Computing and Analyzing Mixed Equilibrium Network Flows with Gasoline and Electric Vehicles

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Abstract: This article addresses a new network equilibrium problem with mode and route choices for the emerging need of modeling regional transportation networks that accommodate both gasoline and electric vehicles. The two transportation modes (or vehicle types) distinguish from each other in terms of driving distance limit and travel cost composition. In view of the advantages (e.g., low fuel expenses and vehicle emissions) and disadvantages (e.g., limited driving range and long charging time) pertaining to driving electric vehicles, it is anticipated that a large number of households/motorists may prefer to own both gasoline and electric vehicles (although, of course, many households/motorists still only own gasoline vehicles (GVs) and some households may choose to own electric vehicles only) in the transition period from the petroleum era to the electricity era. The purpose of this article is to offer a traffic equilibrium modeling tool for networks that serve households/motorists who can choose between gasoline and electric vehicles. Specifically, we present a convex optimization model for characterizing such mixed equilibrium traffic networks with both gasoline and electric vehicles, which are expected to exist for a long period in the future. Two competing solution algorithms, a linear approximation algorithm of the Jacobi type and a quadratic approximation algorithm taking the form of the Gauss–Seidel decomposition, are implemented and evaluated. Experimental results clearly show that, from the model behavior perspective, the produced network flow patterns replicate the anticipated combined mode–route choice results, that is, the higher the distance limit or the gasoline price is, the more travelers choose battery electric vehicles (BEVs) when both BEVs and GVs are available to them; and, from the solution efficiency perspective, the quadratic approximation algorithm exhibits linear convergence and can reach higher solution precision in shorter time.

1 INTRODUCTION

With the promise of achieving reduced petroleum consumption, enhanced energy security, and improved environmental sustainability, electric vehicles have gathered unprecedented interest in recent years. It is anticipated that electric vehicles will capture a significant market share in future years because of the maturity of electric vehicle technologies and the public’s increasing acceptance. A recent study predicted that there will be 13–40 million electric vehicles of a total of 300 million vehicles running on the U.S. roads by 2030 (Addison, 2012). In the current passenger car market, there are two types of electric vehicles in terms of their engine technologies: Plug-in hybrid electric vehicles (PHEVs), which are equipped with gasoline engines and electric motors, and battery electric vehicles (BEVs), which are equipped with only electric motors. Many researchers predicted that BEVs, relying entirely on electricity, will provide an ultimate solution for the electrification of personal transportation (Lin and Greene, 2011).
To individual motorists, the most appealing and visible benefit from driving BEVs compared to gasoline vehicles (GVs) is the lower operating cost. In the United States, for example, the average monetary cost paid for electricity powering a vehicle is approximately one-fourth or fifth of the average cost for gasoline purchase for the same energy output. If we take into account all major travel costs in travel choice analysis, the operating cost is definitely an important element in distinguishing travel choice results between BEVs and GVs and evaluating system performance changes caused by the penetration of BEVs in traffic networks. For this reason, we include the operating cost into the travel impedance evaluation and employ such a generic functional form for evaluating individual travel impedances: \(\text{travel time} + \text{operating cost}\). Meanwhile, we ignore the vehicle depreciation cost because it is difficult to be individually evaluated on the trip level and hence may not directly affect motorists’ travel choice behaviors, and assume that the operating cost is primarily determined by the fuel price, that is, gasoline or electricity price, and has a simple linear relationship with the driving distance. In addition, for simplicity, insurance and taxes tied to the miles driven are not taken into account as travel impedances.

Given the current battery technologies, it typically costs a number of hours to fully charge a BEV, depending on the electricity-charging equipment and battery capacity. For example, a midsize BEV with a 20-kWh lithium-ion battery pack may require 6–8 hours for a full charge with a level 2 charger (providing the 240 VAC charging), and up to 20 hours with a level 1 charger (providing the 120 VAC charging) (Morrow et al., 2008). In face of such relatively long charging time and insufficient charging infrastructures, BEV motorists nowadays (and probably in the foreseeable future) will have to charge their vehicles most of the time by home-based and workplace-based charging facilities and only use their BEVs for trips or tours (i.e., trip chains) with the driving distance shorter than (or equal to) the battery capacity-confined limit. In many metropolitan areas across the world, the driving distance of a round trip between many residential areas and their major international airports is, for example, over 120 or 150 km, whereas the driving range of many electric vehicle models in the current market is less than these numbers. For instance, the driving ranges of Mitsubishi i-MiEV and Roewe E50 are about 100 and 120 km, respectively. As circulated in the driving public, this driving distance limit poses the well-known range anxiety issue: The mental distress or fear of being stranded because the battery runs out of charge (Mock et al., 2010). To take the low-cost advantage and avoid the range anxiety concern with BEVs, a large number of families may prefer to own both GVs and BEVs simultaneously in the transition period from GVs to BEVs, at least until the time the coverage of public charging stations reaches an acceptable level. A recent survey conducted by Graham-Rowe et al. (2013) found that many travelers prefer not to drive BEVs for certain trips or change their travel plans due to the concern about the insufficiency of stored energy for journey completion.

The research problem of interest in this article is a network equilibrium problem for traffic networks with mixed transportation modes or vehicle types discussed above. Specifically, we assume a network of mixed GV and BEV flows in which all motorists have access to GVs and BEVs and make mode and route choices jointly in terms of the travel cost minimization objective and subject to the driving distance limit pertaining to BEVs. By this assumption, we do not mean to expect that all motorists in any real-world network own both GVs and BEVs simultaneously. The real-life condition would be that some travelers have only GVs, then the resulting network equilibrium problem degenerates to the basic traffic assignment problem; some people have only BEVs, this problem becomes the one that has been studied by Jiang et al. (2012). To analyze how the operation cost and range limit impact people’s vehicle and route choices, this study focuses only on those people who own both GVs and BEVs and, for simplicity, assumes traffic networks only comprising motorists who have accessibility to both GVs and BEVs. Such a setting is distinct from the study conducted by Jiang et al. (2013) on a network equilibrium problem of mixed GVs and BEVs in traffic flows, where the mode split is assumed to be fixed. More realistic models can be easily built by combining the basic traffic assignment problem, the distance-constrained traffic assignment problem, and the model developed in this study, and by taking into account the effect of distance limit on trip chains instead of individual trips. Given a trip chain-based network equilibrium model can be simply built based on its trip-based version (see, for example, Maruyama and Harata, 2006), we focus in this article on a trip-based network equilibrium problem with mixed GV and BEV flows and treat it as a building block as more complex trip chain-based problems.

