Qualitative Representation and Reasoning in Cognitive Maps: A Survey from Spatial Cognition Perspective

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Abstract: A comprehensive survey of qualitative representations in cognitive maps is provided in this paper from spatial cognition perspective. The necessity of qualitative representation for robot navigation and map building is discussed with emphasis on different forms of qualitative information represented in different cognitive maps. By discussing the main spatial representations, which are topological representation, directional representation, distance representation, and ordering form of representation, advantages of qualitative representation are concluded as more tolerant to error, more understandable to human and less computation. An expectation is proposed that the combination of qualitative representation and quantitative information is much more practical in robot navigation and exploration.

Keywords: Qualitative representation; reasoning; cognitive map; mobile robot

1 Introduction

The study on robot map plays a significant role in the navigation of mobile robot. A map, which is considered as environment spatial knowledge representation, gives the robot directions when it has to move to somewhere to finish tasks. The traditional robot map has been well developed in form of metrics and topology. These maps, however, have to be accurate and the robot has to know its series of exact locations in a navigation task. This would impose strict restrictions on the accuracy of odometer and sensors of robot, and lead to massive calculations.

Think about it: a man can get his way by instruction: “go straight until the first across, then turn right until you see a bridge”; instead of “moving 500 meters forward, and turning 90 degree”. Human beings always describe directions, locations and distance in a qualitative way, which can also be reasoned to get a new direction and a new location. What’s more, most of people who are familiar with a place like a library can sketch a map of it with ease. This map may not be accurate for the metric information is rough and imprecise, but it is quite useful to a person who is unfamiliar with the place. Here is the question: does the robot have the ability to describe its environment, and reason spatial knowledge?

The cognitive map is used to deal with this kind of problems, which describe the environment in a more qualitative way, similar to the way human does. The central purpose of cognitive map is to help the robot to understand the environment autonomously as well as human instructions. Thus the environment knowledge in cognitive map should be represented in a qualitative way other than a metric way. The purpose of this paper is to discuss main forms of spatial qualitative representation and reasoning, compare and analysis advantages and disadvantages of qualitative representation. A future work is presented after major forms of qualitative representation and approaches reasoning are discussed.

2 Historical Review

The first cognitive map has been proposed by Tolman in the area of psychology and cognition in 1948[1], permitting an encoding of the spatial relations between relevant locations in the environment[2]. Since then cognitive maps have been introduced in navigation, becoming a hot field in cognitive robot. However, the form of cognitive map has not developed into a unified form or structure, due to the structures are determined by different purposes of researches. The mature models of cognitive map are discussed as follows.

In 1995, Chown proposed a cognitive map model called PLAN[3], with the purpose of its application in way finding. Chown believed that a cognitive map serves two functions with regard to wayfinding: representing environment, and the corresponding ability to use the representations to move from places to places within the mapped environments. This can be considered of using the qualitative knowledge of environment unconsciously.

Since 1988, Yeap has developed a way of building and computation for cognitive maps[4]. He focused on the problem that cognitive mapping has to be developed over time in a human way. Humans can rarely remember everything perceived from the start to the end of a journey. The ASR model focused on perception and conception of the outside world, and Yeap identified computational theory which explains what is to be computed and why it should be computed. In 1999, Yeap simplified the algorithm for computing a local environment[5].

Kuipers introduced the “cognitive map” from the field of psychology in his doctoral thesis[6] in 1977, where he
thought the cognitive map as a model of knowledge a person has about the spatial structure of a large-scale environment, and the functions of the cognitive map are to assimilate new information about the environment, to represent the current position, and to answer route finding and relative-position problems. His primary cognitive map, the TUOR model shows how the particular descriptions chosen to represent spatial knowledge support assimilation of new information from local observations into the cognitive map, and how the cognitive map solves route finding and relative-position problems. He then developed the TUOR model in to the SSH model in 2000, with the consideration of reorganizing different levels of the map and focusing ontology of each levels’ own.

