Beamforming for Space Division Duplexing

Damith Senaratne and Chintha Tellambura
Department of Electrical and Computer Engineering,
University of Alberta, Edmonton, AB, Canada.
Email: {damith, chintha}@ece.ualberta.ca

Abstract—Eigenmode transmission in multiple-input multiple-output (MIMO) systems is examined under space division duplexing (SDD). The antennas of each full-duplex node are partitioned to form two antenna banks – one for transmission, the other for reception. Self-interference is suppressed by utilizing the nullspace (or the left nullspace) of corresponding self-interference channel for transmission (or reception). Simulation results are provided on the error performance. Useful insights are obtained on how finite computational precision and quantization errors affect the feasibility of SDD.

Index Terms—MIMO, space division duplexing, eigenmode transmission, null space

I. INTRODUCTION

In this paper, the use of multiple antennas and spatial signal processing to make a wireless terminal full-duplex is considered. Full-duplex wireless communication may be achieved exploiting the degrees of freedom (DoFs) available in time-, frequency- or any suitable dimension. Frequency division duplexing (FDD) and time division duplexing (TDD) techniques are proven; and their applications are ubiquitous. Spectral efficiency being based on the resource utilization in time- and frequency- dimensions, the use of other independent dimensions to achieve duplexing has become attractive, despite the practical challenges. Space division duplexing (SDD) for single-antenna systems has been attempted [1], [2] in this respect, however, with non-spatial techniques for interference suppression. It is with multiple-input multiple-output (MIMO) technology, which supports system nodes with multiple spatial DoFs, that spatial interference suppression became possible.

Full-duplex MIMO repeaters [3] and relays [4], [5] are already receiving the attention, evidently because of the prospects relaying has on extending the coverage of existing/ emerging MIMO compliant cellular and wireless data networks. In a SDD configuration, a given antenna may not transmit and receive simultaneously over the same frequency band. Therefore, the antennas at a node are partitioned to form 2 banks - one dedicated for transmission, and the other, for reception (e.g. \( N_t \) transmit antennas vs. \( N_r \) receive antennas, in Fig. 1). Duplexing is achieved through signal processing techniques that suppress the self-interference the node’s transmission causes on its own reception.

SDD in MIMO wireless channels resembles suppressing near-end crosstalk in digital subscriber lines (DSLs) [6]. However, the spatial channels in wireless systems are virtual, and arise as a result of transmit/ receive beamforming, whereas the wire-pairs of a DSL exist physically. This distinction makes SDD more challenging than crosstalk cancellation.

SDD also pose significant practical challenges in the form of its high amplifier dynamic range requirement, and the high analog-to-digital converter (ADC) resolution requirement. Inspired by new experimental evidence [7] on achieving over 45 dB of spatial interference suppression, SDD techniques are investigated with a renewed interest.

The simplest, and perhaps, the most obvious approach for self-interference cancellation is temporal. It involves assessing and subtracting self-interference from the received signal [4, Sec. III]. Its variant for full-duplex relay nodes is regarding self-interference a ‘feedback’ (as in a control system), and optimizing the relay gain matrix for interference suppression [3], [5], [8].

Spatial interference mitigation is an alternative, in which the transmit precoding matrix \( (w) \) and/ or receiver reconstruction matrix \( (r) \) are chosen such that the self-interference, irrespective of the data being transmitted in either direction, has zero (or negligible) effect at the input of the detector. Such techniques are based on: (i) the additional spatial DoFs transmit (or receive) antennas of a node has [4], [7], [9], or (ii) the orthogonality of distinct spatial modes in the self-interference channel [10]. Joint optimization of transmitter-, relay- and receiver- processing for full-duplex relaying too has been considered [11].

The aim of this paper is exploring spatial self-interference mitigation techniques usable for MIMO SDD eigenmode transmission. The paper is organized as follows: Section II presents the mathematical framework. Numerical results on the performance of selected MIMO SDD configurations are provided in Section III. The conclusion follows, highlighting certain limitations that need to be overcome to realize SDD.

