required for the correct decoding of information, i.e. it is not necessary to encode and forward linearly dependent packets, and on the time stamp of the received packets, i.e. old packets may have already been forwarded by other nodes, or may bring no longer significant information. These individual decisions strongly depend on the decisions made by the other nodes, since each node may decide to retransmit encoded packets at any time, thus generating collisions. As a result, the interactions with the multiple access and flow control have to be taken into account when evaluating network throughput. For this reason, we propose a game theoretic framework, already proven good at analyzing interactions in decision making processes, as a mechanism to control multiple access and network code rate, by means of the distributed selection of transmission probabilities of the network nodes.

We consider a Slotted Aloha system, where network-coded multicast transmissions take place. By setting the appropriate utility function, we find an equilibrium solution where nodes are encouraged to transmit packets depending on how useful their transmissions are, i.e. a node transmission probability increases with the amount of linearly independent packets in its memory and decreases with message age. Game-theoretic equilibrium solutions for non-network-coded Slotted Aloha have been considered in [7] and [8], but in both cases a uniform transmission probability was found at equilibrium. We show that such approach performs worse than our proposed scheme. Moreover, including network coding has the remarkable advantage of eliminating the need for retransmissions; therefore, the Slotted Aloha system is always stable. Simulation results show that the proposed scheme achieves higher throughput and higher detection probability than a Slotted Aloha system with optimal uniform transmission probability.

The paper is organized as follows. Section II describes the system model, with details on the network code and access scheme. Section III describes the game theoretic model to control the retransmission probabilities and consequently the
medium access and the network code rate. Finally, Section IV presents simulation results and Section V summarizes the conclusions.

II. SYSTEM MODEL

We consider a network of wireless terminals where a multicast transmission takes place from one or more source nodes to the rest of network nodes. We model the network as a graph \((\mathcal{V}, \mathcal{E})\), where \(v_i \in \mathcal{V}\), \(i = 1, \cdots, M\) are the network nodes and \(e_{i,j} \in \mathcal{E}\) is a hyperarc denoting the wireless connection \(i\) to a set of nodes \(J \in \mathcal{V}\). A subset of nodes \(S \in \mathcal{V}\) are sources of data packets. We model the wireless channel at the packet level as an erasure channel. Channel losses from node \(i\) to \(j\) in \(J\) may be expressed as

\[
PL_{i,j} = d_{i,j}^{-\alpha}|h_{ij}|^2
\]

where \(d_{i,j}\) is the distance between nodes \(i\) and \(j\), \(\alpha\) is the path loss exponent, and \(h_{ij}\) is the fast fading coefficient, which remains constant during the packet duration. We also assume that a packet is lost whenever the minimum signal strength dictated by the receiver sensitivity is not met. Even if such packets may be received in error, we assume that physical layer error detection mechanisms are able to detect and discard such packet, resulting in a packet erasure. We characterize the packet erasure probability \(\epsilon\) of a link as

\[
\epsilon = Pr(PL_{i,j}Ptx < \rho)
\]

where \(P_{tx}\) denotes transmitted power and \(\rho\) the receiver sensitivity. Assuming Rayleigh-distributed amplitude fading, \(|h_{ij}|^2\) follows an exponential distribution, and therefore the erasure probability may be expressed as a function of distance \(d\) as

\[
\epsilon(d) = 1 - e^{-\frac{\rho}{P_{tx}}}\frac{1}{\pi d^2}
\]

In the following we describe the considered random linear network coding and medium access schemes.

A. Random Linear Network Coding

The network described employs random linear network coding to convey messages from the set of sources to all destinations in a multicast fashion. In order to perform network coding, each source node breaks the data message into \(K\) packets, which are encoded to produce \(L\) coded packets. The source encoding function generates encoded packets as linear combinations of data packets, with coefficients drawn from a finite field, as

\[
t_l = \sum_{k=1}^{K} c_{k,l}s_k
\]

where \(t_l\), \(l = 1, \cdots, L\) is a binary vector denoting the \(l^{th}\) encoded packet, \(c_{k,l}\) a random coefficient, and \(s_k\) \(k = 1, \cdots, K\) the \(k^{th}\) data packet. Each intermediate node buffers packets received through incoming hyperarcs. Then, when a transmission opportunity arises, the node sends out a new linear combination of the packets in its buffer:

