Multi-Robot Cooperative Sensing and Localization

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Abstract
This chapter describes a method for mobile robot localization design based on multi-robot cooperative sensing. A multi-robot cooperative localization system is presented. The system utilizes visual detection and sensor data fusion techniques to achieve mobile robot localization. The visual detection system employs a stereo vision module for both observing other robots and obtaining environmental information. Each mobile robot is able to recognize its teammates by using onboard vision system. The localization error is reduced through the proposed sensor fusion algorithm. The cooperative localization algorithm consists of two stages: serial fusion and parallel fusion. Serial fusion aims to identify the positional uncertainty of an observed robot while parallel fusion reduces its positional error based on Kalman filtering. The multi-robot cooperative localization system has been realized through the client-server architecture. Experimental results are presented to validate the effectiveness of the proposed algorithms.

Keywords: multi-robot system, mobile robot localization, cooperative localization, sensor data fusion, Kalman filtering.

1. Introduction
In recent years, multi-robot systems and distributed autonomous robotic systems have become important research areas in autonomous robotics (Parker, 1998), (Weigel et al., 2002). There are increasing interests in cooperative localization and map-building using a robot team. Traditionally, robot localization and map-building are resolved by a single robot equipped with various perception sensors (Gamini Dissanayake et al., 2001), (Nieto et al., 2002), (Nieto et al., 2003). However, a single mobile robot equipped with several types of sensors will consume much power and is more expensive to construct. A multi-robot system can overcome these drawbacks through cooperative sensing from simpler sensors and map-building will become more efficient by fusing results from each mobile robot. Many efforts have been put to cooperative localization of mobile robots. Several researchers utilize multi-robot cooperative sensing techniques to achieve cooperative localization (Fox et
A Monte Carlo Localization (MCL) algorithm was developed to localize two robots equipped with laser range finders (Thrun et al., 2001). The presented method gathers environmental data fast and can localize individual robot using a laser scanner. A cooperative localization and mapping method was proposed for two robots to localize themselves and for map-building (Rekleitis et al. 2003). The key idea is that one robot carries a target to act as a landmark, and the other equipped with a laser range finder and a camera observes the former for position correction. A Kalman filter based approach was presented to simultaneous localization of a group of robots by sensing the team members and combining position information from all teammates (Roumeliotis & Bekey, 2002). The hypothesis of this method is that the robots can communicate with their teammates and measure their respective poses. A collaborative landmark localization method was presented for multiple robots equipped with vision systems to localize themselves by estimating fixed landmarks (Stroupe & Balch, 2002). The fixed landmarks are established beforehand and the robot needs to record the landmark positions. However, when the number of robots increases to more than two, information from laser scanner will not be able to distinguish each robot.

Vision-based systems have been widely exploited as a robotic perception sensor. They are also useful for distinguishing individual robot and handling the case of more than two robots. A vision-based cooperative mapping and localization method employed stereo vision to build a grid map and localize robots by features in grid map such as corners (Jennings et al., 1999). The key idea is to find corners in a grid map and compare these corners with a-priori landmarks at known positions. Hence, an accurate reference map of the environment is required in the method. A multi-robot MCL approach was presented to achieve the localization of a robot team with improved accuracy (Fox et al., 1999). The results show that the robots can localize themselves faster and with higher accuracy. However, although the MCL algorithm can be used for cooperative localization, it requires transmission of relatively large amount of information. On the other hand, a Kalman filtering approach can reduce not only the transmission of information but also the complexity of computing positional uncertainty. But a drawback of this approach is that the accuracy of the localization results will be decreased compared with the MCL methods. A distributed sensing method was developed based on Kalman filtering to improve the target localization (Stroupe & Martin, 2001). However, this method was only developed for recognizing and localizing a specific target. In a multi-robot cooperation system, it is desirable for each robot to recognize all of its teammates in cooperative localization. An effective algorithm is needed to fuse the observed information and reduce the sensing uncertainty.

Furthermore, when a robot team consists of more than two teammates, the problem of communication, cooperation and recognition among teammates will become increasingly important. In this chapter, we developed a multi-robot cooperative localization scheme exploiting a stereo vision system. This scheme does not restrict the number of robot in a robot team and can collect all observational information of the environment form each robot. We establish a client-server architecture using wireless LAN to integrate the local sensory data from each robot (a client) to a server. The server stores the current position of each robot in order to reduce the transmission of information between robot teammates. The proposed multi-robot cooperative localization algorithm aims to estimate the optimal position and reduce the positional uncertainty of the observed robot collaboratively through Kalman filtering. The localization method consists of two stages, serial fusion and parallel
fusion. The serial fusion is to identify the positional uncertainty of an observed robot, and the parallel fusion takes into account all of the other robots that observe other robots to reduce its positional uncertainty. Recursively, the sensory information by multiple robots can be merged using serial fusion and parallel fusion to obtain the optimal position with minimum positional uncertainty of the observed robot. It is clear that the computation of the proposed fusion algorithms is decentralized.