Fig. 1. A MIMO node transmitting and receiving over same frequency band.
Notation: Given a matrix $A$, its transpose, conjugate transpose, rank, nullity, nullspace, and left nullspace are denoted: $A^T$, $A^H$, rank($A$), nullity($A$), Null($A$), Null($A^T$) respectively [12]. $\{A\}_C(n)$ and $\{A\}_R(m)$ give the sub-matrices of $A$ formed with its first $n$ columns, and first $m$ rows, respectively. $\{A\}_C(m,n)$ is the sub-matrix of $A$ formed with its columns $m$ through $n$. Main diagonal of $A$ is given by diag($A$). $A \in \mathbb{C}^{m \times n}$ denotes that $A$ is an $m \times n$ matrix. The notation $[A_1, A_2]$ represents the concatenation of matrices $A_1$ and $A_2$.

II. MATHEMATICAL FRAMEWORK

A. SDD through nullspace & left nullspace projection

The singular value decomposition (SVD) of a matrix $G \in \mathbb{C}^{m \times n}$ is of the form $G = U \Sigma V^H$, where (i) $\Sigma \in \mathbb{C}^{m \times n}$ is nonnegative real rectangular diagonal; and (ii) $U \in \mathbb{C}^{m \times m}$, $V \in \mathbb{C}^{n \times n}$ are unitary. 

Suppose $G$ does not have full column-rank (i.e. $r = \text{rank}(G) < n$). The columns of $V^{(0)} = \{V\}_C(r+1:n)$ span Null($G$), such that $G V^{(0)} = 0 \in \mathbb{C}^{m \times (n-r)}$. Similarly, Null($G^T$) is spanned by the columns of $U^{(0)} = \{U\}_C(r+1:n)$ such that $(U^{(0)})^H G = 0 \in \mathbb{C}^{(m-r) \times n}$, whenever $G$ does not have full row-rank (i.e. $r < m$). The nullspace and the left nullspace exist simultaneously iff $G$ is rank deficient (i.e. $r < \min(m,n)$).

Suppose $G$ corresponds to the self-interference channel of the MIMO capable node shown in Fig. 1. Given $x$, the symbols to be transmitted, self-interference component at the detector input is given by the term $r G w x$. The interference can be nullified irrespective of $x$, if either of the constraints:

\begin{align}
G w &= 0, \\
r G &= 0,
\end{align}

(can be enforced. The constraints (1a) and (1b) provide three possibilities for implementing SDD at a node.

1) Transmit SDD: Forming $w$ with columns of $V^{(0)}$ enforces (1a). It makes transmitted signal $w x$ to be orthogonal to $G$. This approach requires $G$ not to have full-column rank, a sufficient condition for which is allotting more antennas for transmission than for reception.

2) Receive SDD: Forming $r$ using rows of $(U^{(0)})^H$ enforces (1b). The desired received signal component is forced to be orthogonal to the row space of $G$. This approach requires $G$ to not have full-row rank, guaranteed if the majority of antennas are set aside for reception.

3) Joint Transmit and Receive SDD: Simultaneously enforcing (1a) and (1b) as in reference [4], requires $G$ to be rank deficient. This may only be achieved through proper antenna design and placement (e.g. by arranging a key-hole channel to exist between the antenna banks).

Since $G$ is not bidirectional, ‘Joint Transmit and Receive SDD’ appears redundant. Moreover, it complicates beamforming when two nodes implementing SDD communicate. Hence, we focus only on ‘Transmit SDD’ and ‘Receive SDD’.

B. Eigenmode transmission with SDD

Consider two MIMO capable nodes: Node-$i$, $i \in \{1,2\}$ (see Fig. 2), each having a subset of $M_i$ antennas set aside for transmission, and the remaining $N_i$ antennas dedicated for reception. The transmit (or receive) antennas of a given node need not be physically adjacent.

Suppose the forward MIMO channel from Node-$i$ is $H_i \in \mathbb{C}^{N_i \times M_i}$ for $i, j \in \{1,2\}$, $i \neq j$. Its self-interference MIMO channel $G_i \in \mathbb{C}^{N_i \times M_i}$ may or may not be rank deficient\(^2\). $w_i \in \mathbb{C}^{M_i \times 1}$ and $r_i \in \mathbb{C}^{N_i \times 1}$ are the transmit precoding and receiver reconstruction matrices. $x_i \in \mathbb{C}^{M_i \times 1}$ denotes the signal transmitted by Node-$i$, while $y_i \in \mathbb{C}^{N_i \times 1}$ is the signal it receives. $n_i \in \mathbb{C}^{N_i \times 1}$ is the additive noise component at reception. The received signal at the detector input of each Node-$i$ is then given by

\begin{equation}
\begin{aligned}
y_i &= r_i (H_j w_j x_j + G_i w_i x_i + n_i).
\end{aligned}
\end{equation}

