Propagation of Spectrum Preference in Cognitive Radio Networks: A Social Network Approach

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Abstract—The social behavior in cognitive radio networks is studied using analysis tools in social networks. A recommendation system is proposed for cognitive radio, thus incurring the channel preference propagation in the corresponding random geometric network. A mean field based ordinary differential equation is used to describe the dynamics of the channel preference propagation in cognitive radio networks. The conditional distribution of random geometric graph is studied. The convergence and the steady state of the mean field equation are discussed. Numerical simulations are used to demonstrate the properties uncovered by the analysis.

I. INTRODUCTION

In recent years, cognitive radio has received intensive studies due to its capability of improving the spectrum efficiency. In cognitive radio, secondary users without license are allowed to access the licensed spectrum channel if and only if primary users having the license are not present. Besides the problem of how to sense the primary user, an important question is which channel to sense since there are usually many channels in a licensed frequency band. There have been many studies on how to choose the channel, e.g., using myopic strategies [9] or applying the bandit algorithms [2]. These studies are based on the theories of decisions and controls and apply for the single user case.

An interesting observation on the similarity between cognitive radio network and electronic commerce was discovered in [3]. In cognitive radio, different secondary users can be considered as different customers and different channels can be considered as different commodities. Due to the spatial correlation of spectrum occupancies [8], different secondary users / customers may have similar preferences on different channels / commodities. Similarly to the real life, secondary users can recommend channels to other users, like Alice recommends her favorite shoes to Tracy. For example, when secondary user A finds that channel 1 is available, it can broadcast a recommendation about channel 1 to its neighbors after completing its own data transmission. It has been demonstrated in [3] that such a recommendation system can significantly improve the performance of cognitive radio networks.

The similarity between the recommendation systems in cognitive radio network and in real life motivates us to study the social behavior in cognitive radio networks, i.e., how the channel preference is propagated within the cognitive radio network via the recommendation system, like how the preference on iPad is propagated in the society. Such a study can discover many properties of a recommendation system in cognitive radio network, e.g., how much the impact a new good channel will have on the system, how fast a recommendation can be propagated from one user to another user, whether one channel dominates all users and thus causes congestion on this channel. Although we are unable to answer all the questions in one short paper, we will study a simple mean field model and study the steady state of the system using both analytic and numerical approaches.

It is well known that the social network [1] [5] [7] is a powerful tool to study the propagation of social behaviors. Hence, we apply the social network approach and describe the system using a differential equation, similarly to the SIS model in epidemic propagation [5]. In contrast to many existing studies on social networks whose background is scale free networks or small world networks, a cognitive radio network is a random geometric network. The degrees of neighboring nodes in random geometric networks are strongly correlated, unlike the independent degrees in traditional social networks, which brings a new challenge to our study. Hence, we will analyze the conditional degree distribution in random geometric networks and then apply in the context of cognitive radio. Moreover, in the traditional SIS model of epidemic propagation, the possibility of being infected from no infected neighbors is omitted while this is possible in the context of cognitive radio since a secondary user is able to find the good channel by its own exploitation.

The remainder of this paper is organized as follows. The system model will be introduced in Section II. Then, the social network study on the channel preference propagation will be
II. SYSTEM MODEL

We consider a cognitive radio network with $N$ secondary users and $K$ licensed channels. The locations of these $N$ secondary users are denoted by $X_1, \ldots, X_N$. The network can be represented by a graph with multiple nodes, each of which represents a secondary user, and multiple edges, each of which represent a communication link. The secondary users adjacent in the graph can communicate with each other. Primary users can emerge at any channel at any time slot. A secondary user cannot transmit over a channel that primary users are using. For facilitating the analysis, we assume that the secondary users are randomly distributed in the plane, whose detail will be specified in Section III.

We assume that the secondary users can exchange the information about good channels. For simplicity, we assume that all secondary users are willing to recommend its favorite channel to its neighbors. All the recommendations are assumed to be honest. It is interesting to study the incentives for recommendations and the procedure to detect possible fraud behaviors; however, it is beyond the scope of this paper.