The stereo vision system can be exploited to construct a two-dimensional (2D) map of the environment collaboratively by multiple robots. In the current study, however, the development of map-building algorithm is not emphasized. We will show map-building results in the experimental section only. In the following sections, the presentation will focus on the design and theoretical analysis of the proposed multi-robot cooperative data fusion algorithms for localization purpose. The rest of this chapter is organized as follows. Section 2 describes the overall system architecture of the proposed multi-robot cooperative sensing system. In Section 3, the multi-robot cooperative localization scheme will be presented based on sensor data fusion technique. Section 4 presents the derivations of the Kalman filter recursive fusion formulas to obtain the fusion results. Section 5 gives some interesting experimental results of the proposed method. Finally, Section 6 summarizes the contribution of this work.

2. System Architecture

The presented multi-robot cooperative localization system is shown in Fig 1. The left block represents the environment. The middle part shows blocks of multiple robots; each of it is a client. Each robot captures environmental imagery using an on-board stereo camera module. Image processing algorithms are developed to observe environmental features (such as corners, edges and disparity map, etc.) and the robot’s teammates from the acquired image. Stereo vision techniques are provided to find the depth information of the environment and the distance between each robot. Each robot transforms the local environmental data to a line representation through the line segment block. The upper-right block represents a server for storing the map of the environment and the current Gaussian parameters of each robot’s position. In the server, all detected lines are fused through the line data fusion block, and an environmental map is maintained as line segments. The robot server receives processed local environmental data and integrates all sensory data from each robot.

Each robot utilizes odometry to calculate its Gaussian parameters of current position and updates the corresponding parameters stored in the server. Gaussian parameters of the robot are represented by its current position and the corresponding positional uncertainty. When a robot observes other robots, the serial fusion block (explained later in Section 4) estimates the other robot’s position and positional uncertainty. The parallel fusion block (see Section 4) fuses via Kalman filtering the results from individual serial fusion to obtain a unique and more accurate representation. Note that if a robot does not observe other robots in its sensing region, the serial and parallel fusion processes will not be enabled. In this case, the robot only estimates and updates position and positional uncertainty of itself by odometry. Moreover, the serial and parallel fusions are realized in each robot. In other words, the observed robot receives the serial fusion results from other robots and performs parallel fusion by itself. Therefore, the proposed multi-robot cooperative localization algorithm is decentralized. In the following sections, we will focus the discussion on the design of the presented multi-robot cooperative localization algorithm only.
Remark 1: The main advantage of the discussed multi-robot cooperative localization scheme is that the computation of the localization algorithm is decentralized. However, if the Gaussian parameters of each robot are also decentralized stored in each robot, there will be a large amount of information transmission between all robot teammates due to updating their Gaussian parameters. Because information transmission is also an important factor in multi-robot systems, the external server is helpful to store the Gaussian parameters of each robot and reduce the amount of information transmission. Although the external server seems to be the disadvantage of the presented method, it helps to reduce the information transmission between robot teammates. Another drawback of the presented method is the assumption of Gaussian distribution uncertainty. Thus, in the case of multimodal distribution uncertainty, the proposed method only can provide a suboptimal solution based on a weighted least-square criterion (see Section 4.2). However, this shortcoming can be improved by extending this work by accommodating particle filters.

3. Multi-Robot Cooperative Localization Scheme

Suppose that the robot positional uncertainty can be described by Gaussian distribution, this section presents a method to reduce motion uncertainties of each robot in a robot team. Fig. 2 shows the block diagram of a recursive multi-robot localization system. $M_{Ri}$ denotes the current position of robot $i$, and $C_{Ri}$ is the corresponding positional covariance matrix. Assume that the robot $j$ is observed by robot $i$, where $1 \leq i, j \leq n$ and $i \neq j$, then the estimated position $M_{Eij}$ and the corresponding covariance matrix $C_{Eij}$ can be generated by robot $i$ through its on-board sensor. After merging $(M_{Ri}, C_{Ri})$ and $(M_{Eij}, C_{Eij})$ by serial fusion, the measured position and corresponding covariance matrix of robot $j$, $(M_{Rij}, C_{Rij})$, can be
obtained. The parallel fusion works to merge the current position and corresponding covariance matrix of robot $j$, $(M_{Rj}, C_{Rj})$, with all serial fusion results occurring to robot $j$ to obtain updated parameters of robot $j$, $(M'_{Rj}, C'_{Rj})$. In the next section, we will show that the updated covariance matrix $C'_{Rj}$ can be minimized step-by-step through parallel fusion.

It is easy to see that we separate the multi-robot cooperative localization algorithm into two stages. The first stage is serial fusion, and the second stage is parallel fusion. Table 1 tabulates the purpose and performer of serial and parallel fusions. The performer of serial fusion is the observing robot which observes another robot, and the purpose is to measure the position and the corresponding positional uncertainty of the observed robot. On the other hand, the performer of parallel fusion is the observed robot, which is observed by its teammates. The purpose is to update the current position and reduce positional uncertainty of the observed robot. Therefore, when a robot moves and observes other robots in its sensing region, serial and parallel fusions will recursively occur to localize and reduce positional uncertainties of all robots of the robot team.

