Abstract—In this paper we investigate the performance of two-hop virtual multiple-input-multiple-output systems with random location of nodes. In the scenario we consider the communication between source and destination through the use (following a decode-and-forward approach) of a relay node. A number of ancillary nodes, distributed according to a Poisson point process, are supposed to be distributed around source, relay and destination, with the possibility to create clusters of cooperating nodes. Random fluctuations are accounted for in the channel model which considers both slow and fast fading. Performance is evaluated in terms of outage probability, defined as the probability that the achieved capacity between source and destination is smaller than a given threshold. Finally, the total power consumed by the network for delivering the data is evaluated. Results obtained in the two-hop communication protocol are compared with the single-hop case.

Index Terms—Virtual MIMO, Capacity, Two-Hop, Connectivity.

I. INTRODUCTION

Virtual (also known as distributed) multiple-input-multiple-output (V-MIMO) systems appear as one of the most interesting paradigms for the deployment of future wireless systems [1]–[3]. The key aspect of V-MIMO communication systems is the possibility for the devices, which can be equipped with single or multiple antennas, to create clusters of cooperating nodes. Cooperation between nodes is exploited to obtain the benefits, in terms of spatial diversity, of conventional MIMO. The clusters of cooperating nodes are usually denoted as virtual antenna arrays (VAA), since each VAA acts “virtually” as the set of transmit or receive antennas in a conventional MIMO [1], [3]. Another interesting characteristic of V-MIMO is the possibility to use two different air interfaces for cooperation between devices and data transmission (from source to destination). For example, a short range and low rate air interface (i.e., IEEE 802.15.4) can be used to exchange data within each cluster (intra-VAA communication), whereas a long range and high rate air interface can be used to transmit data from cluster to cluster (inter-VAA communication) [1]–[3].

In [1] and [2] the ergodic capacity of a V-MIMO system for single and two-hop ad-hoc network scenarios, respectively, is derived. [3] considers a multi-hop V-MIMO and derives an optimal resource allocation strategy, in terms of fractional bandwidth and power allocation to each relay. In [4] the diversity gain achieved by a V-MIMO is investigated in realistic indoor propagation environment. In more recent works the concept of V-MIMO has been applied to wireless sensor networks, where the cooperating devices are sensors, equipped with a single antenna element [5]. It is worth noting that the topology of the network considered in the previous works is assumed to be fixed and the issues related to the creation of the VAA are not considered. Furthermore, there exist few works related to connectivity aspects in MIMO systems in the context of ad-hoc networks (i.e., see [6], [7]). In [6] the performance of some spatial diversity techniques including maximal ratio combing MIMO are investigated. In [7], a multiple access scheme with frequency hopping is considered. Also Poisson fields of nodes are studied in several works (i.e., [6], [7]), to the author's knowledge, no article addressing V-MIMO considers connectivity problems which usually arise with the formation of the VAA.

In this article we consider a two-hop V-MIMO system (see Figure 1), where a source node has to transmit data to a destination node via a relay node. In the following, source, relay and destination nodes will be denoted as main nodes. A number of ancillary nodes are located in three areas, \( A_s \), \( A_r \) and \( A_d \), around the main nodes. In particular, ancillary nodes are distributed over the areas according to a Poisson point process (PPP). Three VAA are formed in this scenario: the source VAA (s-VAA), the relay VAA (r-VAA) and the destination VAA (d-VAA). We assume that nodes work in a half-duplex mode and that a decode and forward strategy is implemented at the relay [3]. Random fluctuations of the wireless channel as well as a distance-dependent deterministic path-loss are accounted for in our radio channel model. In our scenario, the main nodes can cooperate only with their ancillary nodes which guarantee a feasible quality of the link. Owing to the randomness nature of the channel, the number of transmit and receive antennas is a random variable and a certain outage probability there exists. We define the outage probability as the probability that the source-destination capacity is lower than a given threshold. The impact of this metric on the different connectivity parameters (such as, the ancillary nodes densities, the channel model parameters, etc..) is investigated. Moreover, the total power consumed by the network for the transmission of the data from the source to the destination, is also evaluated. Results related to the two-
hop communication protocol are compared to those obtained in the single-hop case.