Another important type of vehicles that can address the range anxiety concern while enjoying the low fuel price and vehicle emissions is PHEVs. For the sake of simplicity, we do not explicitly consider PHEVs here since they can be approximately treated as an in-between vehicle type of GVs and BEVs in terms of the technological and economic features (i.e., driving range limit and travel cost composition), or a special type of GVs with a lower operating cost if the varying distance-dependent operation cost is not taken into account.
Joint route and mode choices with GVs and BEVs.

Nevertheless, due to the distance limit, if one chooses to drive a BEV, he or she will have to accordingly choose a route that is shorter than (or with an equal length to) the distance limit, while if choosing to drive a GV, he or she is free of the distance limit worry. Given the lower operating cost with driving BEVs and the driving distance limit imposed on BEVs, the simultaneous mode and route choices will result in such a simple combined mode–route split pattern under equilibrium: Motorists either choose to drive BEVs on routes with the length shorter than or equal to the distance limit, or drive GVs on those routes with the length longer than the limit. The equilibrium choice results on different levels can be illustrated by the tree structure in Figure 1. The corresponding network equilibrium conditions are simply an extension of Wardrop’s (1952) user-equilibrium principle: Each motorist seeks a feasible combination of route and mode to minimize his/her own travel cost; no motorist can reduce his/her travel cost by unilaterally changing route or mode. Under equilibrium, the above combined mode–route flow pattern is a joint result of the driving distance limit and the travel cost function. It should be noted that this special yet simple combined choice mechanism is purely characterized by the deterministic equilibrium conditions, which is different from those widely used travel choice and travel demand models that are dominantly specified by the random utility theory. This simplicity makes the model concise and relatively easy to solve.

In the text below, while elaborating the problem definition and formulation, we will place our research focus on the computation and analysis aspects of network flow patterns and behaviors. Specifically, our interest is in an implementation of two solution methods for the unique mixed-mode traffic equilibrium problem, a comparison of their algorithmic features and computational performance, and an analysis of numerical results and interpretation of the implied behavioral and policy insights, through a set of small- and medium-size example networks. We anticipate that the modeling and solution methods presented in this article, can be used as a stand-alone network analysis tool or integrated into more complex travel demand models for assessing traffic networks with both GVs and BEVs.

The remaining part of this article is organized as follows. The next section presents a convex programming model for the mixed network equilibrium problem and establishes its solution equivalency to the defined mode–route equilibrium conditions, along with a brief overview of previous network equilibrium models involving both mode and route choices. We will then elaborate how two solution algorithms are developed for the proposed mixed network equilibrium problem. One of them is the Frank–Wolfe algorithm, which allows for a parallel treatment of origin-destination (O-D) pairs and a one-to-all constrained shortest path procedure for path generation, whereas another is the projected gradient algorithm, which deals with O-D pairs sequentially and uses a one-to-one constrained shortest path procedure to generate paths. As will be discussed in detail, each of them has certain algorithmic advantages against the other and which one should be chosen in a variety of realistic problem instances and computation environments poses an interesting research question. A set of recommendations regarding the solution performance and algorithm choice will be commented on after we analyze the numerical and computational results from implementing the two algorithms, which simply suggests that, from the behavioral perspective, the produced mode–route network flows replicate the anticipated travel choice pattern, and, from the efficiency perspective, the projected gradient algorithm outperforms the Frank–Wolfe algorithm if the solution precision higher than $1 \times 10^{-3}$ is required even if only a one-to-one constrained shortest path routine is embedded into the former whereas the latter invokes a one-to-all path search routine.

2 PREVIOUS RESEARCH

In general, a network equilibrium problem with multiple vehicle/mode classes cannot be written as a convex mathematical programming model, due to the existence of the asymmetric Jacobi matrix caused by different impacts on travel cost from different vehicle/mode classes (Dafermos, 1972). Instead, it has been formulated as nonlinear complementarity problems (e.g., Florian, 1977; Florian and Nguyen, 1978; Sheffi and Daganzo, 1980; Fisk and Nguyen, 1981).
variational inequality problems (e.g., Friesz, 1981; Dafermos, 1982; Florian and Spiess, 1983; Fernandez et al., 1994; Wu and Lam, 2003), or fixed-point problems (e.g., Ashtiani and Magnanti, 1981; Cantarella, 1997; Bar-Gera and Boyce, 2003). A general review of these techniques used for modeling network equilibrium problems can be referred to in Florian and Hearn (1995).

By using separate traffic and transit subnetworks, asymmetric Jacobian elements may be removed; following this approach, Florian and Nguyen (1978), for example, presented a convex programming model for the combined trip distribution, mode split, and traffic assignment problem. With a similar subnetwork–hypernetwork idea, Sheffi and Daganzo (1980) embedded the probit model into the same problem to specify all travel choices. Though the model is initially represented by a nonlinear complementarity system, it can be potentially represented by an equivalent convex program (see Sheffi and Powell, 1982). By combining the mode split and traffic assignment into a single step, Safwat and Magnanti (1988) eliminated the asymmetric Jacobian issue in constructing their convex programming model for the combined trip generation, trip distribution, mode split, and traffic assignment problem. In another attempt, by a cost-to-time transformation for the link cost function, Lam and Huang (1992a, b) reported an alternative convex programming model for the same problem. As a synthetic review, Oppenheim (1995) summarized a set of convex programming models and provided detailed solution approaches and applications for equilibrium demand models under different behavior assumptions and modeling dimensions. It should, however, be noted that relaxing the asymmetric restriction inevitably leads to a certain level of unrealistic modeling results. To avoid this deficiency, Friesz (1981), by using a set of variational inequalities to represent the user-equilibrium relationship, formulated the combined trip distribution, mode split, and traffic assignment problem that contains the asymmetric restriction into a novel mathematical program. The disadvantage of this formulation, however, is that it is of a nonconvex functional form, where the nonconvexity is caused by the existence of the variational inequality constraints. In addition, in the special case where Jacobi matrix is symmetric, there exists an equivalent convex programming problem (e.g., Abdulaal, 1978; Abdulaal and LeBlanc, 1979; Dafermos, 1971, 1972; Boilé and Spasovic, 2000; Boyce and Xie, 2013).

In our problem, though the two vehicle/mode classes, GVs and BEVs, share the same network, they do not pose different conversion rates from person trips to vehicle trips and occupancy rates in traffic flows. The corresponding terms in the travel time function to the two classes are identical and thus their flow-time impacts on each other are symmetric. This results in the symmetric Jacobi matrix of link costs. As such, the problem proposed in this work can be written as a convex minimization model.

