Wireless Power Meets Energy Harvesting: 
A Joint Energy Allocation Approach

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Abstract—This paper investigates power allocation design for a wireless powered communication system, where one user harvests radio frequency (RF) energy from an energy access point (EAP) that can be used immediately to power its information transmission to a data access point (DAP). The channels from the EAP to the user and from the user to the DAP are assumed to be time-varying over two separate bands but a priori known. Our objective is to maximize the achievable rate at the DAP by jointly optimizing the power allocation at both links, subject to the sum power constraint at the EAP and the energy causality constraint at the user. We study the structural properties of the optimal power allocation, based on which we propose an efficient algorithm to solve the problem. It is shown that by optimizing energy transmission at the EAP, higher rate can be achieved than a conventional energy harvesting system where the user randomly harvests energy from the environment with the same sum power.

I. INTRODUCTION

Conventional energy-constrained wireless networks in general have limited lifetime. Harvesting energy from renewable energy sources becomes a promising approach to prolong the operation time of wireless networks. Wireless communications with energy harvesting (EH) transmitter in fading channels have been considered in [1], [2] that when the battery at a transmitter has infinite storage, the optimal transmission power over slots follows a staircase water-filling (SWF) structure, where the water-levels (WLs) are nondecreasing over slots. This is opposed to the case that the initial battery energy of the user is zero, the optimal transmission power over slots follows a staircase [2] that when the battery at a transmitter has infinite storage, the optimal transmission power over slots is fixed, then it becomes the previous problem studied in [1], [8] by employing a multi-antenna HAP. Unlike in [4]–[7] which assumes perfect channel state information (CSI) available at transmitters, [8] studies the case of imperfect CSI by considering channel estimation. The capacity of large-size WPCN with geographically distributed users is studied in [9] for cellular network and in [10] for cognitive network, respectively, based on the tool of stochastic geometry. Another line of work on joint wireless energy and information transmission has focused on the so-called SWIPT (simultaneous wireless information and power transfer) in downlink, which aims to characterize the achievable performance trade-off between harvesting energy and decoding information from the same signal waveform (see, e.g., [11]–[13]).

In this paper, we consider a new type of WPCN with one user and two APs, namely, the energy AP (EAP) and the data AP (DAP) (see Fig. 1). Instead of having a fixed power supply, the user is embedded with an RF energy receiver to harvest energy from the EAP. The harvested energy is stored in a rechargeable battery to support its information transmission to the DAP. Unlike the prior work [4] where the EAP and DAP are co-located sharing the same antenna with alternating energy transmission and information reception over time, in this paper we assume separated EAP and DAP; as a result, the user can harvest energy and transmit information at the same time, provided that the energy causality constraint [1, 2] is satisfied, i.e., the total energy consumed for data transmission until any given time should be no greater than the total harvested energy so far. We consider energy/information transmission over $K$ time slots, where the channel from the EAP to the user, i.e., the wireless energy transfer (WET) link, and the channel from the user to the DAP, i.e., the wireless information transfer (WIT) link, vary over slots. Our objective is to maximize the achievable rate at the DAP by jointly optimizing the power allocation over both the WET and WIT links. We note that if the transmit power at the EAP is fixed, then it becomes the previous problem studied in [1], [2]. Hence, an additional design freedom is available via the energy allocation at the EAP.

In this paper, we obtain the optimal solution for the joint WET and WIT energy allocation problem. Specifically, for the case that the initial battery energy of the user is zero, the
optimal solution has the following properties. First, WET can only occur in any causally dominating slot, i.e., a slot that has a larger channel power gain than any of its previous slots in the WET link, the first slot is defined to be causally dominating; while WET cannot occur in other slots. Second, the optimal power allocation at the WIT link performs SWF, where every WL can only increase after any causally dominating slot. For the case of arbitrary initial battery energy, the above structural properties may not hold; instead, we propose an efficient algorithm to solve the problem. Numerical results show that the optimal solution achieves higher rate than a conventional EH communication system with random energy arrivals.