II. SYSTEM DESCRIPTION AND SCENARIO
Throughout the article vectors and matrices are indicated by bold, \( \mathbf{I} \) is the identity matrix and \( |A| \) denotes the determinant of \( A \). \( \{a_{i,j}\}_{i,j=1,...,M} \) is an \( M \times M \) matrix with elements \( a_{i,j} = \{A\}_{i,j} \). \( \dagger \) is the operator of conjugation and transposition. Also, \( \mathbb{E}\{\cdot\} \) denotes expectation, and \( \mathbb{P}\{\mathcal{E}\} \) denotes the probability of the event \( \mathcal{E} \).

A. Scenario

We assume that ancillary nodes are spatially distributed in \( A_S \), \( A_R \) and \( A_D \) according to a PPP [8]. With such model the probability of having one node in the infinitesimal area \( \delta A \) is \( \eta \delta A \), where \( \eta \) denotes the nodes' density [8]. As a general case, nodes densities in the three areas may be different: we denote as \( n_S \), \( n_R \) and \( n_D \), the densities of the ancillary nodes distributed around the source, the relay and the destination, respectively. At the beginning of the communication, s-VAA, r-VAA and d-VAA are formed. The main nodes transmit a query to the ancillary nodes, by using the short-range radio interface. Owing to propagation conditions, only a subset of the ancillary nodes can really cooperate with the main nodes. The number of nodes which actually communicate with source, relay and destination is denoted by \( n_S \), \( n_R \) and \( n_D \), respectively, and are called cooperating nodes. We also assume that the distances source-relay and relay-destination are much larger than the distance between a main node and its cooperating nodes. So that the short-range radio interface can be used only to transmit/receive data to/from the main node and its cooperating nodes. Therefore, the number of cooperating nodes is a random variable (r.v.) whose statistical properties depend on the connectivity models we are using and on the spatial distribution of nodes. In particular, when the position of ancillary nodes is distributed according to a PPP, we can apply the following theorem

\textbf{Theorem 1:} Assume a Poisson distribution of nodes in a \( m \)-dimensional space and consider a reference node, denoted by \( R_N \), located somewhere in the scenario. Let \( d \), \( C(d) \) and \( n \) be the euclidean distance between a generic node and \( R_N \), the probability that a generic node is connected with \( R_N \) and the number of nodes which are connected with \( R_N \), respectively. Then, \( n \) is a Poisson r.v..

\textbf{Proof:} The proof is a consequence of the \textit{Marking Theorem} for Poisson processes [8].

As a result of the previous property, the probability distribution of \( n \) is

\[ \mathbb{P}\{n = n_{1}\} \equiv P(n_{1},N) = \frac{N^{n_{1}}}{n_{1}!} e^{-N}, \]

where \( N = \mathbb{E}\{n\} \) depends on the connectivity model chosen.

C. The connectivity model used in the article

In this subsection, we specialize the propagation and connectivity model used in the article. In particular, the channel model we consider accounts for the power loss due to propagation effects including a distance-dependent path loss, the slow and the fast channel fluctuations. We assume that the ratio between the transmit power, \( P_T \) and the received power, \( P_R \), is given by

\[ \frac{P_T}{P_R} = k \cdot d^\beta \cdot \frac{s}{f}, \]

where \( k \) is the propagation coefficient, \( d \) is the distance from the transmitter and the receiver, \( \beta \) is the attenuation coefficient which commonly ranges from 2 to 5, finally, \( s \) and \( f \) are the long-term (shadowing) and the short-term (fast) fading components, respectively. Shadowing is assumed to be log-normally distributed, and Rayleigh fading is considered.
therefore $f$ is exponentially distributed with unitary mean. We recall here that the log-normal p.d.f is given by [9]
\[
 f_s(s) = \begin{cases} 
 \frac{10^{f_{10}}}{\sqrt{2\pi} \sigma} e^{-(10 \log_{10}s - \mu)^2} & \text{for } s \geq 0 \\
 0 & \text{otherwise,} 
\end{cases}
\] (3)
where $\mu$ and $\sigma$ are the mean value and the standard deviation in dB, respectively.