![Fig. 2. Block diagram of recursive multi-robot localization.](image)
4. Proposed Data Fusion Algorithms

In this section, a spatial uncertainty estimation method (Smith & Cheeseman, 1986) is extended to develop serial and parallel data fusion algorithms for the application of multi-robot cooperative localization. Moreover, we will also show that the parallel data fusion algorithm guarantees to provide the minimum updated covariance matrix solution via recursively Kalman filtering.

Table 1. Purpose and performer of serial and parallel fusions.

<table>
<thead>
<tr>
<th>Fusion</th>
<th>Purpose</th>
<th>Performer</th>
</tr>
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<tbody>
<tr>
<td>Serial</td>
<td>Measure the position and the corresponding positional uncertainty of the observed robot.</td>
<td>Observing Robot</td>
</tr>
<tr>
<td>Parallel</td>
<td>Merge the current position and positional uncertainty of the observed robot with all serial fusion results to update the current position and reduce the positional uncertainty of the observed robot.</td>
<td>Observed Robot</td>
</tr>
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Fig. 3. Relative coordinates of two robots.

4.1 Serial fusion

The purpose of serial fusion is to measure the position and positional uncertainty of an observed robot in world coordinate. As shown in Fig. 3, a robot location is described by three parameters \((x_r, y_r, \theta_r)\) in world coordinate frame, where \(x_r\) and \(y_r\) are the \(X\) and \(Y\) coordinates of the robot, \(\theta_r\) is the orientation angle of the robot. The shaded part of Fig. 3 indicates the sensing region of robot 1. \(x_e\) and \(y_e\) denote the relative position of
robot 2 as observed by robot 1. If the relative orientation of robot 2, $\theta_e$, is observed by robot 1, then the observed position and orientation of robot 2 can be estimated as:

$$x = f(M_{Ri}, M_{Eij}) = x_r + x_e \cos \theta_r - y_e \sin \theta_r,$$

$$y = g(M_{Ri}, M_{Eij}) = y_r + x_e \sin \theta_r + y_e \cos \theta_r,$$

$$\theta = h(M_{Ri}, M_{Eij}) = \theta_r + \theta_e,$$

where $M_{Ri} = [x_r \ y_r \ \theta_r]^T$ and $M_{Eij} = [x_e \ y_e \ \theta_e]^T$. In this presentation, the method to observe the relative position and orientation angle between robot 1 and robot 2 will not be discussed for simplicity.

Let $\bar{M}_{Rij} = [\bar{x} \ \bar{y} \ \bar{\theta}]^T$ and $M_{Rij} = [x \ y \ \theta]^T$ denote the ideal and the measured position of the observed robot, respectively; $\bar{M} = [\bar{M}_{Ri} \ \bar{M}_{Eij}]^T$ and $M = [M_{Ri}^T \ M_{Eij}^T]^T$ denote the ideal and the measured system parameters, respectively.

By combining equations (1)-(3) into one and expanding it to a Taylor form, the following result can be obtained:

$$\bar{M}_{Rij} = M_{Rij} + \begin{bmatrix} \cos \bar{\theta}_r & -\sin \bar{\theta}_r & 0 \\ \sin \bar{\theta}_r & \cos \bar{\theta}_r & 0 \\ 0 & 0 & 1 \end{bmatrix} M_{Eij}$$

$$\approx M_{Rij} + J(\bar{M} - M),$$

where $J$ is the Jacobian matrix of $M_{Rij}$ such that

$$J = \frac{\partial}{\partial M} \begin{bmatrix} f(M_{Ri}, M_{Eij}) \\ g(M_{Ri}, M_{Eij}) \\ h(M_{Ri}, M_{Eij}) \end{bmatrix} = \begin{bmatrix} 1 & 0 & -x_e \sin \theta_r - y_e \cos \theta_r & \cos \theta_r & -\sin \theta_r & 0 \\ 0 & 1 & x_e \cos \theta_r - y_e \sin \theta_r & \sin \theta_r & \cos \theta_r & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

Based on (4), the covariance matrix of the measured position of observed robot, $C_{Rij}$, is given by

$$C_{Rij} \equiv E[(\bar{M}_{Rij} - M_{Rij})(\bar{M}_{Rij} - M_{Rij})^T] = J \times E[(\bar{M} - M)(\bar{M} - M)^T] \times J^T.$$

Because parameters $M_{Ri}$ and $M_{Eij}$ are obtained from different sensor systems and uncorrelated, the covariance matrix $E[(\bar{M} - M)(\bar{M} - M)^T]$ becomes such that
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![Image](image-url)

\[
E[(\bar{M} - M)(\bar{M} - M)^T] = \begin{bmatrix} C_{Ri} & 0_{3 \times 3} \\ 0_{3 \times 3} & C_{Eij} \end{bmatrix},
\]