II. SYSTEM MODEL

We consider a wireless powered communication system with one user and two APs, where the user harvests energy from the EAP and sends information to the DAP (see Fig. 1) over two orthogonal bands. We consider energy/data transmission in one block, which is equally divided into $K$ time slots, with each slot being of duration $T$. Let $T = 1$ for convenience. Hence, the two terms energy (per unit slot) and power are used interchangeably in this paper. The slots are indexed in order by $k \in \mathbb{K} = \{1, \ldots, K\}$. During slot $k$, $k \in \mathbb{K}$, the channel power gain from the EAP to the user, i.e., the WIT link, is denoted by $h_k > 0$, and the channel power gain from the user to the DAP, i.e., the WET link, is denoted by $g_k > 0$. It is assumed that all the $h_k$’s and $g_k$’s are a priori known at the beginning of each block transmission, for which we aim to characterize the throughput, similarly as in [1], [2]. The total energy in one block at the EAP is denoted by $Q \geq 0$. The transmission power scheduled during slot $k$ at the EAP is denoted by $p_k \geq 0$. Therefore, the total transmission energy constraint at EAP is $\sum_{k=1}^{K} q_k \leq Q$.

The user contains an energy receiver to harvest energy from the EAP and store the energy in a rechargeable battery to power its information transmitter. We assume the transmission power scheduled by the user during slot $k$ is constant, denoted by $p_k \geq 0$. The stored energy in the buffer battery at time instant $k^-$, i.e., the time instant just before slot $k$, is denoted by $B_k \geq 0$. We assume the energy buffer is unlimited. Also, we assume the stored energy linearly increases with the harvested energy and decreases with the energy consumed by data transmission. Thus, the stored energy is given by $B_{k+1} = B_k + \zeta h_k q_k - p_k$, $k \in \mathbb{K}$, where the initial energy at the buffer battery $B_1 \geq 0$ is known, and $\zeta$ denotes the energy conversion efficiency. Let $\zeta = 1$ for simplicity. The power for information transmission during each slot $i$, $i \in \mathbb{K}$, is no larger than the stored energy at instant $i^-$, i.e., $p_i \leq B_i$, $i \in \mathbb{K}$, which leads to the energy causality constraint

$$\sum_{k=1}^{i} p_k \leq \sum_{k=1}^{i-1} h_k q_k + B_1, \ i \in \mathbb{K}. \quad (1)$$

At the DAP, the receiver noise is modeled as a circularly symmetric complex Gaussian (CSCG) random variable with zero mean and variance $\sigma^2$. Due to frequency orthogonal transmissions of energy and information, the energy signal from EAP will not interfere with the information reception at DAP. Moreover, the gap for the achievable rate from the channel capacity due to a practical modulation and coding scheme is denoted by $\Gamma \geq 1$. The average rate over $K$ slots at the DAP in bps/Hz is thus

$$R = \frac{1}{K} \log_2 \left( 1 + \frac{g_i p_k}{\Gamma \sigma^2} \right). \quad (2)$$

Our objective is to maximize the average rate at the DAP. This leads to the following optimization problem.

$$\max_{\{q_k\}, \{p_k\}} \quad \frac{1}{K} \sum_{k=1}^{K} \log_2 \left( 1 + \frac{g_k p_k}{\Gamma \sigma^2} \right)$$

s.t. \quad $\sum_{k=1}^{K} q_k \leq Q$, \quad (3a)\n
$$\sum_{k=1}^{i-1} p_k \leq \sum_{k=1}^{i-1} h_k q_k + B_1, \ i \in \mathbb{K}. \quad (3b)$$

The optimal solutions are denoted by $q_k^*$ and $p_k^*$, and the maximum average rate by $R^*$. We note that constraint (3a) is satisfied with equality at optimal solution; otherwise, the objective function can be increased by increasing $q_{k-1}$ and $p_k$.