\[
t_n^{l,out} = \sum_{l=1}^{L} c_{l,n}t_l^{l,b}
\]

where \(L\) is the number of packets \(t_l^{l,b}\) buffered in node \(i\). Upon receiving a packet, each network node checks whether it may be obtained as a linear combination of the packets in its buffer. We will refer to new linearly independent packets as innovative packets, as they contribute to the decoding of data packets. Conversely, we refer to linearly dependent ones as non-innovative. Only innovative packets need to be stored in the node buffer, while non-innovative ones may be safely dropped; therefore, the buffer size is limited to \(K\) packets. As packets advance through the network, they undergo linear combinations at each node they traverse. Since all nodes perform random linear combinations, a linear random network code results. In order to be able to decode data packets, the destination node must be able to invert the linear operations on the \(K\) data packets, i.e. it must know what is known as the global encoding vector of each encoded packet. These constitute the generator matrix of the network code. Stacking the encoded packets at the destination into matrix \(T = [t_1, \cdots, t_{ND}]\) (where \(ND\) is the number of coded packets received by the destination), data packets into matrix \(S = [s_1, \cdots, s_K]\), and global encoding vectors into matrix \(G = [g_1, \cdots, g_{ND}]\), data packets are obtained as

\[
S = G^{-1}T
\]

Two practical aspects must be addressed in order to make network-coded transmission implementable and efficient. First, the global encoding vector must be known by all receivers. Second, each node must determine how many encoded packets to send to ensure that all destinations receive enough innovative packets. The easiest method to make \(G\) known to the receiver is by placing the global encoding vector at the header of each encoded packet. If the packet size is large, then the fixed \(K\)-bit overhead incurred by this operation can be considered negligible. Determining the necessary number of packets generated by each network node is a more challenging problem. In fact, this determines the rate of the code, which must be designed to maximize both throughput and detection probability of data packets. We address this problem in Section III.

B. Medium Access

We consider a random access mechanism based on Slotted Aloha. In slotted Aloha, a packet is transmitted whenever it is generated. It is assumed that all packets are of the same size, and are transmitted synchronously in slots (1 packet per slot). If two or more nodes access the channel in the same slot, then a collision occurs. In this paper we assume that, in the event of a collision, no packet can be detected. It has been noted, however, that in practice a subset of the colliding packets may be detected when received signal powers are different (the
flow control as the output of a non-cooperative game, where processes. In this paper we model multiple channel access and mathematical tools to analyze interactions in decision making in wireless networks [7],[8]. Game theory constitutes a set of which has been often used to model distributed multiple access coding guarantees that the Slotted Aloha network is stable

In a standard (i.e. non-network-coded) Slotted Aloha system, the throughput can be easily characterized as a function of the normalized offered load as

\[ T = \lambda e^{-\lambda} \]  

which is maximized for \( \lambda = 1 \). The offered load may be obtained as the sum of individual arrival rates or transmission probabilities, \( \lambda = \sum_{i=1}^{M} p_i \). If the Aloha network has erasure channels with erasure probability \( \epsilon \), then the throughput is

\[ T = (1 - \epsilon) e^{-\lambda} \]  

but it is still maximized for \( \lambda = 1 \), as erasures do not prevent collisions. One of the main problems of Slotted Aloha access is that of stabilizing channel access. Whenever a collision occurs, all nodes involved go into backlogged state, and attempt a retransmission with probability \( p' \). Drift analysis of the resulting backlog Markov model shows that Slotted Aloha may easily fall into an undesired stable equilibrium point with very low throughput [10]. In order to avoid this situation, a very conservative approach with low offered load must be taken, which also results in low throughput.

In a network-coded system, the throughput is determined by the rate at which innovative packets flow from source(s) to destination(s). Therefore, one must ensure that nodes with innovative packets have sufficient channel access opportunities, and that these are not wasted with transmissions of non-innovative packets. We address this issue in the following section.