### 3 PROBLEM FORMULATION AND PROPERTIES

In this study, GVs and BEVs are distinguished only by the driving distance limit and travel cost composition. To simplify the model structure, we assume that the technological and economic characteristics of BEVs and demographic features of BEV motorists are homogeneous in target traffic networks, and GVs and GV motorists as well. Modeling multiple types of BEVs with different driving distance limits can be done in a similar way without adding too much modeling and solution difficulty (see, for example, Jiang et al., 2012).

#### 3.1 Problem formulation

For discussion convenience, we first give the notation of variables and parameters that will be used throughout this work, where subscripts g and e are used to indicate variables or parameters associated with gasoline and electric vehicles, respectively.

Here, \( x_{a,g} \) is GV flow rate on link \( a \); \( x_{a,e} \) is BEV flow rate on link \( a \); \( \rho \) is value of time; \( C_a \) is capacity of link \( a \); \( t_a \) is travel time on link \( a \) which is a continuous, increasing, and continuously differentiable function such as

\[
\begin{align*}
t_a &= t_a^0 \left( 1 + \alpha \left( \frac{x_{a,g} + x_{a,e}}{C_a} \right)^\beta \right) \\
\end{align*}
\]

(1)

where \( t_a^0 \) is the free-flow travel time on link \( a \); and \( \alpha \) and \( \beta \) are function parameters.

Here, \( c_{g} \) is operating cost per mile (or unit operating cost) of a GV; \( c_{e} \) is operating cost per mile (or unit operating cost) of a BEV; and \( c_{a,g} \) is generalized travel cost of a GV on link \( a \).

\[
\begin{align*}
c_{a,g} &= \rho t_a (x_{a,g} + x_{a,e}) + c_g d_a \\
\end{align*}
\]

(2)

Here, \( c_{a,e} \) is generalized travel cost of a BEV on link \( a \).

\[
\begin{align*}
c_{a,e} &= \rho t_a (x_{a,g} + x_{a,e}) + c_e d_a \\
\end{align*}
\]

(3)

Here, \( f_{k,g}^{rs} \) is GV flow rate on path \( k \) between O-D pairs \( (r,s) \); \( f_{k,e}^{rs} \) is BEV flow rate on path \( k \) between O-D pairs \( (r,s) \); \( \delta_{a,k}^{rs} \) is link-path incidence parameter, where \( \delta_{a,k}^{rs} = 1 \) if link \( a \) is contained by path \( k \) between O-D
pairs \((r, s)\), and \(\delta_{r,s}^k = 0\), otherwise; \(q^{rs}\) is travel demand rate between O-D pairs \((r, s)\); \(D\) is driving distance limit of BEVs; \(l^r_k\) is length of path \(k\) between O-D pairs \((r, s)\), where \(l^r_k = \sum_a d_a \delta_{r,s}^k\), and \(d_a\) is distance of link \(a\).

Given the above definition of \(c_{a,g}\) and \(c_{a,e}\), we can easily calculate the cost-flow Jacobi matrix elements of GVs and BEVs and see their equivalence:

\[
\frac{\partial c_{a,g}}{\partial x_{a,g}} = \frac{\partial c_{a,e}}{\partial x_{a,e}} = \rho \alpha \beta t_0 \left( x_{a,g} + x_{a,e} \right) (C_a)^{-1}
\]

which proves that the cost-flow Jacobi matrix of our problem is symmetric. The equivalent convex minimization problem then can be established as below:

\[
\text{min} Z(x(f)) = \sum_a \left[ \rho f_0^s a g + t_0(\omega) d\omega + (x_{a,g} c_g + x_{a,e} c_e) d_a \right]
\]  (4)

subject to

\[
\sum_k \left( f_{k,g}^s + f_{k,e}^s \right) = q^r \ \forall r, s \ (\mu^{rs})
\]  (5)

\[
(D - l^r_k) f_{k,e}^{rs} \geq 0 \ \forall k, r, s \ (\lambda^{rs}_k)
\]  (6)

\[
f_{k,g}^{rs} \geq 0, \ f_{k,e}^{rs} \geq 0 \ \forall k, r, s
\]  (7)

where

\[
x_{a,g} = \sum_{rs} f_{k,g}^{rs} \delta_{a,k} \ \forall a
\]  (8)

\[
x_{a,e} = \sum_{rs} f_{k,e}^{rs} \delta_{a,k} \ \forall a
\]  (9)

where \(\mu^{rs}\) and \(\lambda^{rs}_k\) are dual variables associated with constraints (5) and (6), respectively.

Note that the objective function of the above optimization problem is a simple extension from Beckmann’s transformation (Beckmann et al., 1956), which will result in the optimality conditions equivalent to the extended Wardrop’s equilibrium conditions described above.

### 3.2 Optimality conditions

To prove that the solution of the mathematical program (4)–(9) corresponds to the extended Wardrop’s equilibrium principle for the combined mode–route choice behavior, we first formulate the Lagrangian problem associated with above problem by relaxing constraint (5) and (6):

\[
L(f, \mu, \lambda) = Z(x(f)) + \sum_{rs} \sum_k \mu^{rs} \left[ q^{rs} - \sum_k \left( f_{k,g}^{rs} + f_{k,e}^{rs} \right) \right] - \sum_{rs} \sum_k \lambda^{rs}_k \left( D - l^r_k \right) f_{k,e}^{rs}
\]  (10)

\[
f_{k,g}^{rs} \geq 0, \ f_{k,e}^{rs} \geq 0 \ \forall k, r, s
\]  (11)

\[
\lambda^{rs}_k \geq 0 \ \forall k, r, s
\]  (12)

Then, we check the optimality conditions of the Lagrangian problem:

\[
f_{k,g}^{rs} \left[ \sum_a c_{a,g} \delta_{a,k} - \mu^{rs} \right] = 0 \ \forall k, r, s
\]  (13)

\[
\sum_a c_{a,g} \delta_{a,k} - \mu^{rs} \geq 0 \ \forall k, r, s
\]  (14)

\[
f_{k,e}^{rs} \left[ \sum_a c_{a,e} \delta_{a,k} - \lambda^{rs}_k (D - l^r_k) - \mu^{rs} \right] = 0 \ \forall k, r, s
\]  (15)

\[
\sum_a c_{a,e} \delta_{a,k} - \lambda^{rs}_k (D - l^r_k) - \mu^{rs} \geq 0 \ \forall k, r, s
\]  (16)

\[
\lambda^{rs}_k (D - l^r_k) f_{k,e}^{rs} = 0 \ \forall k, r, s
\]  (17)

\[
(D - l^r_k) f_{k,e}^{rs} \geq 0 \ \forall k, r, s
\]  (18)

\[
\left( f_{k,g}^{rs} + f_{k,e}^{rs} \right) = q^r \ \forall r, s
\]  (19)

\[
f_{k,g}^{rs} \geq 0, \ f_{k,e}^{rs} \geq 0 \ \forall k, r, s
\]  (20)

\[
\lambda^{rs}_k \geq 0 \ \forall k, r, s
\]  (21)