III. OPTIMAL POWER ALLOCATION

First, we investigate the properties for $q_k^*$ and $p_k^*$ for Problem (3). To this end, we define set $D \triangleq \{1\} \cup \{k \in \{2, \ldots, K-1\} : h_k > h_j, \forall 1 \leq j < k\}$. We note that the set $D$ includes the first slot and any slot which has larger channel power gain at the WET link than all its previous slots; hence, the slots in $D$ are called causally dominating slots. For convenience, we also denote set $D$ in order, i.e., $D = \{d_1, d_2, \ldots, d_{|D|}\}$, where $d_i < d_j$ for $i < j$. By the definition of $D$, we have $d_1 = 1$. The complementary set of $D$ is denoted by $D^c$, i.e., $D \cup D^c = \mathbb{K}$ and $D \cap D^c = \emptyset$.

We partition the slot set $\mathbb{K}$ for the WIT link into subsets $D_i \triangleq \{d_i - 1 + 1, \ldots, d_i\}$, $i = 1, \ldots, |D|+1$, referred as the $i$th interval, where we set $d_0 = 0$ and $d_{|D|+1} = K$ for notational simplicity. Thus, $\bigcup_i D_i = \mathbb{K}$ and $D_i \cap D_j = \emptyset$ for $i \neq j$. For the example shown in Fig. 2(a) later, $D = \{1, 5, 23, 27\}$, and the intervals are given by $D_1 = \{1\}$, $D_2 = \{2, \ldots, 5\}$, $D_3 = \{6, \ldots, 23\}$, $D_4 = \{24, \ldots, 27\}$, $D_5 = \{28, \ldots, 40\}$. Fig. 1. A wireless powered communication system with separated EAP and DAP, where the user is equipped with two antennas for energy harvesting and information transmission, respectively, over orthogonal frequency bands.
Proposition 3.1: In Problem 3, \( q_k^* = 0 \) for \( k \in D^c \).

Proof: Please refer to Appendix A.

Proposition 3.1 shows that WET can only occur in the causally dominating slots in \( D \). Intuitively, this is because instead of allocating energy to any slot in \( D^c \), allocating the same amount of energy to an earlier slot in \( D \) which has larger channel power gain at the WET link will result in a larger feasible region for \( \{p_k\} \).

Proposition 3.2: In Problem 3, if \( \{q_k^*\} \) and \( \{p_k^*\} \) satisfy \( \sum_{k=1}^{d_{j-1}} p_k^* = \sum_{k=1}^{d_{j-1}} h_k q_k^* + B_1 \), where \( d_1 \leq d_j \leq d_{D_j} \), i.e., constraint (3b) holds with equality at \( i = d_j \), then we have

\[
\sum_{k \in D_{i+1}} p_k^* = h_k q_k^*, \quad l = j, \ldots, |D|.
\]

Proof: Please refer to Appendix B.

Proposition 3.2 shows that if all the harvested energy is used up at a particular dominating slot, then the harvested energy is used up at later dominating slots.

A. Initial battery energy: \( B_1 = 0 \)

In this subsection, we consider the case \( B_1 = 0 \). With \( B_1 = 0 \) we have \( p_1^* = 0 \), i.e., the first slot is not utilized for information transmission. Hence, constraint (3b) holds with equality at \( i = d_1 \). Define the effective channel power gain as

\[
g_k' = h_d g_k, \quad k \in D_{i+1}, i = 0, \ldots, |D|
\]

where we set \( h_0 \) to 1 for notational simplicity. Define \( \{p_k'\} \) as

\[
p_k' = p_k / h_d, \quad k \in D_{i+1}, i = 0, \ldots, |M|.
\]

From Proposition 3.1 and Proposition 3.2, and using (5) and (6), Problem 3 with \( B_1 = 0 \) is then equivalent to the following problem.

\[
\begin{aligned}
& \text{max.} & & \frac{1}{K} \sum_{k=2}^{K} \log_2 \left( 1 + \frac{g_k' p_k'}{\Gamma \sigma^2} \right) \\
& \text{s.t.} & & \sum_{k=2}^{K} p_k' = Q, \\
&&& \sum_{k \in D_{i+1}} p_k' = q_d, i = 1, \ldots, |D|.
\end{aligned}
\]