III. DISTRIBUTED CONTROL OF RETRANSMISSION PROBABILITIES

In this section, we propose a distributed solution to control transmission probabilities of a network-coded Slotted Aloha network. Our analysis is based on the fundamental assumption that, in network-coded transmission, nodes do not ever go into backlogged state. Whenever a transmission fails, that packet is discarded and a new encoded packet is generated with arrival probability \( p_i, i = 1, \cdots, M \). Therefore, using network coding guarantees that the Slotted Aloha network is stable and greatly simplifies the problem of controlling transmission probabilities. We propose a solution based on game theory, which has been often used to model distributed multiple access in wireless networks [7],[8]. Game theory constitutes a set of mathematical tools to analyze interactions in decision making processes. In this paper we model multiple channel access and flow control as the output of a non-cooperative game, where the players are the nodes of the network and the pure strategies are the actions of transmitting or waiting in a given slot. We will consider that the players use mixed strategies, meaning that they take different actions with different probabilities.

The players make decisions in a distributed fashion, and independently, but they are influenced by the other players decisions. In this context, we are interested in searching an equilibrium point from which no player has anything to gain by unilaterally deviating. This equilibrium point is known as Nash equilibrium. In this section we first formulate the game and then we solve it by finding the equilibrium transmission probabilities at Nash equilibrium.

A. Game-theoretic Modeling

We consider a finite strategic-form game \( \Gamma \), defined as \( \Gamma = \{M, \{\Sigma_i\}_{i \in M}, \{u_i\}_{i \in M}\} \), where:

i) \( M \) is the finite set of players, i.e. nodes in the network.

ii) \( \Sigma_i \) is the set of mixed strategies \( \sigma_i \), associated with player \( i \). \( \Sigma = \times \Sigma_i, i \in M \) is the mixed strategy space. A mixed strategy represents a probability distribution over player \( i \)'s pure strategies \( u_i \in A_i \), i.e. Transmit (T) and Wait (W).

iii) \( u_i : S \rightarrow \mathbb{R} \) is the set of utility (payoff) functions that the players associate with their strategies. For each player \( i \) in game \( \Gamma \), the utility \( u_i \) is a function of \( \sigma_i \), the strategy selected by player \( i \), and of the current strategy profile of the other players, which is usually indicated as \( \sigma_{-i} \). For a finite strategy space, player \( i \)'s utility to profile \( \sigma \) is given by:

\[ u_i(\sigma) = \sum_{a_i \in A_i} \sigma_i(a_i)u_i(a_i, \sigma_{-i}) \]  

When a user transmits, its transmission can either succeed (S), or fail (F). As a result, after taking an action, a player may receive the following payoffs:

- If player \( i \) Transmits AND Succeed, \( u_i = w_i - c \), where \( w_i \) is a weight associated with the status of the buffer of player \( i \) that will be detailed in next subsection, and \( c \) is the transmission cost.
- If player \( i \) Transmits AND Fail, \( u_i = -c \).
- If player \( i \) Waits, \( u_i = 0 \).

B. Equilibrium Transmission Probabilities

Assuming that all players are rational, it is well known that a mixed strategy Nash equilibrium for the game exists. The mixed strategy equilibrium is represented by the transmission probabilities, \( p = (p_1, \cdots, p_M) \) from which no player has incentive to deviate. As a result, at equilibrium, the payoff of each pure strategy, which is part of the mixed strategy with non-zero probability, are the same (this is sometimes referred to as the indifference principle [8]). Based on that, we can obtain the transmission probabilities of the mixed strategy equilibrium. First note that, according to the Slotted Aloha channel access rule, a transmission of node \( i \) succeeds if no other node attempts a transmission in that same slot. This
probability is given by

\[ Pr(tr = 0) = \prod_{j=1, j \neq i}^{M} (1 - p_j) \] (10)

\[ = \prod_{j=1, j \neq i}^{M} A_j \] (11)

where \( A_j = 1 - p_j \). Then, at equilibrium, the payoff of transmitting has to be equal to the payoff of waiting, so that we obtain

\[ w_i Pr(tr = 0) - c = 0, i = 1, \ldots, M \] (12)

which yields

\[ w_i \prod_{j=1, j \neq i}^{M} A_j - c = 0, i = 1, \ldots, M \] (13)

The following proposition provides a solution for the equilibrium transmission probabilities.