Let \(\sum_a c_{a,g} \delta_{a,k} = c_{k,g}^r\) and \(\sum_a c_{a,e} \delta_{a,k} = c_{k,e}^r\) where \(c_{k,g}^r\) and \(c_{k,e}^r\) are the generalized travel costs of driving GVs and BEVs on path \(k\) between O-D pairs \((r, s)\), then conditions (13)-(18) can be rewritten as the following group of systems of equalities/inequalities:

\[
\begin{align*}
\{ c_{k,g}^r = \mu^{rs} \text{ if } f_{k,g}^{rs} > 0 \\
\{ c_{k,e}^r > \mu^{rs} \text{ if } f_{k,g}^{rs} = 0 \ \forall k, r, s
\end{align*}
\]  (22)

\[
\begin{align*}
\{ c_{k,e}^r = \mu^{rs} + l^r_k - D \text{ and } \lambda^{rs}_k = 0 \text{ if } f_{k,e}^{rs} > 0 \\
\{ c_{k,e}^r + \lambda^{rs}_k (l^r_k - D) > \mu^{rs} \text{ if } f_{k,e}^{rs} = 0 \ \forall k, r, s
\end{align*}
\]  (23)
Here, \( \mu_{rs}^\infty \) can be interpreted as the minimum generalized travel cost between O-D pairs (r, s). The term \( \lambda_{k}^{rs}(l_{k}^{rs} - D) \) in the second system for BEV flows is defined as the path out-of-range cost. The equilibrium conditions in (22) show that if a path carries positive BEV flows, it implies that this path is shorter than the distance limit, the out-of-range cost of this path is zero, and the generalized travel cost equals to the corresponding minimum cost. The above two sets of systems for motorist’s route and mode choice behaviors imply the following equilibrium conditions:

1. All the paths between O-D pairs (r, s) are distance-feasible. In this case, \( c_{k,g}^{rs} > c_{k,e}^{rs} \geq \mu_{rs}^\infty \) since \( g > e \). So all motorists will choose BEVs, that is, \( \sum_{k} f_{k,g}^{rs} = q^{rs} \) and \( f_{k,e}^{rs} = 0 \), for all \( k \in K_{rs} \).
2. None of the paths between O-D pairs (r, s) is distance-feasible. In this case, all motorists will choose GVs, that is, \( \sum_{k} f_{k,g}^{rs} = q^{rs} \) and \( f_{k,e}^{rs} = 0 \), for all \( k \in K_{rs} \).
3. Some but not all paths are distance-feasible. In this case, some motorists will choose to drive BEVs on those paths with their travel costs \( c_{k,g}^{rs} \) equal to \( \mu_{rs}^\infty \) among all distance-feasible paths, whereas some motorists will choose to drive GVs on some distance-infeasible paths with their travel costs \( c_{k,g}^{rs} \) equal to \( \mu_{rs}^\infty \). Note that no motorist will choose to drive a GV on any distance-feasible path, since \( c_{k,g}^{rs} > c_{k,e}^{rs} \geq \mu_{rs}^\infty \) for any distance-feasible path \( k \).

4 SOLUTION ALGORITHMS

Recently, Jiang et al. (2012) suggested a Frank–Wolfe procedure for solving the distance-constrained traffic assignment problem, in which a combined preprocessing and label-setting algorithm (PLS) proposed by Dumitrescu and Boland (2003) was used to solve the distance-constrained shortest path problem (CSPP). Though PLS is one of the most efficient procedures for the CSPP, it can only be applied to one-to-one cases, which does not take advantage of the simultaneous all-to-all shortest path search possibility allowed by the Frank–Wolfe framework. A large part of solution efficiency of PLS lies in its preprocessing stage, which makes uses of the information about the cost-based and distance-based shortest paths from the origin to all nodes in the network and from all nodes to the destination. If the preprocessing stage is removed from PLS, the remaining label-setting stage can still be used to solve CSPP with a longer computing time, but its capability of finding one-to-all constrained shortest paths is released.

In this section, two solution algorithms are proposed for solving the mixed network equilibrium problem. The first one employs the Frank–Wolfe algorithmic framework, as similar to the one sketched in Jiang et al. (2012), with a modified one-to-all label-setting algorithm (MLS) for finding distance-constrained shortest paths. The second one is the projected gradient algorithm, which was suggested by Florian et al. (2009) for the prime traffic assignment problem, with the one-to-one PLS mentioned above as the distance-constrained shortest path search engine. For discussion convenience, here we use FW-MLS and PG-PLS as the abbreviation of the two suggested algorithmic schemes, respectively.

It is well known that the Frank–Wolfe algorithm is a linear approximation procedure that typically delivers sublinear convergence performance whereas the projected gradient algorithm is a quadratic approximation procedure that can offer linear convergence performance. Thus, in general, the projected gradient algorithm outperforms the Frank–Wolfe algorithm in solving traffic assignment problems (see, for example, Florian et al., 2009). However, in the Frank–Wolfe procedure, the equilibrium process is conducted after the shortest path search for all O-D pairs is finished, which allows for the implementation of a one-to-all shortest path algorithm (or an all-to-all shortest path algorithm) for the shortest path search; whereas in the projected gradient procedure, the equilibrium process is conducted after the shortest path search for each O-D pair is finished, which, for the efficiency consideration, typically uses the one-to-one shortest path algorithm for the shortest path search. As we know, applying a one-to-all shortest path algorithm for solving the shortest path problem (SPP) between all O-D pairs is typically more efficient than a one-to-one shortest path algorithm. This is especially apparent in our case, in which solving CSPP is required for assigning BEV flows and it dominates the computing time among all algorithmic steps. Following such an algorithmic analysis, we are wondering which of the algorithmic choices, FW-MLS or PG-PLS, are more efficient in solving the mixed network equilibrium problem. This question will be answered in the next section. In this section, we describe the two algorithmic schemes.

4.1 The FW-MLS algorithm

We first sketch the FW-MLS procedure as follows:

**Step 1: Initialization.**

For each O-D pair \((r, s)\), find \( \hat{k} \) of shortest distance \( l_{\hat{k}}^{rs} = \min_{k} l_{k}^{rs} \) and \( \hat{k} \) of minimal cost \( c_{\hat{k},g}^{rs} = \min_{k} (c_{k,g}^{rs}(0)) \).