In this paper, we consider the preference propagation of a single channel. We fix a channel, say 1. Each secondary user has a favorite channel to sense. Each secondary user has two states, 1 and 0, which means that the secondary user favors channel 1 the most and that the secondary user favors another channel, respectively. At time 0, a fraction of the secondary users prefer to sense channel 1. Then, the channel preference is propagated using the following rule. If the state of a secondary user is 1, it does not take the recommendations from other users. However, the secondary user changes its state to 0 with probability $\mu$. Such a change could be due to an occasional degradation of channel 1 due to fast fading. If the state of a secondary user is 0, it randomly chooses one neighbor to exchange information. If the state of the neighbor is 1, the secondary user will change its state to 1 with probability $\lambda$. Even if the neighbor has a state 0, the secondary user may change to state 1 with probability $\phi$, which may be because that the secondary user finds the channel by itself. Obviously, the event of changing the state from 0 to 1 means that the secondary user adopts the recommendation from a neighbor whose favorite channel is 1 and finds that channel 1 is idle; the event of state changing from 1 to 0 means that the secondary user fails in accessing channel 1 and thus changes its channel preference.

Note that the propagation of the channel preference is similar to that of epidemic propagation [7]. The key difference between the two propagation processes include:

- The channel preference is propagated in the real space while the epidemic is dispersed in the abstract space of social network.
- The channel preference can be generated without the recommendation since each secondary user is able to find the channel, while the spontaneous epidemic generation is usually not considered in the study of epidemic propagation.

III. PREFERENCE PROPAGATION

In this section, we study the propagation of channel preference within cognitive radio networks. Due to its similarity to epidemic propagation, we will use the SIS model in complex social networks [7], combining with the conclusions in random geometric networks [6]. We first discuss the degree distribution in the spatially distributed random network. Then, we analyze the dynamics of the propagation using a mean field ordinary differential equation (ODE), which includes the convergence of ODE and the steady state of the solution.

A. Degree Distribution

We assume that the $N$ secondary users are independently and uniformly distributed within a square $S$ with area $AN$, i.e., averagely each secondary user obtains an area of $A^1$. Formally, this means that, for any region $R \in \mathbb{R}^2$, the probability that a given secondary user $n$ falls in region $R$ is given by

$$P(X_n \in R) = \frac{|R \cap S|}{AN}. \tag{1}$$

We assume that two secondary users are neighbors if they are within a distance of $d_{\text{max}}$. Then, the cognitive ratio network topology is determined by the locations of the secondary users, thus forming a random geometric network [6]. For secondary user $n$, its number of edges is called the degree, which is denoted by $k_n$. To analyze the propagation of channel preference within the cognitive radio network, we need to obtain the unconditional distribution of degrees and the conditional distribution of degrees of a secondary user adjacent to another secondary user with a given degree.

Note that the conditional probability is simple for random networks like scale free and small world networks due to the independence of the degrees of two adjacent nodes. However, the degrees of neighboring nodes are obviously correlated in a random geometric network. Consider a node with a large degree, i.e., there are many other nodes within its neighborhood. Then, a neighbor of this node is expected to share many neighboring nodes within this area. Therefore, the degrees of two nodes are positively correlated in a random geometric network, which is disclosed in the following proposition.

Proposition 1: As $N \to \infty$, the distribution of degree converges to a Poisson distribution with expectation $\lambda$, which is given by

$$\lambda = \frac{\pi d_{\text{max}}^2}{A}. \tag{2}$$

Given a secondary user with degree $k$, the probability that an arbitrary neighbor has degree $k'$ is given by

$$P(k'|k) = \int_0^{d_{\text{max}}} P(k'|r) \frac{2\pi r}{d_{\text{max}}^2} dr, \tag{3}$$

where $P(k'|r)$ is the distribution of random variable $r_1+r_2+1$, where $r_1$ and $r_2$ are two independent random variables. The