**Proposition 3.1:** The equilibrium transmission probabilities for the weighted Slotted Aloha network are given by

\[ p_1 = 1 - \left( \prod_{j=1}^{M} \frac{w_j}{c} \right)^{\frac{1}{M-1}} \frac{c}{w_1} \] (14)

\[ p_i = 1 - \left( 1 - p_1 \right) \frac{w_i}{w_1}, i = 2, \ldots, M \] (15)

provided that the following sufficient condition is given on the set of weights \( w_i \)

\[ \max(w_i) < c^{1/(M-1)} \min(w_i)^{1/(M-2)}, i = 1, \ldots, M \] (16)

**Proof:** See appendix.

At this point an important consideration must be made. While a mixed strategy game is guaranteed to have at least one mixed strategy NE, it need not be unique. In addition, other degenerate NE for pure strategies may exist. While some work has been done towards inducing the game to converge to a certain NE ([11] and references therein), we shall not deal with it in this paper. We shall assume that the proposed NE is attainable and leave such considerations for future work.

In order to increase network throughput, we set the transmission weights according to the following function

\[ w_{i,j} = k_1 c + k_2 / (z_i + 1) - k_3 e^{-k_4(t - t_0(s_i))} \] (17)

where \( k_i \) are constants, \( z_i \) is the number of linearly independent packets in node \( i \), and \( t - t_0(s_i) \) corresponds to the number of slots between the current slot \( t \) and the slot when the first packet of that message was received. This function was obtained heuristically to obtain the desired retransmission behavior. Note that a lower weight will result in a higher transmission probability for node \( i \), therefore (17) has the desired effect of increasing the transmission probability when a node has several packets (which may increase its chance to produce innovative packets) and decrease it when the message becomes older, avoiding excessive redundancy.

We are also interested in considering the case where multiple data sources exist. In this case, we assume that packets from different data sources are encoded separately, and that each node competes for channel access in equation (17) based on the total sum of packets in its buffer, and on the arrival time of the most recent message. If a node decides to transmit, a second step consists in selecting which source should be transmitted. In our setup, we propose to select the source with probability proportional to the number of packets available, and exponentially weighted with the message age, as follows:

\[ Pr(\text{transmit source } s) \propto a_{i,s} e^{-(t-t_0(s_i))} \] (18)

**IV. RESULTS**

The network described previously was simulated in a vehicular environment. Network-coded transmission is appropriate in such environment for the transmission of road safety messages, as these are typically multicast and network coding ensures that they are received without delay by eventually all nodes. In order to model a vehicular scenario, we assume that cars are uniformly separated along the road. Erasure probabilities are obtained as the probability of the received signal strength being below the sensitivity threshold of the receiver. Such channel erasure matrix presents a Toeplitz structure as follows

\[
H = \begin{bmatrix}
0 & \epsilon(d) & \epsilon((M-1)d) & \\
\vdots & \ddots & \ddots & \ddots \\
\epsilon(d) & 0 & \epsilon(d) & \epsilon((M-2)d) \\
\epsilon(M-1)d & \cdots & \cdots & \epsilon(d) & 0
\end{bmatrix}
\]

where \( H_{ij} \) is the channel from transmitter \( j \) to receiver \( i \), and \( \epsilon(d) \) denotes the erasure probability for two cars at distance \( d \) found in Section II. The proposed setup is shown in Figure 1. A network of \( M = 10 \) nodes was simulated in order to evaluate the performance of the proposed scheme. It is assumed that each source generates a data message of \( K=10 \) slots. It is also assumed that nodes set their transmission probabilities for a period of 10 slots, and then update them according to the new equilibrium point. The rest of simulation parameters are summarized in Table I. Parameters \( k_1 - k_4 \) are heuristically selected to satisfy (16).

We first compare the decoding probability (the probability of decoding the data message) of the proposed scheme with