(6)

where \( C_{Ri} \equiv E[(\bar{M}_{Ri} - M_{Ri})(\bar{M}_{Ri} - M_{Ri})^T] \) and \( C_{Eij} \equiv E[(\bar{M}_{Eij} - M_{Eij})(\bar{M}_{Eij} - M_{Eij})^T] \) are the covariance matrices of the current position of observing robot and the relative position measured by stereo vision system, respectively; \( 0_{m \times n} \) is a \( m \)-by-\( n \) zero matrix. Based on equations (1)-(6), the serial fusion algorithm to measure the position and positional uncertainty of an observed robot in world coordinate system is summarized as follows:

**Measured Position:**

\[
M_{Rij} = f(M_{Ri}, M_{Eij}) = M_{Ri} + \begin{bmatrix} \cos \theta_r & -\sin \theta_r & 0 \\ \sin \theta_r & \cos \theta_r & 0 \\ 0 & 0 & 1 \end{bmatrix} M_{Eij},
\]

(7)

**Measured Positional Uncertainty:**

\[
C_{Rij} = J \begin{bmatrix} C_{Ri} & 0_{3 \times 3} \\ 0_{3 \times 3} & C_{Eij} \end{bmatrix} J^T.
\]

(8)

Fig. 4. Serial fusion occurs for robot 1, 3 and 4 as each of them observes robot 2.
Figure 4 illustrates the steps of serial fusion for multi-robot localization. In Fig. 4, robot 2 is observed by robot 1, robot 3 and robot 4 at the time instant $k$. Thus the serial fusion of robot 2 occurs for robot 1, 3, and 4 respectively. For example, the global position and uncertainty of robot 1 $(M_{R1}, C_{R1})_k$ and the estimated position and uncertainty of robot 2 from robot 1 $(M_{E12}, C_{E12})_k$ can be merged to obtain the position and uncertainty of robot 2 in world coordinate system, $(M_{R12}, C_{R12})_k$. In the same way, another two positions and uncertainties of robot 2 as observed by robot 3 and robot 4, $(M_{R32}, C_{R32})_k$ and $(M_{R42}, C_{R42})_k$, can also be obtained respectively.

4.2 Parallel fusion

Parallel fusion aims to estimate the optimal position and reduce the positional uncertainty of an observed robot using Kalman filtering. The fusion processing includes two steps. The first step is to estimate the optimal position of an observed robot $M_{Rj}^*$ by minimizing a weighted least-square criterion:

$$J(M) = (M_{Rj} - M)^T C_{Rj}^{-1} (M_{Rj} - M) + (M_{Rij} - M)^T C_{Rij}^{-1} (M_{Rij} - M). \quad (9)$$

More specifically, the first step of parallel fusion algorithm is to find the optimal position that minimizes the performance criterion (9) such that

$$M_{Rj}^* = \arg \min_M J(M) \quad (10)$$

By taking the derivative of (9) with respect to $M$, the necessary condition of local optimal solution of (10) can be obtained as follows

$$(M_{Rj} - M)^T C_{Rj}^{-1} + (M_{Rij} - M)^T C_{Rij}^{-1} = 0. \quad (11)$$

From (11), the local optimal solution of (10) is given by

$$M_{Rj}^* = (C_{Rj}^{-1} + C_{Rij}^{-1})^{-1} C_{Rj}^{-1} M_{Rj} + (C_{Rj}^{-1} + C_{Rij}^{-1})^{-1} C_{Rij}^{-1} M_{Rij}$$

$$= M_{Rj} + (C_{Rj}^{-1} + C_{Rij}^{-1})^{-1} C_{Rj}^{-1} (M_{Rj} - M_{Rj})$$

$$= M_{Rij} + (C_{Rj}^{-1} + C_{Rij}^{-1})^{-1} C_{Rj}^{-1} (M_{Rj} - M_{Rj}) \quad (12)$$

Let $K_{i} = (C_{Rj}^{-1} + C_{Rij}^{-1})^{-1} C_{Rj}^{-1}$ and $K_{j} = (C_{Rj}^{-1} + C_{Rij}^{-1})^{-1} C_{Rj}^{-1}$ denote Kalman gain matrices, expression (12) is simplified such that
Therefore, expression (13) provides the optimal fusion result for the positional estimation of parallel fusion algorithm based on the performance criterion (9).

The second step of parallel fusion is to minimize the covariance matrix of the updated position of the observed robot:

\[
C_{Rj}^* = E[(\bar{M}_{Rj} - M_{Rj}^*)(\bar{M}_{Rj} - M_{Rj}^*)^T].
\]  

(14)

Substituting (13) into (14), we can obtain the following two results

\[
C_{Rj}^* = E\{[(\bar{M}_{Rj} - M_{Rj}) - K_{ij}(M_{Rij} - M_{Rj})][(\bar{M}_{Rj} - M_{Rj}) - K_{ij}(M_{Rij} - M_{Rj})]^T\}
\]

\[
= C_{Rj} - C_{Rij}K_{ij}^T + K_{ij}C_{Rj} + K_{ij}C_{Rij}K_{ij}^T + K_{ij}C_{Rij}K_{ij}^T,
\]  