Suppose $s_i$ spatial modes need to be facilitated from each Node-$i$ to the other. This requires

\begin{equation}
\text{rank}(H_i) \geq s_i,
\end{equation}

and, either of

\begin{align}
\text{nullity}(G_i) &\geq s_i, \quad \text{or} \quad (4a) \\
\text{nullity}(G_i^T) &\geq s_j, \quad (4b)
\end{align}

to be satisfied for $i, j \in \{1,2\}, i \neq j$.

1) Case: Transmit SDD implemented at both nodes:

Design requirements: A necessary, but not sufficient condition for (3) is having $N_i \geq s_i$. The requirement (4a) can be met, irrespective of rank($G_i$), by ensuring that $(M_i - N_i) \geq s_i$. Where $H_i$s are not rank-deficient, the requirements are satisfied for $(M_i - s_i) \geq N_i \geq s_j$.

Example 1: Having $M_i = 4$ and $N_i = 2$, for instance, guarantees 2 spatial modes in each direction, provided $H_i$s in $\{1,2\}$ are not keyhole channels. If communications were only from Node-1 to Node-2, each node would have had 6 DoFs; but SDD yields only 4 spatial modes.

\(^2\)Rank deficiencies in $G_i$s would lessen the spatial DoFs SDD costs.
Beamforming matrices: Suppose the SVDs: \( \mathbf{G}_i = \mathbf{U}_i \Sigma_i \mathbf{V}_i^H \) hold for \( i \in \{1, 2\} \). The columns of each \( \mathbf{V}_i^{(0)} = \{ \mathbf{v}_i \}_{\mathbb{C}^{\text{rank}\left(\mathbf{G}_i\right)+1:M_i}} \) span \( \text{Null}(\mathbf{G}_i) \). Define \( \hat{\mathbf{H}}_i = \mathbf{H}_i \mathbf{V}_i^{(0)} \) for \( i \in \{1, 2\} \), and let their SVDs be \( \hat{\mathbf{H}}_i = \mathbf{Q}_i \Lambda_i \mathbf{W}_i^H \).

The choice of \( \mathbf{w}_i = \mathbf{V}_i^{(0)} \} \mathbf{W}_i \} \mathcal{C}(s_i) \) and \( \mathbf{r}_j = \{ \mathbf{Q}_j^H \} \mathcal{R}(s_i) \) therefore produces the spatial modes in both directions.

Remarks:
- The effective MIMO channel \( \hat{\mathbf{H}}_i \) is \( N_j \times \text{nullity}(\mathbf{G}_i) \), and no longer \( N_j \times M_i \). This implies reduced diversity orders. Since \( \text{rank}(\hat{\mathbf{H}}_i) \leq \min(\text{rank}(\mathbf{H}_i), \text{nullity}(\mathbf{G}_i)) \) a loss of multiplexing gain too is apparent.
- Under Rayleigh fading, each \( \mathbf{H}_i \) would be a complex Gaussian random matrix; \( \mathbf{H}_i \mathbf{V}_i \) would have the same distribution since \( \mathbf{V}_i \) is unitary. Therefore, \( \hat{\mathbf{H}}_i \) would also be complex Gaussian irrespective of the distribution of \( \mathbf{G}_i \). This premise makes performance analysis of MIMO SDD under Rayleigh fading straightforward.
- Channel estimation may be easily performed, for example, by TDD the pilot signals, and estimating each \( \mathbf{G}_i \), \( \mathbf{H}_i \) pair while \( \text{Node-}i \) transmits the pilots, for \( i, j \in \{1, 2\}, i \neq j \).
- Transmit SDD requires that each Node-\( i \) (i) receives channel state information (CSI) for the forward channel \( \hat{\mathbf{H}}_i \) from \( \text{Node-}j \); (ii) computes \( \mathbf{w}_i, \mathbf{r}_j \) as outlined above; and (iii) conveys \( \mathbf{r}_j \) and the gains \( \text{diag}(\Lambda_i) \) back to \( \text{Node-}j \).