$^1$It is easy to extend to general non-uniform distribution case.
random variable $r_1$ has a Binomial distribution $B(n, \rho(r))$, where $n = k - 1$ and

$$\rho(r) = \frac{2d_{max}^2 \cos^{-1} \left( \frac{r}{2d_{max}} \right)}{\pi d_{max}^2} - \frac{\sqrt{d_{max}^2 - \left( \frac{r}{2d_{max}} \right)^2}}{\pi d_{max}^2}. \quad (4)$$

The random variable $r_2$ has a Poisson distribution with expectation $\lambda'(r)$, which is given by

$$\lambda'(r) = \frac{d_{max}^2 \left( \pi - 2 \cos^{-1} \left( \frac{r}{2d_{max}} \right) \right)}{d_{max}^2} - \frac{\sqrt{d_{max}^2 - \left( \frac{r}{2d_{max}} \right)^2}}{d_{max}^2}. \quad (5)$$

B. Propagation Dynamics

Although the propagation procedure has been significantly simplified, the analysis is still complicated. A rigorous analysis concerns the ergocity of contact process in interacting particle systems [4] and needs mathematical tools in nonequilibrium statistical mechanics like coupling and duality. Therefore, we adopt the mean field approach in [5] and model the dynamics as a deterministic ODE.

1) ODE Description: We consider continuous time and denote by $x_k(t)$ the proportion of the secondary users with degree $k$ and state 1. The dynamics of $x_k(t)$ is given by

$$\dot{x}_k(t) = -\lambda x_k(t) + \mu (1 - x_k(t)) \left( \phi + \sum_{n=1}^{\infty} x_n(t) P(m|k) \right), \quad (6)$$

for $k = 1, 2, \ldots$. The first term on the right hand side is the proportion of secondary users with degree $k$ changing its preference from channel 1 to other channels while the second term is the proportion of secondary users with degree $k$ beginning to prefer channel 1 due to the advice from other secondary users and the spontaneous discovery of the channel.

2) Convergence: It is difficult to analyze the dynamics of the ODE in (6) due to its nonlinearity and the infinite number of equations. For the nonlinearity, we can use Lyapunov function to analyze the convergence. For the number of equations, we truncate the equations by assuming that each secondary user can choose at most two neighbors, thus reducing the number of equations to two. The following proposition shows that, for the truncated system, the ODE converges to a stationary point when certain conditions are satisfied.

Proposition 2: Suppose that each secondary user has at most two neighbors and $\phi = 0$, then the ODE in (6) converges to a stationary point as $t \to \infty$ if

$$\lambda > \mu \max \{P(1|1), P(2|2)\}, \quad (7)$$

and

$$\sqrt{(\lambda - \mu P(1|1))(\lambda - \mu P(2|2))} > \frac{\mu(P(2|1) + P(1|2))}{2}. \quad (8)$$

The proof is based on Laypunov function and is given in Appendix B. From the proposition, we find that a sufficiently large $\lambda$, i.e., the rate that a secondary user changes its current channel preference, results in the convergence. Unfortunately, we are still unable to obtain the conditions of convergence for the general case. In all our numerical simulations, the dynamics of the ODE always converge.

3) Steady State: Now, we assume that the ODE in (6) always converges. At the steady state, we have $\dot{x}_k = 0$, for $k = 1, 2, \ldots$. Similarly to [5] [7], we define

$$\theta_k = \sum_{n=1}^{\infty} x_n P(m|k), \quad (9)$$

whose physical meaning is the probability that an arbitrary neighbor of a secondary user with degree $k$ prefers channel 1. Then the steady state condition $\dot{x}_k = 0$ implies

$$x_k = \frac{\mu(\theta_k + \phi)}{\lambda + \mu(\theta_k + \phi)}, \quad (10)$$

as well as

$$\theta_k = \sum_{m=1}^{\infty} \frac{\mu(\theta_m + \phi)}{\lambda + \mu(\theta_m + \phi)} P(m|k). \quad (11)$$

Then, the steady state of the ODE is determined by the equations (10) and (11). The following proposition shows the upper bound of the proportion of users preferring channel 1.

Proposition 3: There always exist a solution for Eq. (11).