We define $L = k \cdot d^2 \cdot s$ as the averaged (with respect to fast fading) loss (in linear scale). By introducing the logarithmic scale, we obtain
\[
 L[dB] = k_0 + k_1 \ln d + S[dB],
\] (4)
where $k_0 = 10 \log_{10} k$, $k_1 = \beta \frac{10}{\ln 10}$, and $\sigma^2$ is a Gaussian random variable, with mean $\mu = 0$ and variance $\sigma^2$ ($S \sim N(0, \sigma^2)$).

Note that, for each air interface (intra-VAA and inter-VAA) we could have different power transmission ($P_T$) and propagation parameters ($k_0$, $k_1$, $\sigma$), so we define the probability that two nodes are connected, $C(d)$, as the probability that $L < L_{th}$; therefore $C(d) = \mathbb{P}(L < L_{th})$, where $L_{th}$ represents the maximum loss tolerable by the communication system, and $d$ is the distance between the two nodes. The threshold $L_{th}$ depends on transmit power and receiver sensitivity.

For the sake of simplicity, we assume that the areas $A_S$, $A_R$ and $A_D$ are circular (with center in the main nodes) with radius $r_S$, $r_R$ and $r_D$, respectively. In particular, when the channel model of eq. (4) is used, the mean value of the number of nodes in $A_S$ for which $L < L_{th}$, is denoted by $N_S$, and can be written as [10]
\[
 N_S = n_S \pi \left[ e^{2L_{th}/k_1 - 2k_0/k_1 + 2\sigma^2}/k_1^2 + \Psi \left( L_{th} - k_0/k_1 + 2\sigma^2/k_1, r_S \right) \right],
\] (5)
where
\[
 \Psi(a_1, b_1, r) = r^2 \exp(a_1 - b_1 \ln r) - e^{a_1 + b_1 \ln r} \Phi(a_1 - b_1 \ln r + 2b_1),
\] (6)
and $\Phi(x) \triangleq 1/(\sqrt{2\pi}) \int_{-\infty}^{x} e^{-u^2/2} du$, $a_1 = (L_{th} - k_0)/\sigma$, and $b_1 = k_1/\sigma$.

$N_R$ and $N_D$ (i.e. the mean number of nodes in $A_R$ and $A_D$ for which $L < L_{th}$) can be easily obtained from (5) by using the couple of values $(n_R, r_R)$ or $(n_D, r_D)$ instead of $(n_S, r_S)$. Finally, the parameters $k_0, k_1, \sigma$ and $L_{th}$ in (5) and (6) refer to intra-VAA transmission.

III. ERGODIC CAPACITY EXPRESSIONS FOR V-MIMO

In this article we assume that the receiver has perfect knowledge of the channel state, whereas the transmitter knows only the average loss (path-loss and shadowing) [1]–[3]. The received signal at the $\ell$th hop can be written as
\[
 y_{\ell} = \sqrt{P_G} H_{\ell} b_{\ell} + n_{\ell},
\] (7)
where $y_{\ell}$ is a $(n_R + 1)$ (for $\ell = 1$) or $(n_D + 1)$-dimensional (for $\ell = 2$) vector. $P_G$, $H_{\ell}$, $b_{\ell}$, $n_{\ell}$ are the averaged (over fast fading) power received by a given node of r-VAA (or d-VAA) when transmitted by a node of s-VAA (or r-VAA), the fast fading channel matrix, the transmitted symbol vector and the thermal noise vector, respectively. We assume $\mathbb{E}\{b_{\ell}b_{\ell}^H\} = I$, and $\mathbb{E}\{n_{\ell} \cdot n_{\ell}^H\} = \sigma_n^2 I$, where $\sigma_n^2$ is the thermal noise power per antenna element. We consider a flat uncorrelated Rayleigh environment so that the elements of $H_{\ell}, h_{ij}(\ell)$, can be modelled by a collection of i.i.d complex-valued Gaussian r.v.’s having $\mathbb{E}\{h_{ij}(\ell)\} = 0$ and unitary mean $\mathbb{E}\{|h_{ij}(\ell)|^2\} = 1$. Since the distances source-relay and relay-destination are much larger than the distance between a main node and its cooperating nodes, the averaged power ($P_{\ell}$) received by a node in r-VAA ($\ell = 1$) or d-VAA ($\ell = 2$) does not depend on the specific transmit node.