If \( l_{\hat{k}}^{rs} > D \), then

Let \( \Phi_{rs} = 1 \), where \( \Phi \) is a binary variable, \( \Phi_{rs} = 1 \) indicating that there is no distance-feasible path between
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Table 1
Route composition and length of the Nguyen–Dupuis network

<table>
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<tr>
<th>O-D</th>
<th>Route</th>
<th>Node sequence</th>
<th>Length</th>
<th>O-D</th>
<th>Route</th>
<th>Node sequence</th>
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</table>

Fig. 2. The Nguyen–Dupuis network.

Step 2: Update.
Calculate the updated generalized link costs according to $c_{a,g}^n = \rho (x_{a,g}^n + x_{a,e}^n) + c_{g}d_{a} + c_{a,e} = \rho (x_{a,g}^n + x_{a,e}^n) + c_{g}d_{a} + c_{a,e}(y_{a,g}^n, y_{a,e}^n, \forall a)$.

Step 3: Direction finding.
For each O-D pair $(r, s)$
Find the updated least cost path $\hat{k}$ of $c_{k,r}^s = \min_k \{c_{k,r}^s \}$. 
If $\Phi_{x}^s = 1$, then
Assign all the travel demand between this O-D pair to $\hat{k}$. Update the GV flows on those links used by $\hat{k}$.
Else
Find the new distance-constrained least-cost path $\hat{k}$ of minimal cost
$c_{k,e}^s = \min_k \{c_{k,e}^s : \hat{l}^s_k \leq D \}$. 
If $c_{k,e}^s \leq c_{k,g}^s$, then
Assign all vehicles to $\hat{k}$. Update the BEV flows on those links used by $\hat{k}$.
Else
Assign all the demand to $\hat{k}$. Update the GV flows on those links used by $\hat{k}$.
End if
End if
End for
This yields auxiliary flow $\{y_{a,g}^n\}$ and $\{y_{a,e}^n\}, \forall a$.

Step 4: Line search.
Apply any of the interval reduction line search method to find the optimal value of $\theta$ by solving

$$
\min_{0 \leq \theta \leq 1} \sum_{a} \left[ \rho \int_{0}^{l_{e}(\omega)} \left( x_{a,g}^n + \theta (x_{a,g}^n + y_{a,g}^n) - (x_{a,g}^n + y_{a,g}^n) \right) t_{e}(\omega)d\omega + d_{a}\left( x_{a,g}^n + \theta (y_{a,g}^n - x_{a,g}^n) \right) c_{g} + (x_{a,e}^n + \theta (y_{a,e}^n - x_{a,e}^n)) c_{e} \right]
$$

(24)
Step 5: Move.
Set \( x_{n+1}^{a,g} = x_{n}^{a,g} + \theta \left( y_{n}^{a,g} - x_{n}^{a,g} \right) \), \( x_{n+1}^{a,e} = x_{n}^{a,e} + \theta \left( y_{n}^{a,e} - x_{n}^{a,e} \right) \), \( n = n + 1 \).

Step 6: Convergence test.
If the preset convergence criterion is not met, set \( n = n + 1 \) and go to step 2; otherwise, stop and \( \{ x_{n+1}^{a,g}, x_{n+1}^{a,e} \} \) is the set of equilibrium link flows.

In step 3, SPP and CSPP need to be solved repeatedly for GV and BEV flows, respectively. The MLS procedure described below is for solving CSPP. As for SPP, Dijkstra’s (1959) algorithm is employed here.

The MLS sketched here is based on the label-setting algorithm of Desrochers and Soumis (1988). The key algorithmic device is node labels: The \( k \)th label for node \( i \), \((C_k^i, D_k^i)\), represents the \( k \)th path from the origin node to node \( i \) and is composed of two attributes: \( C_k^i \) is the cost of traveling along this path from the origin node to node \( i \) and \( D_k^i \) is the length of this path. An important notion in this algorithm is the label dominance or efficiency, which is originally from multicriterion SPPs. In our case, it is defined as:

**Definition 1.** A label \((C_k^i, D_k^i)\) is said to be dominated by another label \((C_l^i, D_l^i)\) if \( C_k^i > C_l^i \) and \( D_k^i \geq D_l^i \) or \( C_k^i \geq C_l^i \) and \( D_k^i > D_l^i \). A label \((C_k^i, D_k^i)\) is said to be efficient if it is not dominated by any other label on the same node and \( D_k^i \leq D \).

The labeling process in the MLS procedure can be briefly described as follows. The process moves forward from the origin node to all other nodes and updates the set of labels on those nodes so that only efficient labels/paths are maintained. It will end when all the efficient labels are generated. At the end of process, for each node, among the set of efficient labels, the one with the least cost is the optimal label and the corresponding path is the optimal path. For our problem, we do not need to find the optimal path from an origin node to every other node. Instead, we only need to find the optimal path between those O-D pairs that has at least one distance-feasible path. Besides, we also know that if \( C_k^i \) is the minimum cost among all labels on all nodes, then the \( k \)th path from the origin to node \( i \) is the optimal path between them. Therefore, once we find the optimal paths from the origin to all the destinations that are reachable by BEVs from this origin, the algorithm...
The relationship of the mode split and the unit operation costs of the Nguyen–Dupuis network.

ends. If we define \( L(i) \) as the set of labels on node \( i \), \( SE \) as the set of checked eligible node and label pairs, \( FN \) the set of nodes that are reachable from origin node \( r \), and \( OL \) the set of nodes the optimal path from origin to which has been found, then the pseudo-code of MLS can be given as follows:

Initialize \( L(r) = \{(0, 0)\} \), \( L(i) = \emptyset \), \( \forall i \in N \setminus \{r\} \), \( SE = \{(r, (0, 0))\} \), \( OL = \{r\} \), and \( FN = \{s|\Phi^s = 0\} \).

While \( FN \not\subset OL \):

Choose a pair of node and label \((i, (C_i^k, D_i^k))\) from \( SE \) so that the \( C_i^k \) is the minimum among all labels in \( SE \).

Set \( SE = SE - \{(i, (C_i^k, D_i^k))\} \).

For all nodes \( j \) such that \((i, j) \in A \):

If \( D_j^k + d_{ij} \leq D \), then

Delete all labels in \( L(j) \) that are dominated by \((C_i + c_{ij}, D_i + d_{ij})\).

If \((C_i + c_{ij}, D_i + d_{ij})\) is not dominated by any label in \( L(j) \), then

Set \( L(j) = L(j) \cup (C_j + c_{ij}, D_j + d_{ij}) \).

Set \( SE = SE \cup (j, (C_j + c_{ij}, D_j + d_{ij})) \).