We recognize that the optimization over \( \{p_k'\}, k = 2, \ldots, K \) is then a conventional water-filling (WF) problem, because \( \{q_d\} \) can be arbitrarily chosen and thus the last constraint becomes redundant. The optimal \( \{p_k'\} \) is then obtained by the so-called WF power allocation

\[
p_k' = \left( \frac{1}{\lambda K \ln 2} - \frac{\Gamma \sigma^2}{g_k} \right)^+, k = 2, \ldots, K
\]

where \((a)^+ = \max(0, a)\). \( \lambda \) satisfies \( \sum_{k=2}^{K} p_k' = Q \). The water-level (WL) is given by \((\lambda K \ln 2)^{-1}\). From (6) and (8), the optimal \( \{p_k'\} \) for Problem 3 with \( B_1 = 0 \) is obtained as

\[
p_k' = \left( \frac{h_d}{\lambda K \ln 2} - \frac{\Gamma \sigma^2}{g_k} \right)^+, k \in D_{i+1}, i = 1, \ldots, |D|.
\]

Next, we consider the general case of \( B_1 \geq 0 \). We note that in Problem 3, \( \{q_k^*\} \) and \( \{p_k^*\} \) satisfy constraint (3b) with equality at the last slot \( K = d_{D_{i+1}} \); otherwise, the objective function can be increased by increasing \( p_K \). Let \( d_x, 1 \leq x \leq |D| + 1 \), denote the first slot index in \( D \cup \{K\} \) such that \( \{q_k^*\} \) and \( \{p_k^*\} \) satisfy constraint (3b) with equality, i.e.,

\[
\sum_{k=1}^{d_x-1} p_k^* < h_d q_d^* + B_1, i = 1, \ldots, x - 1,
\]

\[
\sum_{k=1}^{d_x-1} p_k^* = h_d q_d^* + B_1.
\]
Lemma 3.1: For Problem (3), \( q_{d_k} = 0 \) for \( k < x - 1 \). The optimal \( \{q_k^*\} \) and \( \{p_k^*\} \) satisfy \( \sum_{k=1}^{d_x} p_{k}^{*} = h_{d_{x-1}} q_{d_{x-1}} + B_1 \).

Proof: Please refer to Appendix C.

Similar to the case of zero initial battery energy, we define the effective channel power gain as
\[
g'_k = \begin{cases} h_{d_{x-1}} g_k, & k = 1, \ldots, d_x, \\ h_{d_k} g_k, & k \in D_{i+1}, i = x, \ldots, |D|. \end{cases} \tag{12} \]

Define \( \{p_k'\} \) as
\[
p_k' = \begin{cases} p_k / h_{d_{x-1}}, & k = 1, \ldots, d_x, \\ p_k / h_{d_k}, & k \in D_{i+1}, i = x, \ldots, |D|. \end{cases} \tag{13} \]

In the following lemma, we show that Problem (3) is equivalent to a problem with optimizing variables \( \{p_k'\} \).

Lemma 3.2: Problem (3) is equivalent to the following problem.

\[
\begin{align*}
\max_{\{p_k'\}} & \quad \frac{1}{K} \sum_{k=1}^{K} \log_2 \left(1 + \frac{g'_k p_k'}{\Gamma \sigma^2} \right) \\
\text{s.t.} & \quad \sum_{k=1}^{K} p_k' \leq Q + B_1 / h_{d_{x-1}}, \tag{14a} \\
& \quad \sum_{k=1}^{d_x} p_k' \leq B_1 / h_{d_{x-1}}, \tag{14b} \\
& \quad \sum_{k=1}^{d_x} p_k' \geq B_1 / h_{d_{x-1}}. \tag{14c}
\end{align*}
\]

The optimal \( \{p_k'\} \) for (3) is obtained by (13); the optimal \( \{q_k\} \) for (3) is obtained by
\[
q_j = \begin{cases} \sum_{k=1}^{d_x} p_k' - B_1 / h_{d_{x-1}}, & j = d_x - 1, \\ \sum_{k \in D_{i+1}} p_k', & j = d_i, i = x, \ldots, |D|, \\ 0, & \text{otherwise}. \end{cases} \tag{15}
\]

Proof: Please refer to Appendix D.

Problem (14) is solved by the following proposition.