The mean (with respect to fast fading fluctuations) capacity in the two-hop case, $C^{(2)}$, is the minimum between the mean capacity of the first link (from the source to the relay) and of the second link (from the relay to the destination) [2]. Therefore, by assuming $n_{s}, n_{R}$ and $n_{D}$ cooperating nodes, the source-destination ergodic capacity can be written as
\[
 C^{(2)}_{n_{s}, n_{R}, n_{D}} = \frac{1}{2} \min \left\{ C^{(1)}_{n_{S}, n_{R}}(\rho), C^{(1)}_{n_{R}, n_{D}}(\rho) \right\},
\] (8)
where the term $1/2$ reflects the fact that half of the resources (in the time or frequency axes) are spent for the transmission from source to relay and half for the transmission from relay to destination. $C^{(1)}_{n_{S}, n_{R}}(\rho)$ is the mean capacity of a MIMO channel with $n_1 + 1$ transmit (the main node plus $n_1$ cooperating nodes) and $n_2 + 1$ receive antennas and $\rho_1$ is the signal-to-noise ratio, defined as $\rho_1 \triangleq P_1/\sigma_n^2$. In the single-hop case, the expression for the capacity can be easily written as $C^{(1)}_{n_{S}, n_{D}}(\rho)$.

The mean capacity of MIMO in Rayleigh fading channels has been extensively studied in the past years, here we use a closed form expression which was derived in [11]
\[
 C^{(1)}_{n_{S}, n_{D}}(\rho) = \frac{\min K}{\ln 2} \sum_{n_1=1}^{n_{max}} \sum_{n_{min}=1}^{n_{max}} \left( -1 \right)^{n_1 + n_{max} - n_{min}} |\Omega| \times \rho^{n_{max} + n_{min} - n_{max} + 1} F(n_1 + n_2 - 1, n_{max} - n_{min}, 1/\rho),
\] (9)
where $n_{min} = 1 + \min\{n_1, n_2\}$, $n_{max} = 1 + \max\{n_1, n_2\}$, $K = \prod_{i=1}^{n_{max}} (n_{max} - i)! \prod_{j=1}^{n_{min}} (n_{min} - i)!$, the $(i, j)^{th}$ element of $\Omega$ is
\[
 \omega_{i, j} = \frac{(\alpha_{i, j}^{(n)(m)}) + n_1 n_2}{n_{max} - n_{min}},
\] (10)
and
\[
 \alpha_{i, j}^{(n)(m)} \triangleq \begin{cases} 
 i + j - 2 & \text{if } i < n_1 \text{ and } j < m \\
 i + j & \text{if } i \geq n_1 \text{ and } j \geq m \\
 i + j - 1 & \text{otherwise},
\end{cases}
\] (11)
and
\[
 F(a, d) \triangleq (a - 1)! \frac{1}{e^a} \sum_{k=1}^{a} \frac{\Gamma(-a + k, d)}{d^k},
\] (12)

2This definition is different from the usual definition of signal-to-noise ratio in conventional (co-located) MIMO systems, which is given by $N_T P_h/\sigma^2$.
where $\Gamma(\alpha, x)$ is the incomplete Gamma function [12].

IV. OUTAGE PROBABILITY ANALYSIS

Since the number of cooperating nodes is a r.v., there exists a certain probability that the source-destination mean capacity, $C^{(2)}$, is lower than a given value, $C_0$, which depends on the specific application considered. In such scenario, a useful performance metric is the outage probability, $P_{\text{out}}$ defined as

$$P_{\text{out}} = \mathbb{P}\{C^{(2)} < C_0\}$$

which can be evaluated as

$$P_{\text{out}} = \sum_{s=0}^{M_S} \sum_{r=0}^{M_R} \sum_{d=0}^{M_D} \mathbb{P}\{n_S = s, n_R = r, n_D = d\}$$

$$\times I\left(C^{(2)}_{s,r,d}, C_0\right),$$

(13)

where $\mathbb{P}\{n_S = s, n_R = r, n_D = d\}$ is the probability that there are $s$, $r$, and $d$ cooperating nodes at the source, relay, and destination, respectively. Finally, the indicator function $I(x, y)$ is defined as

$$I(x, y) = \begin{cases} 1 & x < y \\ 0 & \text{otherwise} \end{cases}.$$  

(14)