Moreover, the steady proportion is upper bounded by

$$x_k \leq \frac{\mu(\theta_\infty + \phi)}{\lambda + \mu(\theta_\infty + \phi)}, \quad \forall k, \quad (12)$$

where

$$\theta_\infty = \frac{-\left(\lambda + \mu \phi - \mu\right) + \sqrt{(\lambda + \mu \phi - \mu)^2 + 4\mu \phi}}{2}. \quad (13)$$

IV. Numerical Results

In this section, we use numerical simulations to demonstrate the discussion on the channel preference propagation. We drop 500 secondary users within a 5km x 5km square area and assume that the maximum communication distance $d_{max} = 500m$.

Figure 2 shows three realizations for the evolution of the proportion of secondary users preferring channel 1 when $\lambda = 0.05$, $\mu = 0.1$ and $\phi = 0.01$. We observe that the evolution becomes stable and fluctuates around a certain value after about 50 time slots. This demonstrates the convergence of the channel preference propagation, as motivated by the conclusion in Prop. 2.

We also tested the case when $\lambda = 0.11 > \mu = 0.1$. The results are shown in Fig. 3, where $\phi = 0$, 0.01 (two realizations for each case). We observe that, when $\phi = 0$, the proportion converges to zero, i.e., the preference propagation finally dies out. When $\phi > 0$, the proportion fluctuates since the secondary users can find the channel by themselves.

In Fig. 4, we show the proportion $x_k$ as a function of degree $k$, based on 500 realizations of the preference propagation. Note that there are some fluctuations due to the limited number of realizations. We observe that $x_k$ increases with $k$ and converges to a certain value soon as $k$ increases. We also plotted the upper bound of $x_k$, i.e., $x_\infty$ in (12). We observe that the upper bound matches the numerical results quite well, despite some fluctuations. Therefore, the conclusion in Prop. 3 can be used to evaluate the proportion when $k$ is sufficiently large.
Finally, we plot the upper bound in (12) with respect to
different $\lambda$ and $\mu$ by fixing $\phi = 0.01$. We observe that when
$\mu$ is large, the steady proportion increases almost linearly as
$\lambda$ decreases. When $\mu$ is small, the proportion keeps almost
constant and suddenly increases to 1 when $\lambda$ is decreased.

V. CONCLUSIONS

In this paper, we have studied the channel preference prop-
agation in cognitive radio networks with a spatially random
deployment. A simple channel preference propagation rule has
been proposed. A mean field based ODE has been adopted to
describe the dynamics of the preference propagation. The con-
tditional degree distribution in random geometrical networks
has been obtained. The convergence of the mean field ODE
has been proved for a special case. The steady state of the
ODE has been analyzed and an upper bound for the steady
state proportion has been obtained. The analytic conclusions
have been demonstrated by numerical simulation results.

Our future work will address the following two challenges
for more practical situations:

- The competition among different channels.
- The possible collision if multiple secondary users prefer
the same channel.

APPENDIX A

PROOF OF PROP. 1

Proof: The proof for the unconditional distribution is
straightforward and is very similar to that of Theorem 4.1
in [6]. The only difference is that the scaling law of the
communication range is different. First, the edge case can be
asymptotically ignored since the probability that the neigh-
borhood (with radius $d_{\text{max}}$, denoted by $\Omega$) of an arbitrary
secondary user is completely contained in the square $S$ is given by

$$P(\Omega \subset S) = 1 - \frac{2d_{\text{max}}\sqrt{AN} + 2d_{\text{max}}(\sqrt{AN} - d_{\text{max}})}{AN} \rightarrow 1, \quad N \rightarrow \infty.$$  

Therefore, we can assume that the neighborhood of a
secondary user is completely within the square $S$. Since the
locations of the secondary users are mutually independent, the probability that an arbitrary secondary user \( X_n \) falls in a neighborhood \( \Omega \) is given by
\[
P(X_n \in \Omega) = \frac{\pi d_{\text{max}}}{N A}.
\] (15)

Then, the number of secondary users falling within the neighborhood \( \Omega \) satisfies a binomial distribution \( B(N, \frac{\pi d_{\text{max}}}{N A}) \). When \( N \to \infty \), it converges to a Poisson distribution with expectation in (15). This concludes the proof of the unconditional probability.