2) Case: Receive SDD implemented at both nodes:

Design requirements: Where \( \mathbf{H}_i \), \( \mathbf{S} \) are not rank-deficient, the requirements (3), (4b) are satisfied for \( (N_i - s_i) \geq M_i \geq s_i \).

Beamforming matrices: Suppose the SVDs: \( \mathbf{G}_i = \mathbf{U}_i \Sigma_i \mathbf{V}_i^H \) hold for \( i \in \{1, 2\} \). Each \( \mathbf{U}_i^{(0)} = \{ \mathbf{u}_i \}_{\mathbb{C}^{\text{rank}(\mathbf{G}_i)+1:N_i}} \) would span \( \text{Null}(\mathbf{G}_i^T) \). Define \( \hat{\mathbf{H}}_i = \mathbf{U}_i^{(0)} \mathbf{H}_i \mathbf{V}_i^{(0)} \) for \( i, j \in \{1, 2\}, i \neq j \); and let their SVDs be \( \hat{\mathbf{H}}_i = \mathbf{Q}_i \Lambda_i \mathbf{W}_i^H \).

Choosing \( \mathbf{w}_i = \{ \mathbf{w}_i \}_i \mathcal{C}(s_i) \) and \( \mathbf{r}_j = \{ \mathbf{U}_j^T \mathbf{Q}_j \} \mathcal{R}(s_i) \) would yield the desired spatial modes.

Remarks:
- The effective channel \( \hat{\mathbf{H}}_i \) is nullity \( \{ \mathbf{G}_i^T \} \times M_i \). Moreover, 

\[
\text{rank}(\hat{\mathbf{H}}_i) \leq \min\{\text{rank}(\mathbf{H}_i), \text{nullity}(\mathbf{G}_i^T)\}
\]

- A loss of diversity and multiplexing gains results in.
- Swapping the transmit/ receive role of each antenna should convert a given Receive SDD configuration to a Transmit SDD configuration exhibiting equivalent error performance, and vice versa. Receive SDD appears simpler in practice, since it requires only the \( \mathbf{w}_i \)s to be exchanged over the channel as an overhead.

3) Case: Transmit SDD implemented at one node, and Receive SDD at the other:

Without a loss of generality, suppose that Node-1 implements Transmit SDD, while Node-2 implements Receive SDD.

The requirements (3) and (4) are met if \( (M_1 - s_1) \geq N_1 \geq s_2 \) and \( (N_2 - s_1) \geq M_2 \geq s_2 \). The effective channel for

3Defining \( \hat{\mathbf{H}}_i = \mathbf{H}_i \{ \mathbf{v}_i^{(0)} \}_i \mathcal{C}(s_i) \), using \( s_i \) columns from \( \mathbf{v}_i^{(0)} \) too is possible here. It would however yield lower diversity orders.

![Fig. 3. SNR vs. average SER curves in either direction of \( \{M_1, N_1\}_{s_1} \leftrightarrow \{M_2, N_2\}_{s_2} \} \) MIMO SDD configurations.](image)
Rayleigh fading (just as corresponding $H_i$ does). The loss of diversity gains is implicit. Since only 5 spatial modes are facilitated with 11 antennas at Node-1, and 8 antennas at Node-2, a loss of 3 spatial DoFs is also apparent. These losses represent the cost of SDD; the benefit is, obviously, the duplexing capability.

From a mathematical point of view, the SDD techniques we have examined suppress the self-interference perfectly. That is not so in practice, when finite computational precision (in transmitter- and receiver- signal processing) and/or quantization errors (at analog-to-digital conversion) are in effect.

Fig. 5 depicts approximately\(^4\), how the number of significant digits of computation affects the average SER, using the \{4, 2\}_2\rightarrow\{4, 2\}_2 MIMO SDD configuration. \(10^5\)-point Monte-Carlo simulation has been used; other assumptions are as same as before. The SER floors hint the presence of un-mitigated interference. Apparently, self-interference does not get suppressed for precisions less than 6-digits. The effect of truncation errors is evident even at 6-digit precision. However, the error performance improves rapidly as the number of significant digits of computation increases beyond a threshold, that depends on the ratio of transmit and receive signal strengths (note: \(\log_{10}(\sqrt{100\ dB}) = \frac{1}{2}\log_{10}(10^{10}) = 5\)).

Low resolution of the ADC is another concern. It gives rise to quantization errors, and hence, to increased average SERs.