(15)

\[
C_{Rj}^* = E\{[(\bar{M}_{Rj} - M_{Rj}) - K_{ij}(M_{Rij} - M_{Rj})][(\bar{M}_{Rj} - M_{Rj}) - K_{ij}(M_{Rij} - M_{Rj})]^T\}
\]

\[
= C_{Rij} - C_{Rij}K_{ij}^T + K_{ij}C_{Rj} + K_{ij}C_{Rij}K_{ij}^T + K_{ij}C_{Rij}K_{ij}^T.
\]  

(16)

From (15), the minimum covariance matrix can be obtained by taking the derivative of (15) with respect to \(K_{ij}\) and setting to zero such that

\[
\frac{dC_{Rj}^*}{dK_{ij}} = -2C_{Rj} + 2K_{ij}(C_{Rj} + C_{Rij}) = 0.
\]  

(17)

Solving (17) for \(K_{ij}\) gives

\[
K_{ij} = C_{Rj}^{-1}(C_{Rj} + C_{Rij})^{-1} = (C_{Rij}^{-1} + C_{Rj}^{-1})^{-1}C_{Rj}^{-1}.
\]  

(18)

Expression (18) is the Kalman gain equation. Substituting (18) into (15), the minimum covariance matrix becomes

\[
C_{Rj}^* = C_{Rj} - C_{Rij}K_{ij}^T + K_{ij}C_{Rj} = C_{Rj} - K_{ij}C_{Rj} = (I - K_{ij})C_{Rj}.
\]  

(19)

Using the same procedure discussed above, similar result also can be obtained from (16) such that

\[
K_{ij} = C_{Rij}^{-1}(C_{Rj} + C_{Rij})^{-1} = (C_{Rij}^{-1} + C_{Rj}^{-1})^{-1}C_{Rj}^{-1},
\]  

(20)

\[
C_{Rj}^* = C_{Rij} - K_{ij}C_{Rj} = (I - K_{ij})C_{Rj}.
\]  

(21)

Based on equations (13) and (18)-(21), the parallel fusion algorithm to estimate the optimal position and minimum positional uncertainty of an observed robot in world coordinate system is summarized as follows:

\[
M_{Rj}^* = M_{Rj} + K_{ij}(M_{Rij} - M_{Rj}) = M_{Rij} + K_j(M_{Rj} - M_{Rij}).
\]  

(13)
Kalman Gain Matrix:

\[ K_{ij} = C_{Rj} (C_{Rj} + C_{Rij})^{-1} = (C_{Rj}^{-1} + C_{Rij}^{-1})^{-1} C_{Rj}^{-1}, \]  

(22)

Updated Position:

\[ M_{Rj}^* = M_{Rj} + K_{ij} (M_{Rij} - M_{Rj}), \]  

(23)

Updated Covariance Matrix:

\[ C_{Rj}^* = C_{Rj} - K_{ij} C_{Rj} = (I - K_{ij}) C_{Rj}. \]  

(24)

Or, Kalman Gain Matrix:

\[ K_j = C_{Rij} (C_{Rj} + C_{Rij})^{-1} = (C_{Rj}^{-1} + C_{Rij}^{-1})^{-1} C_{Rj}^{-1}, \]  

(25)

Updated Position:

\[ M_{Rj}^* = M_{Rij} + K_j (M_{Rj} - M_{Rij}), \]  

(26)

Updated Covariance Matrix:

\[ C_{Rj}^* = C_{Rij} - K_j C_{Rij} = (I - K_j) C_{Rij}. \]  

(27)

The symbol “◦” is employed to represent the parallel fusion operation. For instance, we can simplify the presentation of parallel fusion equations (22)-(24) as:

\[ M_{Rj}^* = M_{Rj} \circ M_{Rij}, \]  

(28)

\[ C_{Rj}^* = C_{Rj} \circ C_{Rij}. \]  

(29)

Similarly, the presentation of parallel fusion equations (25)-(27) can be simplified as:

\[ M_{Rj}^* = M_{Rij} \circ M_{Rj}, \]  

(30)

\[ C_{Rj}^* = C_{Rij} \circ C_{Rj}. \]  

(31)

Compare equations (28)-(29) with (30)-(31), it is clear that the parallel fusion operation has commutative property. Moreover, because of \[ I - K_{ij} = C_{Rij} (C_{Rj} + C_{Rij})^{-1} \] and
\[ I - K_j = C_{Rj} \left( C_{Rj} + C_{Rij} \right)^{-1}, \] the updated covariance matrix becomes

\[ C_{Rj}^* = C_{Rij} \left( C_{Rj} + C_{Rij} \right)^{-1} C_{Rj} = C_{Rj} \left( C_{Rj} + C_{Rij} \right)^{-1} C_{Rij}. \] (32)

Expression (32) leads to the fact that the matrix norm of updated covariance matrix, \( \|C_{Rj}^*\| \), will not be larger than that of covariance matrices \( C_{Rj} \) and \( C_{Rij} \) (Golub & Van Loan, 1996). More specifically, expression (32) leads to the following result

\[ \|C_{Rj}^*\| \leq \min(\|C_{Rj}\|,\|C_{Rij}\|). \] (33)