Kuipers has worked in the area of qualitative reasoning (QR) since 1984, and he argues that cognitive mapping – building and using symbolic models of the large-scale spatial environment - is a highly appropriate domain for qualitative reasoning research. In the work of Kuipers’s, qualitative representation makes the robot understand the environment in a human-like way, and abstract the environment structure into high levels for simpler reasoning. Continued works on cognitive map after Kuipers also focus on the representations of environment qualitative knowledge and spatial reasoning. One of the researches is focused on the qualitative representations labeled by semantic information. In Galindo’s cognitive map, qualitative knowledge and semantic information have developed into their own reasoning layers. With the correspondence of each layer, a multi-level cognitive map is built. Skubic represents the spatial qualitative knowledge of human into the form of semantic, and applies it in human-computer dialogue, providing robots with human-like spatial language capabilities. The research on semantic information is beyond our survey, which will not be further discussed in this paper.

3 Why Qualitative

The traditional maps for robot navigation have been well developed in form of metric map and topology map. Metric map is suited when it is necessary for the robot to know its location accurately in terms of metric coordinates. This is depended on the high accuracy and precision of robot odometer and sensors, which is difficult to achieve with existing technology. Topology map, which is known for its high compactness, is lack of other form of environment knowledge except topology. Hence it is always used with the subsidiary of other maps like metric map. What more, these kind of spatial descriptions need the complete knowledge of the environment. However, the knowledge of environment can not been acquired completely with ease. Qualitative knowledge is foundational to commonsense knowledge, and hence to most kinds of knowledge that humans possess. “On the left of” and “in the direction of” are instances of information which have no exact value. They convey a rather loose meaning which may be interpreted in different ways depending on the context.

4 The Forms of Qualitative Representation

The form of qualitative representation in cognitive maps varies from different spatial information needed in navigation. Following a classification by Feksa, spatial information about object configuration can be divided into three distinctive modalities: directional information, distance information, and topological information.

- Directional information represents information about the direction to an object. This can be expressed in qualitative relations (e.g. “left of”), or just in qualitative intervals (e.g. “in the direction of the north east”).
- Distance information describes the distance between two objects either in a relative manner by means of comparison (e.g. “farer than”) or in terms of a qualitative interval (e.g. “far”, “near”).
- Topological information denotes knowledge about connectivity. In cognitive maps, connectivity of “local maps” or “distinctive places” helps spatial reasoning.

There are other forms of qualitative representation which are used for more than navigation. The ordering information, or “the route”, simulates episodic memory of human beings. The semantic expression of map focuses on the dialog between human and robot, which is also qualitative in common sense. This article will focus on different forms of qualitative representation by studying its purposes and functions in cognitive map.

4.1 Topology in Spatial Qualitative Representation

Topological information is often considered to describe the connectivity of regions and places, and plays significant role in spatial reasoning as well as in path planning or way finding. The most prominent approach has been done by Kuipers. He suggests in his Spatial Semantic Hierarchy to interpret connectivity of distinctive places as applicability of action primitives that allow moving the robot from one place to another.

The SSH model is consist of five levels with their own ontology. The sensory and control levels deal with continuous sensing of a continuous world, and produce continuous behavior. The transition to the causal level abstracts continuous behavior to discrete states and actions. The transition to the topological level does an abduction, hypothesizing places and paths to account for observed states and actions. A global metrical map is created by merging local geometric maps as linked by the topological description. “Quantitative information is useful at every level when it is available, but effective behavior is often possible with only qualitative knowledge.”
The topological map describes the environment as a collection of places, paths and regions, linked by topological relations such as connectivity, order, boundary and containment. Places, paths and boundary regions are created from experience represented as a sequence of views and actions. They are created by abduction, positing the minimal additional set of places, paths, and regions required to explain the sequence of observed views and actions. The topological information is reasoned in terms of first-order logic.

A place describes part of the environment as a zero-dimensional point. A place may lie on zero or more paths, and may also be defined as the abstraction of a region.

A path describes part of the environment, for example a street in a city, as a one dimensional subspace. The two directions along a path are \( \text{dir} = +1 \) and \( \text{dir} = -1 \).

A region represents a two-dimensional subset of the environment. A region may be defined by one or more boundaries, by a common frame of reference, or by its use in an abstraction relation.

Spatial relationships are defined:
- \( \text{at} \text{(view, place)} \) : view is seen at place
- \( \text{along} \text{(view, path, dir)} \) : view is seen along path in direction dir.
- \( \text{on} \text{(place, path)} \) : place is on path.
- \( \text{order} \text{(path, place1, place2, dir)} \) : the order on path from place1 to place2 is dir.
- \( \text{right_of} \text{(path, dir, region)} \) : path, facing direction dir, has region.
- \( \text{left_of} \text{(path, dir, region)} \) : on its right (respectively left).
- \( \text{in} \text{(place; region)} \) : place is in region.