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d )</td>
<td>20 m</td>
</tr>
<tr>
<td>( P_{tx/p} )</td>
<td>37 dB</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>2</td>
</tr>
<tr>
<td>( c )</td>
<td>0.1</td>
</tr>
<tr>
<td>( k_1 )</td>
<td>10</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>0.1</td>
</tr>
<tr>
<td>( k_3 )</td>
<td>0.1</td>
</tr>
<tr>
<td>( k_4 )</td>
<td>0.2</td>
</tr>
</tbody>
</table>
respect to a benchmark system with uniform transmission probability. It is assumed that a packet can be decoded if \( K \) linearly independent combinations of source packets are received. In this simulation, node 1 generates a data message at slot 0. The benchmark system was optimized to maximize packet throughput by setting \( p = 1/M \), which results in a total normalized offered load \( \lambda = 1 \). Note that this transmission probability maximizes the throughput of Slotted Aloha, therefore maximizes detection probability with uniform transmission probability allocation. Figure 2 shows that the decoding failure probability of the proposed scheme decreases much faster than for a system with uniform probability, and is inferior to 1% for less than 130 slots. The benchmark scheme needs over 380 slots to achieve the same performance. Therefore, we increase the throughput by almost 200%. In the next figures we analyze the performance of the proposed system with multiple data sources. In the simulations, node one becomes an active data source at slot 1 and node 3 becomes an active data source at slot 100. Both data messages have a length of 10 slots. Figure 3 shows the strategies followed by players 1 and \( M \) for data sources 1 and 2. We can see how the transmission probability is highest for node 1 source 1 at the beginning, as node 1 is the only player in the game since it is the only player having packets. As time goes by, packets propagate to other nodes which become also players, and the transmission probability of node 1 decreases. Eventually, packets reach node \( M \), which then starts transmitting with non-zero probability. When source 2 starts transmitting packets, the transmission probability for the message from source 1 decreases to zero, as the message becomes outdated. This behavior is desirable because, as the message becomes older, it can be safely assumed that it has already been detected by almost all nodes, and then resources can be concentrated on the more recent message.

Figure 4 shows the number of received encoded packets corresponding to the two sources, and for users 2 and \( M \). In the figure, we can see how packets propagate along the network. First the number of packets circulated around node 2 for source 1 is very high. Then, as transmission probability decreases, the curve saturates. Again, this is a desirable behavior since by then all nodes around node 2 have decoded the packet. The activity then focuses on the message from source 2 (node 3). Similar behavior can be observed around node \( M \), with a delay corresponding to the time it takes for packets to propagate through the network.
V. Conclusion

In this paper we proposed a solution to control multiple access and packet flow in wireless, network-coded multicast transmission. In the setup we considered a Slotted Aloha network-coded network. In order to optimize the flow of innovative packets and maximize data detection probability, we propose a distributed scheme to control transmission probabilities of the network nodes. A game-theoretic framework was used to model the system, and transmission probabilities were found for the game equilibrium point. Simulation results show that, following this setup, higher network throughput and detection probability can be achieved compared to a benchmark system with optimum uniform transmission probability.

VI. Acknowledgment

The authors wish to thank Dr. Mischa Dohler for helpful discussions during the execution of this work.

REFERENCES


APPENDIX

In order to solve (13), we divide all $M$ equations by $\prod_{j=2}^{M} A_j$, which yields

$$\frac{\prod_{j=1, j \neq i}^{M} A_j - d_i}{\prod_{j=2}^{M} A_j} = \frac{A_i - d_i}{A_i} = 0$$

where $d_i = c/w_i$, and in the second equality we used the equation for $i = 1$. Solving for $A_i$ we have

$$A_i = A_1 \frac{d_1}{d_i}$$

Substituting again in the first equation ($i = 1$) we have

$$d_1 = \prod_{j=2}^{M} A_j = \prod_{j=2}^{M} A_i \frac{d_i}{d_j} = (A_1 d_1)^{M-1} d_1 \prod_{j=1}^{M} \frac{1}{d_j}$$

Now solving for $A_1$ we obtain

$$A_1 = \left( \prod_{j=1}^{M} d_j \right)^{\frac{1}{M-1}} \frac{1}{d_1}$$

from which transmission probabilities $p_i = 1 - A_i$ can be obtained.

Now we wish to find a sufficient condition on the transmission cost and the set of weights that ensures that convergence transmission probabilities $p_i$ are always contained in $[0, 1]$. First note that the condition is equivalent to $A_i \in [0, 1]$ for $i = 1, \ldots, M$. A sufficient condition for $A_i > 0$ is that $d_i > 0$, which, in turn, implies that $w_i > 0$. For $A_i < 1$, we write

$$A_i = A_1 \frac{d_1}{d_i} < 1$$

$$d_i > A_1 d_1$$

Substituting for $A_1$ we have

$$c/w_i > \left( \prod_{j=1}^{M} d_j \right)^{\frac{1}{M-1}} \frac{1}{d_1} d_1$$

After some algebraic manipulations we obtain

$$w_i < c \cdot \left( \prod_{j=1, j \neq i}^{M} w_j \right)^{\frac{1}{M-2}}$$

Finally, a sufficient condition for (25) to hold is given in (16).