Therefore, the parallel fusion operation also has associative property, which guarantees that the final fusion result has minimum matrix norm irrelevant to the order of fusion operation. These two properties are helpful to generalize the parallel fusion algorithm into the sensor fusion procedure of multiple robots. Suppose that robot j is observed by robot i at instant \( k \), where \( 1 \leq i, j \leq n \), and \( i \neq j \), we can write the parallel fusion formula such that:

\[
\begin{align*}
(M_{Rj})_{k+1} & = (M_{Rj} \circ M_{R1j} \circ M_{R2j} \circ \ldots \circ M_{Rnj})_k, \\
(C_{Rj})_{k+1} & = (C_{Rj} \circ C_{R1j} \circ C_{R2j} \circ \ldots \circ C_{Rnj})_k,
\end{align*}
\] (34) (35)

where \( (M_{Rj})_{k+1} \) and \( (C_{Rj})_{k+1} \) represent the updated global position and the corresponding covariance matrix of robot j at next instant \( k+1 \), respectively. In equations (34) and (35), any two terms of \( (M_{Rij}, C_{Rij})_k \), \( 1 \leq i, j \leq n \), can be chosen to perform the parallel fusion. The output of the first fusion is used as the input of next step, and then the optimal position of robot j with minimum covariance matrix can be obtained after several iterations.

Figure 5 depicts the complete steps of parallel fusion. As the case shown in Fig. 5(a), there are three sets of parameters of serial fusion obtained from other robots, which observe robot 2. Using the proposed parallel fusion scheme, the parameters of serial fusion \( (M_{R12}, C_{R12})_k \), \( (M_{R32}, C_{R32})_k \), \( (M_{R42}, C_{R42})_k \) and \( (M_{R22}, C_{R22})_k \) can be merged in three subsequent steps as depicted in Fig. 5(b). A new positional parameters of robot 2 at instant \( k+1 \), \( (M_{R22}, C_{R22})_{k+1} \), is obtained and updated accordingly. We write parallel fusion formula following the representation in Fig. 5(b):
\( \begin{align*} 
(M_{R2})_{k+1} &= (M_{R12} \circ M_{R32} \circ M_{R42} \circ M_{R2})_k, \\
(C_{R2})_{k+1} &= (C_{R12} \circ C_{R32} \circ C_{R42} \circ C_{R2})_k. 
\end{align*} \) (36)
\( \begin{align*} 
(M_{R2})_{k+1} &= (M_{R24} \circ M_{R32} \circ M_{R12} \circ M_{R2})_k \\
(C_{R2})_{k+1} &= (C_{R24} \circ C_{R32} \circ C_{R12} \circ C_{R2})_k. 
\end{align*} \) (37)

We can also rewrite (36) and (37) to another form using the commutative and associative properties:
\( \begin{align*} 
(M_{R2})_{k+1} &= (M_{R24} \circ M_{R32} \circ M_{R12} \circ M_{R2})_k \\
(C_{R2})_{k+1} &= (C_{R24} \circ C_{R32} \circ C_{R12} \circ C_{R2})_k. 
\end{align*} \) (38)
\( \begin{align*} 
(M_{R2})_{k+1} &= (M_{R24} \circ M_{R32} \circ M_{R12} \circ M_{R2})_k \\
(C_{R2})_{k+1} &= (C_{R24} \circ C_{R32} \circ C_{R12} \circ C_{R2})_k. 
\end{align*} \) (39)

In the following section, the performance of the proposed serial and parallel fusion algorithms will be validated by practical experiments.

5. Experimental Results

The developed system has been implemented on two experimental mobile robots H1 and H2 developed in our lab. Each robot has an on-board industrial PC (IPC), which is connected to the internet via wireless LAN. On top of the robot, a stereo vision module was installed in order to estimate the position of the observed robot and build the local map of the experimental environment. The robots were equipped with a motor control card for driving two independent wheels. Fig. 6 shows the robots H1 and H2 in the experiments. Figs. 7(a) and (b) show the experimental environment in the lab.

Two experiments were conducted to verify the effectiveness of the proposed method of cooperation localization. One experiment used a single robot H1 to build a map. The second experiment used two robots, H1 and H2, to cooperatively build an environmental map with multi-robot localization. In these two experiments, the robots are set to only have four
orientation angles: 0°, 90°, 180°, and 270°. Therefore, the orientation of the robot can be supposed to be known without uncertainty. This assumption leads a simplified implementation of the proposed algorithm, which does not consider the information of robot orientation. For instance, expressions (7) and (8) can be simplified such that

\[
M_{Rij} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f(M_{Ri}, M_{Eij}) \\ g(M_{Ri}, M_{Eij}) \end{bmatrix} = M_{Ri} + \begin{bmatrix} \cos \theta_r & -\sin \theta_r \\ \sin \theta_r & \cos \theta_r \end{bmatrix} M_{Eij},
\]

(40)

\[
C_{Rij} = J \begin{bmatrix} C_{Ri} & 0_{2 \times 2} \\ 0_{2 \times 2} & C_{Eij} \end{bmatrix} J^T,
\]

(41)

with

\[
M_{Ri} = \begin{bmatrix} x_r \\ y_r \end{bmatrix}, \quad M_{Eij} = \begin{bmatrix} x_e \\ y_e \end{bmatrix}, \quad \text{and} \quad J = \begin{bmatrix} 1 & 0 & \cos \theta_r & -\sin \theta_r \\ 0 & 1 & \sin \theta_r & \cos \theta_r \end{bmatrix}.
\]

However, the performance and convergence of the proposed algorithm will not be influenced. In the following, these two experimental results are compared and discussed.