Proposition 3.3: For Problem (14), the optimal \( \{p_k'\} \) is either given by
\[
p_k' = \left(1 + \frac{\Gamma \sigma^2}{g_k'} \right)^{-1}, k = 1, \ldots, K \tag{16}
\]
where \( \lambda \) satisfies \( \sum_{k=1}^{K} p_k' = Q + B_1 / h_{d_{x-1}} \); or given by
\[
p_k' = \begin{cases} \left(\frac{1}{\lambda K \ln 2} - \frac{\Gamma \sigma^2}{g_k'} \right)^+, & k = 1, \ldots, d_x, \\ \left(\frac{1}{\lambda K \ln 2} - \frac{\Gamma \sigma^2}{g_k'} \right)^-, & k = d_x + 1, \ldots, K, \end{cases} \tag{17}
\]
where \( \lambda \) and \( \mu \) satisfy \( \sum_{k=1}^{d_x} p_k' = B_1 / h_{d_{x-1}} \) and \( \sum_{k=d_x+1}^{K} p_k' = Q \).

Proof: Please refer to Appendix E.

To summarize, Problem (3) can be solved as follows: for each \( d_x, 1 \leq x \leq |D| + 1 \), solve Problem (14) to obtain \( \{q_k\} \), \( \{p_k\} \), and the objective value, denoted by \( R(d_x) \). The optimal \( d_x \) is then obtained by the \( d_x \) which achieves the largest rate \( R(d_x) \) and the corresponding power allocation \( \{q_k\} \) and \( \{p_k\} \) satisfy the constraints (3b) and (3d).

IV. NUMERICAL EXAMPLE

The transmission power at EAP is assumed to be \( Q = 1 \) Watt (W) or 30 dBm. The distance from EAP to the user is fixed to be 1 m, which results in approximately \(-30 \) dB of signal power attenuation at a carrier frequency 900 MHz. The distance from the user to DAP is denoted by \( d \), with approximately \((-30 - 30 \log_{10} d) \) dB of signal power attenuation. The channel for both WET and WIT links is assumed to be independent and identically distributed (i.i.d.) Rayleigh fading channel. We assume that the receiver noise power at DAP is \( \sigma^2 = -70 \) dBm, and \( \Gamma = 9 \) dB. \( B_1 \) is set to be zero. For comparison, we consider the system in [1], [2] with random energy arrivals at the EH user. In particular, the EAP in Fig. 4 is replaced by an ambient RF transmitter which is oblivious of the WET link, and hence its transmit power over time is random to the user, since it is adapted to its own information transmission link (to another receiver). In simulation, the transmit power at the ambient transmitter \( q_k, k = 1, \ldots, K - 1 \) are randomly generated by the uniform distribution over \([0, 1]\), and then are normalized such that \( \sum_{k=1}^{K} q_k = Q \); \( q_K \) is set to be zero. Hence, during each slot \( k \) a random energy \( h_k q_k \) arrives at the user. With given \( \{h_k q_k\} \), the achievable rate and optimal \( \{p_k\} \) are obtained according to [1], [2]. The achievable rate by this system is obtained from averaging the results from 1000 realizations of \( \{q_k\} \).

Fig. 3 shows the achievable rates by different schemes versus the distance \( d \). It is observed that larger rates are achieved by optimizing the energy allocation at the EAP than by the system with random energy arrivals, and the performance gap is considerably large when the information transmission distance \( d \) is small.

V. CONCLUSION

This paper obtained the optimal power allocation policy for a wireless powered communication system, where a user harvests energy from the EAP to power its information transmission to the DAP. The achievable rate at the DAP is maximized by jointly optimizing the power allocation over both WET and WIT links. Numerical results show that the system achieves...
higher rate than a conventional EH communication system with random energy arrivals, thus demonstrating the benefit of the new wireless powered system with controllable EH source.