Owing to the presence of the limitation on the number of cooperating nodes, $n_S$, $n_R$, and $n_D$ do not have Poisson distribution. However, their distribution can be easily obtained from (1) as

$$Q(s, N_S) = \begin{cases} P(s, N_S) & \text{for } s < M_S \\ 1 - \sum_{s=0}^{M_S-1} P(l, N_S) & \text{for } s = M_S, \end{cases}$$

(15)
equivalent expressions can be written for $Q(r, N_R)$ and $Q(d, N_D)$. Being $n_s$, $n_r$ and $n_d$ independent r.v.s, (13) can be re-written as

$$P_{\text{out}} = \sum_{s=0}^{M_S} \sum_{r=0}^{M_R} \sum_{d=0}^{M_D} Q(s, N_S)Q(r, N_R)Q(d, N_D)$$

$$\times I\left(C^{(2)}_{s,r,d}, C_0\right).$$

(16)

Note that, with the definition of the signal-to-noise ratio given in this article, $C^{(2)}_{s,r,d}(\rho) = C^{(1)}_{s,r,d}(\rho)$. So that, in the case of $\rho_1 = \rho_2$, expression (8) can be simplified as

$$\bar{C}^{(2)} = \frac{1}{2} C^{(1)}_{n_S,n_R,n_D}(\rho),$$

(17)

where $n_M = \min\{n_S, n_R\}$. The expression for the $P_{\text{out}}$ becomes

$$P_{\text{out}} = \sum_{r=0}^{M_R} \sum_{m=0}^{M_D} Q(r, N_R)\mathbb{P}\{n_M = m\}$$

$$\times I\left(\frac{1}{2} C^{(1)}_{n_S,n_R,n_D}(\rho), C_0\right),$$

(18)

and the distribution of $n_M$ can be written as

$$\mathbb{P}\{n_M = m\} = Q(m, N_S)Q(m, N_D) + Q(m, N_S)$$

$$\times \sum_{\nu=m+1}^{\min\{M_S, M_D\}} Q(\nu, N_D) + Q(m, N_D) \sum_{\nu=m+1}^{\min\{M_S, M_D\}} Q(\nu, N_S).$$

(19)

When $\eta_S = \eta_D$, $n_S$ and $n_D$ becomes i.i.d., in that case $\mathbb{P}\{n_M = m\}$ can be simplified as

$$Q(m, N)^2 + 2Q(m, N) \sum_{\nu=m+1}^{\min\{M_S, M_D\}} Q(\nu, N),$$

(20)

where $N = N_S = N_D$. Starting from (16), the outage probability for the single-hop case can be easily written as

$$P_{\text{out}} = \sum_{s=0}^{M_S} \sum_{d=0}^{M_D} Q(s, N_S)Q(d, N_D) I\left(C^{(1)}_{s,d}(\rho), C_0\right).$$

(21)

V. CONSIDERATIONS ON POWER CONSUMPTION

The total power spent by the network to deliver the data from the source to the destination, depends on the power spent by each node participating in the communication. In this article, we neglect the power spent by the network for performing cooperation (i.e., we do not consider the power spent for intra-V AA transmissions) and focus only on inter-V AA transmission. To have a unique performance metric, we denote as $\mathbb{E}\{P_{\text{tot}}\}$ the averaged (with respect to fast and slow fading, and to the number of cooperating nodes) power spent by all the active nodes in the network. $\mathbb{E}\{P_{\text{tot}}\}$ can be written as

$$\mathbb{E}\{P_{\text{tot}}\} = \mathbb{E}\left\{ \sum_{s=0}^{M_S} (1 + Q(s, N_S)) P_T^{(1)} \right\}$$

$$+ \mathbb{E}\left\{ \sum_{r=0}^{M_R} (1 + Q(r, N_R)) P_T^{(2)} \right\}$$

$$= \mathbb{E}\left\{ P_T^{(1)} \right\} (\mathbb{E}\{n_S\} + 1) + \mathbb{E}\left\{ P_T^{(2)} \right\} (\mathbb{E}\{n_R\} + 1),$$