Fig. 6. Front view of robots H1 and H2. H1 on the left is with a blue ball and H2 with a red ball. Both robots are equipped with (1) a blue or red color ball, (2) stereo vision module with two CMOS cameras, (3) motion control module, (4) wireless LAN card, (5) on-board IPC.
Remark 2: The covariance matrices $C_{Ri}$ and $C_{Eij}$ in (41) are calibrated by practical testing for each robot. In the experiments, the covariance matrices for each robot are calibrated by

Robot H1: $C_{R1}^{(k+1)} = C_{R1}^{(k)} + \begin{bmatrix} 0.0596 & 0 \\ 0 & 0.0248 \end{bmatrix}$ and $C_{E12} = \begin{bmatrix} 0.0430 & 0 \\ 0 & 0.0532 \end{bmatrix}$

Robot H2: $C_{R2}^{(k+1)} = C_{R2}^{(k)} + \begin{bmatrix} 0.0478 & 0 \\ 0 & 0.0513 \end{bmatrix}$ and $C_{E21} = \begin{bmatrix} 0.0595 & 0 \\ 0 & 0.0793 \end{bmatrix}$

where $C_{Ri}^{(k)}$ denotes the positional covariance matrix of robot $R_i$ at time step $k$ and will be accumulated step-by-step when the robot travels due to the odometry measurement noise.

Experiment 1: Single robot localization and map-building
In this part, robot H1 was commanded to build a map of the environment alone. The localization of the robot is using the information from odometry only such that

$$x_r^{(k+1)} = x_r^{(k)} + v_r^{(k)} \cos \theta_r, \text{ and } y_r^{(k+1)} = y_r^{(k)} + v_r^{(k)} \sin \theta_r,$$

where $v_r^{(k)}$ is the linear velocity of the robot at time step $k$. The linear velocity is calculated by counting encoder signals from two driving wheels. Because of $\theta_r = \{0^\circ, 90^\circ, 180^\circ, 270^\circ\}$, the angular velocity of the robot can be ignored in the experiments.
Fig. 8 shows the experimental results. In Fig. 8, the dotted lines represent actual environmental shape. Solid lines show the map established by the robot H1. Six positions A, B, C, D, E and F were recorded as the robot moved around the environment. Table 2 shows the position parameters of H1 of these points. In Table 2, it is observed that as H1 moves through point A to F subsequently, the standard deviations of \( x \) and \( y \) coordinates become larger and larger. This means that H1 is getting lost of its position estimation because the accumulated error from dead reckoning. From Table 2, we also see that the standard deviations of from A to F are increasing. This implies the error between the actual and estimated positions becomes larger as the robot travels further.

**Experiment 2: Cooperative visual localization and map-building**

In this part, robots H1 and H2 worked together to cooperatively localize each other and built a map of the environment. Fig. 9 shows the experimental results. In Fig. 9(a), seven positions of H1 were recorded. In Fig. 9(b), five positions of H2 were recorded. The black points are the recorded trajectory of H1 and H2, respectively. The dotted lines are actual environmental shape. Solid lines represent the maps established by H1 and H2, respectively. In this experiment, H1 was observed by H2 at point D of H1. The robot H2 was observed by H1 at points C and E of H2.

![Image](image.png)

**Fig. 8.** Map-building result of single robot H1.

The position estimation results of H1 and H2 in experiment 2 are shown in Table 3 and 4, respectively. From Table 3, we see that when H1 moved through point A, B and C subsequently, the standard deviations of \( x \) and \( y \) coordinates increase. However, when H1 moved to point D and was observed by H2, parallel fusion of H1 occurred. So the position uncertainty of robot H1 and the error between actual and estimated position became smaller as expected. The similar results also can be confirmed for robot H2. As shown in Table 4, since H2 was observed by H1 at points C and E, the position uncertainty and the positional error were reduced at point C and E as expected.
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Table 2. Position parameters of H1 in experiment 1.

<table>
<thead>
<tr>
<th>Actual</th>
<th>Estimated</th>
<th>Standard</th>
<th>Move</th>
<th>Move</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position(cm)</td>
<td>Position(cm)</td>
<td>Deviation</td>
<td>Counts</td>
<td>Distance(cm)</td>
</tr>
<tr>
<td>A (37.98)</td>
<td>(40.10)</td>
<td>$\sigma_x=4.001$</td>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>B (38.4)</td>
<td>(43.49,9.94)</td>
<td>$\sigma_x=8.169$</td>
<td>11</td>
<td>270</td>
</tr>
<tr>
<td>C (95,28)</td>
<td>(104.5,37.83)</td>
<td>$\sigma_x=12.348$</td>
<td>22</td>
<td>540</td>
</tr>
<tr>
<td>D (-60,240)</td>
<td>(-75.72,253.63)</td>
<td>$\sigma_x=14.885$</td>
<td>33</td>
<td>930</td>
</tr>
<tr>
<td>E (0,60)</td>
<td>(-14.09,83.92)</td>
<td>$\sigma_x=16.808$</td>
<td>41</td>
<td>1170</td>
</tr>
<tr>
<td>F (-17,-47)</td>
<td>(-32.36,-64.83)</td>
<td>$\sigma_x=18.148$</td>
<td>46</td>
<td>1320</td>
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Table 3. Position parameters of H1 in experiment 2.