**APPENDIX A**

**PROOF OF PROPOSITION 3.1**

The harvested energy during the last slot \( K \) is available at instant \((K+1)^3\), therefore cannot be used for information transmission during the considered block \( K \). Hence, \( q_k^* = 0 \). Next, we prove \( q_k^* = 0 \) for \( k \in \mathcal{D}^c, k \neq K \). For any power allocation \( \{q_k\} \) and \( \{p_k\} \) that satisfy the constraints (3a) and (3b), assume there exists a slot \( i \in \mathcal{D}^c, i \neq K \) with \( q_i > 0 \). By the definition of set \( \mathcal{D} \), there exists a slot \( 1 \leq j < i \) such that \( h_j > h_i \). We construct a power allocation strategy \( \{\hat{q}_k\} \) and \( \{\hat{p}_k\} \) given by

\[
\hat{q}_k = \begin{cases} 
q_j + q_i, & k = j, \\
0, & k = i, \\
q_k, & \text{otherwise}.
\end{cases}
\]

\[
\hat{p}_k = \begin{cases} 
p_k + (h_j - h_i)q_i, & k = i, \\
p_k, & \text{otherwise}.
\end{cases}
\]

It can be verified that \( \{\hat{q}_k\} \) and \( \{\hat{p}_k\} \) satisfy the constraints (3a) and (3b). Since \( h_j \geq h_i \) and \( q_i > 0 \), \( \{\hat{q}_k\} \) and \( \{\hat{p}_k\} \) with \( \hat{q}_i = 0 \) achieve no smaller rate than that by \( \{q_k\} \) and \( \{p_k\} \). Therefore, \( q_k^* = 0 \) for \( k \in \mathcal{D}^c \).

**APPENDIX B**

**PROOF OF PROPOSITION 3.2**

We prove that if \( \{q_k^*\} \) and \( \{p_k^*\} \) satisfy (3b) with equality at \( i = d_j \), where \( 1 \leq j \leq |\mathcal{D}| \), then they satisfy (3b) with equality at \( i = d_{j+1} \), i.e.,

\[
\sum_{k=1}^{d_{j+1}} p_k^* = \sum_{k=1}^{d_{j+1}} h_k q_k^* + B_1.
\]

Note that (19) is satisfied for \( j = |\mathcal{D}| \); otherwise, the objective function in Problem (3) can be increased by increasing \( p_K \).

Next, we prove (18) for the case \( 1 \leq j \leq |\mathcal{D}| - 1 \) by contradiction. Let \( \{q_k^*\} \) and \( \{p_k^*\} \) denote the optimal solutions for Problem (3), then \( \{q_k^*\} \) and \( \{p_k^*\} \) satisfy the constraints (3a) and (3b). Assume that \( \{q_k^*\} \) and \( \{p_k^*\} \) do not satisfy (18), i.e.,

\[
\Delta \triangleq \sum_{k=1}^{d_{j+1}-1} h_k q_k^* + B_1 - \sum_{k=1}^{d_{j+1}+1} p_k^* > 0.
\]

Since \( \sum_{k=1}^{d_{j+1}} p_k^* = \sum_{k=1}^{d_{j+1}} h_k q_k^* + B_1 \), from Proposition 3.1 we have

\[
\Delta = h_{d_j} q_{d_j}^* - \sum_{k=d_j+1}^{d_{j+1}} p_k^*.
\]

Now, we construct a power allocation strategy \( \{\hat{q}_k\} \) and \( \{\hat{p}_k\} \) given by

\[
\hat{q}_k = \begin{cases} 
q_{d_j} - \frac{\Delta}{h_{d_j}}, & k = d_j, \\
q_{d_j+1} + \frac{\Delta}{h_{d_j}}, & k = d_{j+1}, \\
q_k^*, & \text{otherwise}.
\end{cases}
\]

\[
\hat{p}_k = \begin{cases} 
p_{d_j}^* + \frac{(h_{d_j+1} - h_{d_j})\Delta}{h_{d_j}(K - d_{j+1})}, & k = 1, \ldots, d_{j+1}, \\
p_k^*, & k = d_{j+1} + 1, \ldots, K.
\end{cases}
\]