where the two terms of the sum refer to the total averaged power spent by the s-V AA and the r-V AA, respectively, being $\mathbb{E}\left\{ P_T^{(1)} \right\}$ the averaged power used by each node of the s-V AA, and $\mathbb{E}\left\{ P_T^{(2)} \right\}$ the averaged power used by each node of the r-V AA. $P_T^{(1)}$ and $P_T^{(2)}$ can be calculated by recalling that the power control at the transmitter exploits the knowledge of path loss and shadowing to obtain a target signal-to-noise ratio at the receiver ($\rho_1$ or $\rho_2$). For a fixed value of $\rho_1$ we obtain the transmit power used by each s-V AA node

$$P_T^{(1)} = \rho_1 k d_1^5 \sigma_S^2 s,$$

(22)

where $d_1$ is the source-relay distance. $P_T^{(2)}$, the averaged transmit power used by each node at the r-V AA, is obtained by eq. (22) by using $\rho_2$ instead of $\rho_1$ and $d_2$ (the relay-destination distance) instead of $d_1$.

Finally, we can derive the averaged transmit power used by each s-V AA node, by calculating the expectation of $P_T^{(1)}$ with respect to shadowing

$$\mathbb{E}\left\{ P_T^{(1)} \right\} = \rho_1 k d_1^5 \sigma_S^2 \int_0^{\infty} s f_s(s) ds$$

$$= \rho_1 k d_1^5 \sigma_S^2 \frac{1}{2} \left(\frac{\alpha}{2}\right)^{\frac{\alpha}{2}} \Gamma\left(\frac{\alpha}{2}, \frac{\alpha}{2}\right),$$

(23)
where \( f_s(s) \) is given by eq. (3). Similarly, \( \mathbb{E}\{P_T^{(2)}\} \) can be obtained by (23) by replacing \( \rho_1 \) with \( \rho_2 \). Note that the parameters \( k \), \( \beta \), \( \sigma \) and \( \sigma^2 \) in (23) refer to the inter-VAA transmission.

VI. NUMERICAL RESULTS

In this Section the behavior of the complementary outage probability, \( P_{\text{in}} \equiv 1 - P_{\text{out}} \), is shown by varying different scenarios and system parameters. Results are obtained by setting, if not otherwise specified, the following parameters: \( r_S = r_R = r_D = 10 \) [m], \( \sigma = 4 \) [dB]; \( \sigma^2 = 8 \cdot 10^{-15} \) [W] and \( M_S = M_R = M_D = 10 \). We consider two different channel models for intra-VAA and inter-VAA communication. In the first case, we set \( k_0 = 41 \) dB, \( k_1 = 13.03 \) (\( \beta = 3 \)), and \( L_{\text{th}} = 92 \) dB (an IEEE 802.15.4-like air interface is used as a short-range interface [8]); whereas we set \( k_0 = 15 \) dB and \( k_1 = 17.37 \) (\( \beta = 4 \)) for the inter-VAA transmissions \( L_{\text{th}} \) is not fixed in this case, since we assume that the s-VAA and the r-VAA so that the r-VAA and the d-VAA are always connected). In the following, we will consider \( \rho_1 = \rho_2 = \rho \) and, except Figure 4, the densities of ancillary nodes is fixed at \( \eta_S = \eta_R = \eta_D = \eta \).

Figure 2 reports \( P_{\text{in}} \) as a function of \( \rho \), for different values of \( \rho \), having set \( \eta = 5 \cdot 10^{-4} \) [m\(^{-2}\)]. As expected, \( P_{\text{in}} \) decreases by increasing \( \rho \) and the curves are translated by increasing \( \rho \). Note that the step behavior of the curve can be explained by observing that \( \bar{C}_{\text{in}}^{(2)}(\eta_S, \eta_R, \eta_D) \) is a function of the three discrete random variables \( \eta_S, \eta_R \) and \( \eta_D \).

In Figure 3 \( P_{\text{in}} \) as a function of \( \sigma \) for different values of \( C_0 \) is shown, other parameters are \( \rho = 10 \) dB and \( \eta = 10^{-5} \) [m\(^{-2}\)]. The Figure shows that by increasing \( \sigma \), \( P_{\text{in}} \) increases. The beneficial (from the \( P_{\text{in}} \) point of view) effect of \( \sigma \) can be explained by observing that the presence of the shadowing leads to an increase of the number of cooperating nodes [10] and therefore, the average number of virtual antennas of V-MIMO gets larger with \( \sigma \).