<table>
<thead>
<tr>
<th>Actual</th>
<th>Estimated</th>
<th>Standard</th>
<th>Move</th>
<th>Move</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position(cm)</td>
<td>Position(cm)</td>
<td>Deviation</td>
<td>Counts</td>
<td>Distance(cm)</td>
</tr>
<tr>
<td>A (40,-10)</td>
<td>(40.01,-10)</td>
<td>$\sigma_x=3.904$</td>
<td>1</td>
<td>10</td>
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<tr>
<td>B (33,115)</td>
<td>(40.34,120.1)</td>
<td>$\sigma_x=11.023$</td>
<td>16</td>
<td>280</td>
</tr>
<tr>
<td>C (100,31)</td>
<td>(110.5,40.3)</td>
<td>$\sigma_x=13.835$</td>
<td>26</td>
<td>430</td>
</tr>
<tr>
<td>D (-14,-35)</td>
<td>(-11.11,38.64)</td>
<td>$\sigma_x=3.710$</td>
<td>28</td>
<td>530</td>
</tr>
<tr>
<td>E (-76,225)</td>
<td>(-82.41,231.19)</td>
<td>$\sigma_x=8.108$</td>
<td>41</td>
<td>740</td>
</tr>
<tr>
<td>F (-5,70)</td>
<td>(-11.81,64.83)</td>
<td>$\sigma_x=12.158$</td>
<td>52</td>
<td>910</td>
</tr>
<tr>
<td>G (-12,-70)</td>
<td>(-21.31,78.29)</td>
<td>$\sigma_x=14.368$</td>
<td>60</td>
<td>1090</td>
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</tbody>
</table>
Table 4. Position parameters of H2 in experiment 2.

<table>
<thead>
<tr>
<th></th>
<th>Actual Position(cm)</th>
<th>Estimated Position(cm)</th>
<th>Standard Deviation</th>
<th>Move Counts</th>
<th>Move Distance(cm)</th>
<th>Observed by</th>
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<tbody>
<tr>
<td>A</td>
<td>(-28,-100)</td>
<td>(-30,-100)</td>
<td>(\sigma_x=5.385)</td>
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<td>30</td>
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<td></td>
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<td></td>
<td>(\sigma_y=6.196)</td>
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<td></td>
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<tr>
<td>B</td>
<td>(103,-91)</td>
<td>(110.49,-100)</td>
<td>(\sigma_x=8.462)</td>
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<td>190</td>
<td>No</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>(\sigma_y=9.306)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>(58,-92)</td>
<td>(60.2,-95.18)</td>
<td>(\sigma_x=3.878)</td>
<td>13</td>
<td>240</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(\sigma_y=4.356)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>(34.94)</td>
<td>(41.2,108.18)</td>
<td>(\sigma_x=11.885)</td>
<td>28</td>
<td>570</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(\sigma_y=12.043)</td>
<td></td>
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<tr>
<td>E</td>
<td>(38.40)</td>
<td>(41.09,43.92)</td>
<td>(\sigma_x=5.180)</td>
<td>34</td>
<td>740</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(\sigma_y=5.354)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 10. Cooperative map-building result of H1 and H2.

Therefore, the experimental results validate that the proposed cooperative localization algorithm effectively reduces the position error of each robot in the cooperative robotic system.

Fig. 10 shows the integrated result of Fig. 9 constructed in the robot server through cooperation of robots H1 and H2. In Fig. 10, the gray lines represent actual environmental shape. Solid lines represent the global map established by robot H1 and H2 collaboratively. Because the position error of each robot was reduced by the cooperative localization, the error of the map also became smaller compared with the case in Fig. 8 without sensor fusion.

6. Conclusions

In this chapter, we presented a multi-robot cooperative localization scheme which does not restrict the number of robots in the system and guarantees the optimal fusion results with minimum covariance matrix. In this scheme, each robot is equipped with a stereo vision
system to recognize the individual robot and find important features in environment. This is advantageous compared to a robotic system with sonar or laser sensors, where the robots have practical limitations in recognizing more than two robot-teammates. From practical experiments, we observe that parallel fusion can effectively reduce robot positional uncertainty as expected. Using serial and parallel fusion, one can increase the accuracy of robot localization and reduce the errors in map-building. In the future, we will first extend the experiments to a more complex environment with more robots. Theoretical improvements will also be investigated, such as the information transmission issue between each robot teammate, and how often the robots need to observe each other.

7. Acknowledgment

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8. References