It can be verified that \( \{\hat{q}_k\} \) and \( \{\hat{p}_k\} \) satisfy the constraints (3a) and (3b). Since \( \Delta > 0 \) and \( h_{d_{j+1}} > h_{d_j} \), the power allocation \( \{\hat{q}_k\} \) and \( \{\hat{p}_k\} \) achieve larger rate than \( \{q_k^*\} \) and \( \{p_k^*\} \), which contradicts the assumption that \( \{q_k^*\} \) and \( \{p_k^*\} \) are optimal for (3). Therefore, \( q_k^* \) and \( p_k^* \) satisfy (19). By induction,

\[
\sum_{k=1}^{d_{l+1}} p_k^* = \sum_{k=1}^{d_{l+1}} h_k q_k^* + B_1, \ l = j, \ldots, |\mathcal{D}|.
\]

It follows from (22) that

\[
\sum_{k=d_{l+1}} h_k q_k^* = h_{d_l} q_{d_l}^* + B_1, \ l = j, \ldots, |\mathcal{D}|
\]

which completes the proof of Proposition 3.2.

**APPENDIX C**

**PROOF OF LEMMA 3.1**

For the case \( 1 \leq x \leq 2 \), from (11), \( \sum_{k=1}^{d_x} p_k^* = h_{d-x} q_{d-x}^* + B_1 \) is satisfied. For the case \( 2 < x \leq |\mathcal{D}| + 1 \), we first prove \( q_{d_x}^* = 0 \) for \( 1 \leq k \leq x - 2 \) by contradiction. Assume there exists \( q_{d_x}^* > 0 \) for \( 1 \leq j \leq x - 2 \). Define

\[
\Delta \triangleq \min_{i=j+1, \ldots, x-1} \left( \sum_{k=1}^{i} h_{d_k} q_{d_k}^* + B_1 - \sum_{k=1}^{i+1} p_k^* \right).
\]

From (10), we have \( \Delta > 0 \). We construct a power allocation strategy \( \{\tilde{q}_k\} \) and \( \{\tilde{p}_k\} \) given by

\[
\tilde{q}_k = \begin{cases} 
q_k^* - \min \left( q_{d_x}^*, \Delta/h_{d_x} \right), & k = d_j, \\
q_k^* + \min \left( q_{d_x}^*, \Delta/h_{d_x} \right), & k = d_{x-1}, \\
q_k^*, & \text{otherwise}.
\end{cases}
\]

\[
\tilde{p}_k = \begin{cases} 
p_k^* + (h_{d_{x-1}} - h_{d_x}) \min \left( q_{d_x}^*, \Delta/h_{d_x} \right), & k = d_x, \\
p_k^*, & \text{otherwise}.
\end{cases}
\]

It can be verified that \( \{\tilde{q}_k\} \) and \( \{\tilde{p}_k\} \) satisfy the constraints (3a) and (3b). Since \( h_{d_{x-1}} > h_{d_x} \), \( q_{d_x}^* > 0 \), and \( \Delta > 0 \), the power allocation \( \{\tilde{q}_k\} \) and \( \{\tilde{p}_k\} \) achieve larger rate than \( \{q_k^*\} \) and \( \{p_k^*\} \), which contradicts the assumption that \( \{q_k^*\} \) and \( \{p_k^*\} \) are optimal for (3). Therefore, \( q_{d_x}^* = 0 \) for \( 1 \leq k \leq x - 2 \). From (11), we have \( \sum_{k=1}^{d_x} p_k^* = h_{d-x} q_{d-x}^* + B_1 \). The proof of Lemma 3.1 then completes.