**Example 2:** If each element of $G_i$s is zero mean complex Gaussian with $2\sigma^2$ variance, each of the real and imaginary components of the self-interference may lie in the \((-8\sigma, 8\sigma)\) range, at \(\text{erf}(8/\sqrt{2}) = 0.999999999999998\) (i.e. practically 1) probability. An \(n\)-bit linear quantizer for that range would be able to resolve only up to \(\Delta = 16\sigma \sqrt{2}/\pi\). Corresponding quantization error \(\frac{\Delta}{2}\) needs to be insignificant with respect to both the desired signal and the self-interference.

The effect of quantization is severe than that of finite computational precision, because linear quantization at a wide dynamic range is required for SDD to function properly. Self-interference dominates the received signal; hence, the dynamic range depends on the $G_i$s. Linear quantization is required since the self-interference is additive.

Fig. 6 illustrates the effect the quantization errors the 10-, 12-, 14- and 16-bit ADCs introduce on the average SER.

Assumptions: Elements of $H_i$s have unit variance, while those of $G_i$s have 40 dB variance\(^5\). Midtread quantization at a dynamic range of 16\(\sigma\) is considered, where $\sigma = \sqrt{10^4/2}$. 10 data symbols, per spatial mode, per channel realization are assumed; along with 10 pilot symbols, per transmit antenna, per channel realization. Least square method is used for channel estimation.

\(^4\)Approximate, because the internal precision of MATLAB’s ‘svd’ routine was not restricted. Inputs and outputs of the routine were nevertheless truncated to have the desired number of significant digits.

\(^5\)An order of separation above 40 dB is not achievable with the ADC resolutions considered. Additional \(K\) dB separation would approximately require extra $\frac{1}{2}\log_2(10^{10.1K})$ bit precision at the ADC.
$10^6$-point Monte-Carlo simulation has been used; and other assumptions are as with Fig. 3. Error rates improve with the number of bits the ADCs output per sample. An abrupt degradation of error performance can be seen in the first spatial mode (i.e. $k = 1$) as the precision reduces from 12-bits to 10-bit. A likely reason for it is having $\log_2(16\sigma) = 10.1439$. Moreover, an error floor, which is common with systems affected by SNR invariant errors, can be seen in the average SER curves.

Quantization of the pilot symbols gives rise to channel estimation errors, which significantly influence the error rates. Fig. 7 confirms the fact for a 14-bit ADC, and the same MIMO SDD configuration and assumptions as with Fig. 6, by comparing the error performance for the following cases:

i) Ch Est + Qnt D&P: both the data and pilots (used for channel estimation) quantized;
ii) Ch Est + Qnt D: data quantized, but not the pilots;
iii) Perf CSI + Qnt D: data quantized, perfect CSI assumed;
iv) Ch Est + No Qnt: neither data nor the pilots quantized;
v) Perf CSI + No Qnt: with perfect CSI and no quantization.

The curves corresponding to cases i) and ii) highlight the degradation of the performance quantization of pilots induces. Quantization induced channel estimation errors set an error floor in both the spatial modes. Quantization of data appears to have a less significant effect; an error floor is apparent only with $k = 1$. The cases iv) and v) let the effect of channel estimation errors be assessed in isolation. An error floor does not appear, evidently because the least square method of estimation improves with the SNR. To sum, the non-availability of perfect CSI appears to be the main contributing factor for errors when ADC resolution is coarse.

**IV. CONCLUSION**

Beamforming for eigenmode transmission over MIMO space division duplexing was examined. Associated loss of diversity and multiplexing benefits was highlighted. Further insights were obtained on the adverse effects of finite computational precision and quantization errors on the error rate.

General purpose ADCs operating above $10^7$ samples per second do not currently have resolutions beyond 16-bits [13], [14]. Improving both the sampling rate and the resolution appears to be challenging due to high data rates, and other factors such as synchronization and jitter. This, along with non-linearities in the amplifiers makes suppressing self-interference greater than $40\, \text{dB}$ challenging at present. Amount of self-interference suppression required could be manageable for short-range links; it may be reduced further by using directional antennas. But SDD, as was discussed here, may not be feasible in general until the hardware limitations are overcome.

**ACKNOWLEDGMENT**

This work is supported in part by the Alberta Ingenuity Fund through the iCORE ICT Graduate Student Award.

**REFERENCES**