End if

End if

End for

End while

4.2 The PG-PLS algorithm

Then we sketch the PG-PLS procedure below. The PLS subroutine is directly from Dumitrescu and Boland (2003) and thus not repeated here.

Step 0: Initialization.

For each O-D pair \((r, s)\), find \( \hat{k} \) of shortest distance \( l_k^r \) and \( \hat{k} \) of minimum cost \( c_{k,g}^r = \min_k\{c_{k,g}^r(0)\} \).

If \( l_k^r > D \), then

Let \( \Phi^s = 1 \). Assign all the travel demand between \( r \) and \( s \) to \( \hat{k} \). Add \( \hat{k} \) to \( K_{r,s}^+ \), the set of paths with positive GV flows.

Else

Find the distance-constrained least cost path \( \hat{k} \) of minimal cost \( c_{k,g}^r = \min_k\{c_{k,g}^r(0) : l_k^r \leq D\} \).

If \( c_{k,g}^r \leq c_{\hat{k},g}^r \), then

Assign all the demand to \( \hat{k} \). Add \( \hat{k} \) to \( K_{r,s}^+ \).

End if

End if

The above procedure yields an initial solution \( \{f_{k,g}^r\}, \{f_{k,g}^s\}, \{x_{a,g}\}, \{x_{s,g}\}, K^+_{r,s} = \{k|f_{k,g}^r > 0\} \) and \( K_{r,s}^+ \).

While the converge criterion is not satisfied

For each O-D pair \((r, s)\):

Step 1: Direction finding.

Compute the descent direction

\[
\begin{align*}
\bar{b}_{k,g}^r &= (\bar{c}_{k,g}^r - c_{k,g}^r), \forall k \in K_{r,s}^+ \\
\bar{b}_{k,e}^r &= (\bar{c}_{k,e}^r - c_{k,e}^r), \forall k \in K_{r,s}^+
\end{align*}
\]

where \( \bar{c}_{k,g}^r \) is the average cost of all paths in set \( K_{r,s}^+ \) and \( K_{r,s}^+ \).

If \( \max_{k \in K_{r,s}^+ \cup K_{r,s}^+} |\bar{b}_{k,g}^r| < \varepsilon \), then

Go to step 4.

End if

Step 2: Line search.

Find the optimal step size \( \theta^* \) which is the solution of the subproblem

\[
\min \sum_a \left\{ \rho \int_0^\infty (x_{a,g}^r + x_{a,e}^r) + \theta (\bar{x}_{a,g}^r + \bar{x}_{a,e}^r) \right\} t_a(\omega) d\omega \\
+ d_a \left( x_{a,g}^r + \bar{x}_{a,g} + \theta \bar{x}_{a,g} \right) c_g \\
+ \left( x_{a,e}^r + \bar{x}_{a,e} + \theta \bar{x}_{a,e} \right) c_e \right\}
\]

subject to \( 0 \leq \theta \leq \min(\frac{\bar{b}_{k,g}^r}{\bar{b}_{k,e}^r}, 0, k \in K_{r,s}^+ \cup K_{r,s}^+) \)

(b) The unit operation cost of BEVs

Fig. 5. The unit operation cost of GVs and the unit operation costs of the Nguyen–Dupuis network.
Fig. 6. The relationship of path flows and the unit operation costs of the Nguyen–Dupuis network.

where \( y_{a,g}^r = \sum_{k \in K_{rs,g}^+} \delta_{a,k}^r b_{k,g}^r \) and \( y_{a,e}^r = \sum_{k \in K_{rs,e}^+} \delta_{a,k}^e b_{k,e}^e \). Two sets of link flow variables \( \bar{x}_{a,g} = \sum_{od \neq rs} \sum_{k \in K_{od,g}^+} \delta_{a,k}^{od} f_{k,g}^{od} \) and \( \bar{x}_{a,e} = \sum_{od \neq rs} \sum_{k \in K_{od,e}^+} \delta_{a,k}^{od} f_{k,e}^{od} \) are, respectively, the fixed GV and BEV flows of all other O-D pairs of the network.

**Step 3: Move.**

Update path flows and link flows

\[
\begin{align*}
    f_{k,g}^r &= f_{k,g}^{rs} + \theta b_{k,g}^{rs}, \forall k \in K_{rs,g}^r \\
    f_{k,e}^r &= f_{k,e}^{rs} + \theta b_{k,e}^{rs}, \forall k \in K_{rs,e}^r
\end{align*}
\]
\[ x_{a,g}^{rs} = x_{a,g}^{r} + \theta y_{a,g}^{rs}, \forall a \]
\[ x_{a,e}^{rs} = x_{a,e}^{r} + \theta y_{a,e}^{rs}, \forall a \]

If a path \( f_{k,g}^{rs} \) (\( f_{k,e}^{rs} \)) diminishes to zero, then

Eliminate this path, that is, \( K_{rs,g}^{+} (K_{rs,e}^{+}) = K_{rs,g}^{+} (K_{rs,e}^{+}) - k \).

End if

Step 4: Update the path set.
Comput the shortest path \( \hat{k} \) of minimal cost \( c_{\hat{k},g}^{rs} = \min_{k} \{ c_{k,g}^{rs} \} \).

If \( \Phi_{k,g}^{rs} = 1 \) and \( c_{\hat{k},g}^{rs} < \min_{k \in K_{rs,g}^{+}} \{ c_{k,g}^{rs} \} \), then

Add path \( \hat{k} \) to the set of active paths \( K_{rs,g}^{+} \) and return to step 1.

Else

Compute the constrained shortest path \( \bar{k} \) of minimal cost \( c_{\bar{k},g}^{rs} = \min_{k} \{ c_{k,g}^{rs} : l_{k}^{g} \leq D \} \)

End if

If \( c_{\bar{k},g}^{rs} \leq c_{\bar{k},g}^{rs} \) and \( c_{\bar{k},g}^{rs} \leq \min_{k \in K_{rs,g}^{+}} \{ c_{k,g}^{rs} \} \), then

Add path \( \bar{k} \) to the set of active paths \( K_{rs,g}^{+} \) and return to step 1.

Else if \( c_{\bar{k},g}^{rs} < c_{\bar{k},g}^{rs} \) and \( c_{\bar{k},g}^{rs} < \min_{k \in K_{rs,g}^{+}} \{ c_{k,g}^{rs} \} \), then

Add path \( \bar{k} \) to the set of active paths \( K_{rs,g}^{+} \) and return to step 1.

Else

Stop the procedure.