For the conditional probability, fix secondary user 1 and consider its neighborhood \( \Omega_1 \). It is easy to show that the probability density function of the distance \( r \) from a secondary user falling in \( \Omega_1 \) to secondary user 1 is given by \( p(r) = \frac{2r}{d_{\text{max}}} \).

Then, suppose that secondary user 2 falls in \( \Omega_1 \) and the distance is \( r \), as illustrated in Fig. 6. Denoting by \( \Omega_2 \) the neighborhood of secondary user 2, we discuss the secondary users falling in \( \Omega_2 \) for the two sets \( \Omega_1 \cap \Omega_2 \) and \( \Omega_2 - \Omega_1 \) (i.e., the elements in \( \Omega_2 \) but not in \( \Omega_1 \)), separately.

- Set \( \Omega_1 \cap \Omega_2 \): All the secondary users in \( \Omega_1 \cap \Omega_2 \) are the neighbors of both secondary users 1 and 2. Then, the number of secondary users in \( \Omega_1 \cap \Omega_2 \) is binomially distributed with \( k - 1 \) (secondary user 2 excluded) trials and success probability \( \rho(r) \). Obviously, \( \rho(r) \) should be equal to the ratio of the areas of \( \Omega_1 \cap \Omega_2 \) and \( \Omega_1 \). It is easy to verify that the area of \( \Omega_1 \cap \Omega_2 \) is equal to the numerator of (4).

- Set \( \Omega_2 - \Omega_1 \): It is easy to verify that the area of \( \Omega_2 - \Omega_1 \) is given by the numerator of (5). The distribution of the number of secondary users falling in \( \Omega_2 - \Omega_1 \) follows the same argument as that of (2).

APPENDIX C
PROOF OF PROP. 3

Proof: According to the conclusion in Prop. 1, the conditional degree distribution \( P(k'|k) \) increases with \( k' \) when \( k \) is fixed. Therefore, \( \theta_k \) is an increasing function of \( k \) since

\[
\theta_k - \theta_{k'} = \sum_{m=1}^{\infty} \frac{\mu \theta_m + \phi}{\lambda + \mu \theta_m + \phi} (P(m|k) - P(m'|k)) > 0,
\] (17)

when \( k > k' \).

To prove the existence of the root of Eq. (11), we construct an iteration, starting from
\[
\theta_1(0) = \theta_2(0) = \ldots = 1.
\] (18)
Substituting them into Eq. (11), we obtain \( \{\theta_k(1)\}_k \). Obviously, we have \( \theta_k(1) < \theta_1(0) \), \( \forall k \). We substitute \( \{\theta_k(1)\}_k \) into (11) again and obtain \( \{\theta_k(2)\}_k \). Since \( \frac{\mu(\theta_k + \phi)}{\lambda + \mu(\theta_k + \phi)} \) is an increasing function and \( \theta_k(1) < \theta_1(0) \), we have \( \theta_k(2) < \theta_k(1) \). We repeat this procedure, which generates a decreasing sequence \( \{\theta_k(l)\}_{l=0,1,2,...} \) for each \( k \). Since \( \theta_k \) is lower bounded (always positive), each sequence converges to a limit, thus forming a root of Eq. (11).

To prove the upper bound, we notice that the sequence \( \{\theta_k\}_{k=1,2,...} \) is an increasing sequence. Since this sequence is bounded (\( \theta_k < 1 \)), it converges to a limit \( \theta_\infty \), which satisfies
\[
\theta_\infty = \frac{\mu(\theta_\infty + \phi)}{\lambda + \mu(\theta_\infty + \phi)},
\] (19)
whose reasonable solution is given in (13). Since \( x_k \) is an increasing function of \( \theta_k \), the upper bound in (12) is obtained due to the fact \( \theta_k < \theta_\infty \).

REFERENCES