Figure 4 shows the impact of the distribution of the ancillary nodes. The Figure plots \( P_{\text{in}} \) as a function of \( \eta_S = \eta_D \), for different values of \( \eta_R \). The Figure has been obtained by setting \( C_0 = 5 \) [bit/s/Hz] and \( \rho = 10 \) dB. The curves saturate at a given value, which increases by increasing \( \eta_R \). This behavior can be explained by recalling that when the density of the ancillary nodes at the relay is low, the number of receive antennas used in the first hop (which coincides with the number of transmit antennas used in the second hop) is small. This effect on the capacity is shown in eq. (18), where the capacity is written as a function of \( \eta_R \) and on the minimum between \( \eta_S \) and \( \eta_D \). Since the capacity is limited by the minimum between the number of transmit and receive antennas, the value of \( P_{\text{in}} \) does not reach 1 even if \( \eta_S \) and \( \eta_D \) (but not \( \eta_R \)) become very large. This latter consideration suggests us a simple way for the dimensioning of the system: once the application fixes the minimum acceptable value of \( P_{\text{in}} \), the minimum number of the density of ancillary nodes at the relay can be easily obtained from Figure 4. The previous figure can be also useful to evaluate the minimum value of \( \eta_S = \eta_D \), which leads to the requested \( P_{\text{in}} \).

Since the number of cooperating nodes has an impact on the overall amount of energy consumed for inter- and intra-VAA transmissions, it is reasonable to introduce a limit on the number of cooperating nodes. In Figure 5 the minimum number of \( M_S = M_R = M_D \) which allows to obtain \( P_{\text{in}} \geq 0.9 \) is shown as a function of \( C_0 \) for different values of \( \rho \). Here, nodes’ density is \( \eta = 6.5 \cdot 10^{-3} \) [m\(^{-2}\)]. This Figure can be useful, for dimensioning purposes, to obtain the limit on the number of cooperating nodes which should be imposed to satisfy the application requirement also to minimize the energy consumption.

Finally, in Figure 6, \( P_{\text{in}} \) as a function of \( \mathbb{E}\{P_{\text{los}}\} \), for different values of \( \eta \), is shown for \( C_0 = 5 \) [bit/s/Hz] and \( d_1 = d_2 = 300 \) [m]. The single and the two-hop communication protocols are compared. As expected, the increase in the value of \( P_{\text{in}} \) is obtained at the cost of an increasing of the total power spent by the network. Once again the curves saturate, since, owing to the values considered for \( C_0 \) and \( L_{\text{th}} \), \( P_{\text{in}} \) cannot reach 1 even if nodes’ density increases. For what concerns the comparison between two communication protocols, we can deduce that for low values of \( \eta \), the single-hop protocol allows to obtain larger \( P_{\text{in}} \). When nodes’ density increases, the two-hop protocol can exploit the additional degrees of freedom given by r-VAA and outperforms the single-hop case.

VII. CONCLUSION

In this paper, the performance of a two-hop V-MIMO system has been studied in the presence of randomness of nodes’ location. The impact of the standard deviation of shadowing and of nodes’ density has been investigated. Finally, the comparison between single and two-hop communication protocol, in terms of tradeoff between outage probability and total power consumed by the nodes, shows that the two-hop protocol outperforms the single-hop case when nodes’ density increases.

ACKNOWLEDGMENT

This work was supported by the European Commission in the framework of the FP7 Network of Excellence in Wireless Communications NEWCOM++ (contract n. 216715).

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Fig. 2. The complementary outage probability, $P_{in}$, as a function of $C_0$, for different values of $\rho$.

Fig. 3. The complementary outage probability, $P_{in}$, as a function of $\sigma$, for different values of $C_0$.

Fig. 4. The complementary outage probability, $P_{in}$, as a function of $\eta_S = \eta_D$, for different values of $\eta_R$.

Fig. 5. The maximum number of antennas $M_S = M_R = M_D$ as a function of $C_0$, for different values of $\rho$.

Fig. 6. The complementary outage probability, $P_{in}$, as a function of $E\{P_{tot}\}$ for different values of $\eta$, in the one-hop and two-hop cases.