End if

End for

End while

5 NUMERICAL AND COMPUTATIONAL ANALYSIS

This section contains a presentation of the numerical and computational analysis results we obtained from applying the above two algorithms to the mixed network equilibrium problem in a few synthetic and realistic example networks. The purpose of this analysis is of both the behavioral and computational aspects: (1) to assess the impacts on the network performance from the distance limit and fueling price; (2) to compare the computational performance of the two algorithms in solving this type of traffic assignment problems. Both the algorithms are coded in C++. We used a Windows 7–based PC equipped with a Core 2 Duo CPU E7500 processor running at 2.93 GHz as the computing platform. All the computing times reported below are the result from using only one core of the processor of the computer.

5.1 Evaluation of network flows
To look into the GV and BEV link flow patterns given different driving distance limits, a small network shown in Figure 2 is used as an illustrative example. This network was originally used by Nguyen and Dupuis (1984). The network includes 19 links, 13 nodes, and 4 O-D pairs. Nodes 1 and 4 represent origin zones; nodes 2 and 3 represent destination zones.

The original network supply and demand information from Nguyen and Dupuis (1984) is all used here. Moreover, the free-flow travel time of each link is used as a proxy of the link length and additional parameter values such as the value of time \( \rho = 4 \) and the unit operation costs \( c_{g} = 4 \) and \( c_{e} = 1 \) are suggested by the authors. Due to the small size of the network, we can enumerate all its paths and identify the distance feasibility of these paths a priori, as given in Table 1. Such information is very useful in understanding the combined mode–route choice result under the driving distance limit.

The remaining numerical evaluation for this small network is mainly focused on how the driving distance limit and unit operating cost influence the mode and route choices and network flows. The evaluation is conducted by a scenario-based sensitivity analysis.

Given the all integer values of the path lengths (see Table 1), we developed a set of problem scenarios, each of which corresponds to an integer value of the distance limit \( D \) ranging from 29 to 36. By evaluating the traffic flow pattern for each integer value of the distance limit and aggregating the result, we then obtain a complete profile of network flows over the given range. It is noted that 29 is the lower bound of the distance limit for the existence of at least one distance-feasible path in the network, that is, all motorists choose to drive GVs if the distance limit is less than 29, while 36 is the upper bound of the distance limit for the existence of GV flows, that is, all motorists choose to drive BEVs if the distance limit is equal or greater than 36. Note that though when \( D \geq 36 \), no motorist will choose GVs, the network flow pattern could still be changed with the change of the distance limit. This change is purely the result of the changing route availability for BEV motorists.

First, Figure 3 shows the overall mode split in the network with the change of the distance limit. We can see that in the distance limit range from 29 to 36, more paths become distance-feasible and more motorists choose to drive BEVs with the increase of the distance limit. The proportions of BEVs and GVs show an increasing and decreasing relationship with the distance limit, respectively. Second, we then analyze the impacts of the distance limit on the mode and route choices. Figure 4 gives a complete combined mode–route flow split result
for all O-D pairs. For clarity, we omitted those zero-flow mode–route choices in the figure. Despite the variety of mode–route flow patterns across different O-D pairs, all plots in the figure clearly show that all route flows pertain exclusively to one mode under any given distance limit value. In other words, no route simultaneously carries both GV and BEV flows. The underlying reason, as we discussed earlier, is the difference of the electricity-charging price and gasoline-fueling price. From the perspective of reflecting the reality, this result sounds very abrupt. One way to relax such a restricted one-mode route flow phenomenon is to use the random utility theory instead of the deterministic equilibrium principle to specify travel choices in the model. Note that the impact of the distance limit on the mode choice and path choice results has a piecewise effect, due to the discrete nature of individual path lengths.

On the other hand, we investigate the impact of the unit operation costs on the mode and route choices and network flows still in the small network given in Figure 1. For simplicity, the distance limit \( D \) is fixed at 32 in all scenarios of the following sensitivity analysis.

Different from the distance limit that causes the change of network flows in a noncontinuous way, the unit operating costs presents a continuous relationship with network flows. Figure 5 is the evaluation result from the varying unit operation costs of GVs and BEVs, which are the proxies of the gasoline-fueling price and electricity-charging price, respectively. The mode split tendency is fully compliant with the expected result: The higher the gasoline-fueling price or the lower the electricity-charging price, the more people switch from GVs to BEVs. The same phenomenon also occurs on the path level, as shown in Figure 6.

### 5.2 Evaluation of computational performance

This section focuses on a numerical analysis of the computational performance of the two solution algorithms. Two larger networks, the Sioux Falls network (see Figure 7) and Anaheim network (see Figure 8), are used for this purpose. The Sioux Falls network is a small-size transportation network with 24 nodes, 76 links, and 576 O-D pairs. The Anaheim network is a medium-size one with 416 nodes, 914 links, and 1,444 O-D pairs. The complete information of the two networks can be found at the following transportation network problem website: http://www.bgu.ac.il/~bargera/tntp/. Additional parameters required for evaluating the mixed GV-BEV flows in these networks are given as: The value of time is \( \rho = US\$10/hour \), and the unit operation costs of GVs and BEVs are \( c_g = US\$0.16/mile \) and \( c_e = US\$0.04/mile \), respectively.
Computing and analyzing mixed equilibrium network flows

(a) $D = 0$

(b) $D = 20$

(c) $D = +\infty$

Fig. 9. Convergence performance of FW-MLS and PG-PLS for the Sioux Falls network.

(a) $D = 0$

(b) $D = 20$

(c) $D = +\infty$

Fig. 10. Convergence performance of FW-MLS and PG-PLS for the Anaheim network.
Note the distance limit values used in these examples are adjusted in reference to the size of these networks so that the distance constraint does make significant impacts on network flow patterns. In the real world, their values could range from about 40 to 250 miles. These example networks are used to purely illustrate the impact of distance limit on the algorithms’ performance instead of replicating the real-world situations.

The convergence performance results from applying FW-MLS and PG-PLS to solve the two example networks are discussed, as depicted in Figures 9 and 10, respectively, given different distance limit values ranging from 0 to infinity. The convergence precision of these computational experiments is set as the relative gap between the total travel time over the network and the total travel time if all flows are assigned to shortest paths no more than $1 \times 10^{-6}$ unless 1,000 iterations are reached. In either of the algorithms, an iteration is defined as the computational process of completely executing the path generation and equilibrium approximation steps of all O-D pairs once. For example, in the pseudo-code of PG-PLS, an iteration means a process of executing all the algorithmic steps starting from clause for to clause end for.