**APPENDIX D**

**PROOF OF LEMMA 3.2**

We prove the equivalence between Problems (3) and (14). It is sufficient for us to prove that given optimal solution \( \{q_k\} \) and \( \{p_k\} \) for Problem (3), \( \{\tilde{p}_k\} \) obtained by (13) is optimal for Problem (14); given optimal solution \( \{\tilde{p}_k\} \) for Problem (14), \( \{p_k\} \), \( \{q_k\} \) obtained by (13) and (15) is optimal for Problem (3). For convenience, the optimal value of Problems (3) and (14) are denoted by \( R^* \) and \( R' \), respectively.
Given optimal solution \( \{g_k\}, \{q_k\} \) for Problem (3), then \( \{g_k\}, \{q_k\} \) satisfy constraints (3a) and (3b). We obtain \( \{p_k\} \) by (13). Since \( g_k p_k = g_k' p_k' \), the average rate achieved by \( \{p_k\} \) equals to \( R^* \). Next, we prove that \( \{p_k\} \) is a feasible solution for Problem (14). From Lemma 3.1 (10), and (13), \( \{p_k\} \) satisfy constraints (14b) and (14c). From Proposition 3.2 and (11), \( \{q_k\}, \{p_k\} \) satisfy
\[
\sum_{k \in D_{i+1}} p_k^* = h_d g_d, \quad l = x, \ldots, |D|.
\]
From Proposition 3.1 Lemma 3.1 (13), and (24), we have
\[
\sum_{k=1}^{K} p_k^* \leq \sum_{i=x-1}^{|D|} q_d + \frac{B_1}{h_d - 1} \leq Q + \frac{B_1}{h_d - 1}.
\]
It follows that \( \{p_k^*\} \) satisfy constraint (14a); thus, \( \{p_k^*\} \) is a feasible solution for Problem (3). Therefore, the average rate achieved by \( \{p_k^*\} \) is no larger than \( R^* \); i.e.,
\[
R^* \leq R'^* \tag{26}
\]
where the equality holds if and only if \( \{p_k^*\} \) is optimal for Problem (14).

Given optimal solution \( \{p_k^*\} \) for Problem (14), then \( \{p_k^*\} \) satisfy constraints (14a), (14b), and (14c). We obtain \( \{p_k\}, \{q_k\} \) by (13) and (15). Since \( g_k p_k = g_k' p_k' \), the average rate achieved by \( \{q_k\}, \{p_k\} \) equals to \( R' \). Next, we prove that \( \{q_k\}, \{p_k\} \) is a feasible solution for Problem (3). From (14a) and (15), \( \{q_k\} \) satisfy constraint (3a). From (13), (14b), and (15), \( \{q_k\} \) and \( \{p_k\} \) satisfy constraint (3b). Therefore, \( \{q_k\}, \{p_k\} \) is a feasible solution for Problem (3). It follows that the average rate achieved by \( \{q_k\}, \{p_k\} \) is no larger than \( R^* \); thus,
\[
R' \leq R^* \tag{27}
\]
where the equality holds if and only if \( \{q_k\}, \{p_k\} \) is optimal for Problem (3).

From (26) and (27), \( R^* = R' \). Therefore, given optimal solution \( \{q_k\}, \{p_k\} \) for Problem (3), \( \{p_k\}^* \) obtained by (13) is optimal for Problem (14); given optimal solution \( \{p_k\}' \) for Problem (14), \( \{p_k\}, \{q_k\} \) obtained by (13) and (15) is optimal for Problem (3). The proof of Lemma 3.2 completes.

APPENDIX E
PROOF OF PROPOSITION 3.3

Problem (14) is a convex optimization problem, and thus can be optimally solved by applying the Lagrange duality method. The Lagrangian of Problem (14) is given by
\[
\mathcal{L}(\{p_k^*\}, \lambda, \delta, \mu) = \frac{1}{\mathcal{K}} \sum_{k=1}^{\mathcal{K}} \log_2 \left(1 + \frac{g_k p_k^*}{\Gamma g^2}\right) + \lambda \left(Q + \frac{B_1}{h_d - 1} - \sum_{k=1}^{\mathcal{K}} p_k^*\right) + \delta \left(B_1 \frac{d_x - 1}{h_d - 1} - \sum_{k=1}^{d_x - 1} p_k^*\right) + \mu \left(\sum_{k=1}^{d_x} p_k^* - \frac{B_1}{h_d - 1}\right)
\]
where \( \lambda, \delta, \) and \( \mu \) are the non-negative dual variables associated with the corresponding constraints in Problem (14).

The necessary and sufficient conditions for \( \{p_k^*\} \) and \( \lambda, \delta, \mu \) to be both primal and dual optimal are given by the Karush-Kuhn-Tucker (KKT) optimality conditions: \( \{p_k^*\} \) satisfy all the constraints in Problem (14), and

REFERENCES