In Figure 9, it is found that for the Sioux Falls network, both the FW-MLS and PG-PLS procedures cost less than 20 seconds to reach the convergence gap of $1 \times 10^{-3}$. This convergence level poses virtually a threshold for us to evaluate the relative performance of the two algorithms: When the required convergence criterion is lower than this threshold (e.g., $1 \times 10^{-2}$), FW-MLS converges faster than PG-PLS; when the convergence criterion is higher than this threshold (e.g., $1 \times 10^{-5}$), PG-PLS apparently outperforms FW-MLS. From Figure 9, we can see that the convergence curves of the FW-MLS and PG-PLS algorithms clearly show the difference between the sublinear and linear convergence behaviors. A similar phenomenon can be observed from Figure 10 for the Anaheim network. However, the convergence threshold is distributed between $1 \times 10^{-4}$ and $1 \times 10^{-5}$, depending on the distance limit. Overall, PG-PLS still significantly outperforms FW-MLS for high-precision solutions.

We are also interested in the impacts of the distance limit on the computational performance. It is well known that if the distance limit $D = 0$, which corresponds to the mode choice result that all motorists choose to drive GVs, or $D = +\infty$, which corresponds to the mode choice result that all motorists choose to drive BEVs, the path generation phase does not involve any CSPP but is completely powered by solving SPPs. It is well known that solving SPP is typically much faster than solving a CSPP of the same size; the former is a polynomial-time problem whereas the latter has the exponential-time complexity (Dumitrescu and Boland, 2003). Given that the path generation phase is the most time-consuming step of the algorithms, we can reasonably assume that the computing times for the case of $D = 0$ or $+\infty$ approximately represent the lower end of the computing times under different distance limit values. But the computational performance of these two extreme cases could be different for two reasons. First, when $D = 0$, only an SSP in terms of $c_{k,g}^{rs}$ is required to be solved whenever a new path is generated because $\Phi^r g = 1$ for every O-D pair; when $D = +\infty$, two SSPs, in terms of $c_{k,g}^{rs}$ and $c_{k,e}^{rs}$ respectively, need to be solved. (Note that finding the shortest path in terms of $c_{k,g}^{rs}$ is a part of the preprocessing phase of the constrained shortest path search. In the case, the length of the obtained shortest path is distance-feasible, it means that the subsequent constrained shortest path search is not necessary.) Second, even travelers face the same feasible route set in the two cases, the travel costs of those routes are different because of different operation costs; as a result, the optimal path set could be different. This, for example, can be seen from the optimal path sets connecting O-D pair (1, 3) in the Nguyen–Dupuis network when $D = 0$ and $D = +\infty$ (see Table 2). As for any other distance limit value between 0 and $+\infty$, its influence on the computing time may be analyzed in two aspects: (1) the number of generated SPPs and CSPPs for the path generation phase; and (2) the tightness of the distance constraint in generated CSPPs. In the former aspect, a very low or very high distance limit implies potentially a lower number of CSPPs to be solved across O-D pairs (i.e., a lower number of O-D pairs are associated with both the SSP and CSPP); in the latter aspect, a lower distance limit means a tighter constraint for the CSPP, which results in a lower number of dominance checks in the labeling process of the MLS or PLS algorithm, whereas a higher distance limit means a looser constraint for the CSPP, which might lead to a rapid conclusion of the optimal

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<th>Optimal path</th>
<th>Path flow</th>
<th>Path cost</th>
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solution in the preprocessing step of PLS. The combined effect of the distance limit on either the FW-MLS or PG-PLS algorithm, however, is difficult to predict in an analytical way. Moreover, the computing efficiency of either of the algorithms not only depends on the path generation phase but also the equilibrium approximation phase. In view of such algorithmic complexity, we resort to a numerical analysis.

The computational results of the Sioux Falls network show the complexity of the relationship between the computing time and the distance limit (see Figure 9). For the purpose of comparison, we purposely set the scale of the x-axis in all the figures to be the same (from 0 to 140 seconds). We can see that the computing time of using PG-PLS or FW-MLS with \( D = 0 \) or \(+\infty\) is significantly lower than that with \( D = 20 \). This is what we expected, as when \( D = 0 \) or \(+\infty\), only SPPs need to be solved while when \( D = 20 \), time-consuming CSPPs need to be solved for some O-D pairs. The computing time with \( D = +\infty \) is slightly higher than \( D = 0 \) due to the reason we discussed previously, that is, one SPP is solved to generate a new path in the former case while two for the latter case. From the results of the Anaheim network (see Figure 10), similar results were found for FW-MLS among all three scenarios, that is, the computation times with \( D = 0 \) and \( D = +\infty \) are similar, in which the time in the latter is slightly higher than that in the former, and both of them are much lower than that with \( D = 20 \), which costs more than 4 hours to finish 1,000 iterations.

6 CONCLUSIONS

This article formulates, solves, and evaluates a special network equilibrium problem with mode and route choices. The transportation modes (or vehicle types) included in this problem are exclusively GVs and BEVs, which are distinguished in terms of their driving distance limit and travel cost composition. A convex programming model is proposed in the simplest form of its type for evaluating such mixed GV–BEV transportation networks that are anticipated to exist for a long period in the future. It provides us with a simple, fundamental tool to understand transportation network flow changes due to the switch of motorists between gasoline and electric vehicles. Given its simple structure, one may readily extend its functionalities (e.g., accommodating GVs, BEVs, and PHEVs, modeling other dimensions of travel choices, incorporating time-dependent factors and traffic dynamics, taking into account supply and demand uncertainties, or considering other battery recharging opportunities in addition to home and workplace charging facilities) to capture other proutrudent transportation network phenomena possibly arising during the transition period from the gasoline era to electricity era.

Our focus in this article is on the solution method and numerical analysis. Specifically, we implemented two competing algorithms, namely, FW-MLS and PG-PLS. FW-MLS is a linear approximation algorithm of the Jacobi type and it takes advantage of dealing with O-D pairs in a parallel manner and using a one-to-all constrained shortest path procedure to generate paths. PG-PLS is a quadratic approximation algorithm of the Gauss–Seidel type, which treats O-D pairs in a sequential manner and uses a one-to-one constrained shortest path procedure for path generation. The computational results we obtained from the implementation of these two algorithms for a number of synthetic and realistic networks clearly show that, on the model behavior aspect, the generated mode–route flow patterns are consistent with our hypothesis that a higher distance limit or a higher gasoline price increases the usage frequency of BEVs when both BEVs and GVs are available to motorists; on the solution efficiency aspect, FW-MLS typically achieves the convergence state faster than PG-PLS for low-precision solutions (e.g., solutions with their convergence gaps larger than \( 1 \times 10^{-3} \)), whereas PG-PLS is preferred to FW-MLS for high-precision solutions (e.g., solutions with their convergence gaps smaller than \( 1 \times 10^{-5} \)).

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