Place and path are abducted from views and actions, and regions and boundaries are abstracted by travel actions of robot. The Touro machine can deduce time-independent assertions of topological or local metrical relations with input of sequences of views and actions. The axioms of Touro machine are as the following:

\[
\begin{align*}
\text{current\_place (p)} \land \text{current\_view (v)} & \rightarrow \text{at (v, p)} \\
\text{current\_path (p)} \land \text{current\_direction (d)} \land \text{current\_view (v)} & \rightarrow \text{along (v, p, d)} \\
\text{current\_place (p)} \land \text{current\_path (path)} & \rightarrow \text{on (p, path)}
\end{align*}
\]

As is shown in Fig 1, each of the six possible pairs of landmarks is connected by straight lines, which partitions the plane into 18 regions. If an agent is somewhere located in region R, then it will see landmark 2 left of landmark 3. In this way, the spatial relations of regions and locations can be present by the order of a landmark, which is shown in Fig 2.

4.2 Directional Information in Spatial Qualitative Representation

According to Moratz[21], three types of directional reference systems can be distinguished: Intrinsic reference describes relative position of an object (the referent) to another one (the relatum) by referring to intrinsic properties of the relatum, e.g. their front or back. Relative reference system uses a position of third entity as reference. In this case, the position of the observer or another agent (used as origin) is relevant for the identification of the object. Absolute reference system uses cardinal world directions as reference.

Stolzenburg assumes that there is a certain number of natural or artificial landmarks in the environment, that they abstract by points without dimension[22]. That means landmark is considered as the complete qualitative representation of environment. Since regions and locations of objects can be identified by landmarks without reference to the observer or cardinal directions, landmarks form a kind of intrinsic reference system, given by the straight lines between pairs of landmarks.
The landmark has been used in localization with cyclic ordering and aspects. However, Stolzenburg finds it difficult to deal with exploration problem when only qualitative information is available. In the application of navigation, a qualitative notion of distance is needed, which is expressed as the number of regions that has to be passed minimally while going from one place to another.

Moratz also adds information of distance to the directional representation[23]. Besides, he adopts intervals distance/orientation-intervals representation instead of complete non-number qualitative representation, and then combines distance/orientation-intervals propagation with a path-based representation scheme which is based on the generalized Voronoi graph of the robot’s free space. The combination of qualitative approach and metrical approach benefits each other. Qualitative calculi can represent imprecise spatial knowledge. Metric representations are good at distinguishing different spatial entities. Moratz uses a point and a reference direction as anchor and has four additional parameters: \( r_{\text{min}}, r_{\text{max}}, \phi_{\text{min}} \) and \( \phi_{\text{max}} \). Let \( d \) is set of polar vector vectors of polar vector vectors \( (r', \phi') \):

\[
\{ (r', \phi') \mid r'_{\text{min}} \leq r' \leq r'_{\text{max}} \land \phi'_{\text{min}} \leq \phi' \leq \phi'_{\text{max}} \}.
\]

These quantitative intervals can be propagated along paths. Then the respective reference directions are determined by the direction through adjacent points on the path.

### 4.3 Ordering Information in Spatial Qualitative Representation

Ordering information represents a sequence of locations according to their projection on an arbitrarily chosen linear reference, e.g. a coordinate axis or a circle[24]. Schlieder believes that ordering information allows us to answer some questions concerning a point’s position, even though this information is abstracted from metric relationships[18]. He uses the fact that linear ordering is the one-dimensional specialization of a general n-dimensional concept of point ordering and gives the reasoning of interval relations which is consisted by linear ordering of points.

Wagner puts the ordering information in a more egocentric way other than traditionally allocentric[19]. By the use of ordering information, which is based on a description of how landmarks can shift and switch, Wagner generates an extended panoramic representation (EPR).

### 5 Qualitative Reasoning in Cognitive Maps

Two main classes of methods have been used in qualitative reasoning[16]. The first consists of complete qualitative representations and are usually based on formal logics. And the second class consists of consists of (semi)quantitative approaches and are often based on fuzzy set or probability theories.

#### 5.1 Complete Qualitative Representation and Formal Logics

Qualitative representation and reasoning by formal logics has been well developed with its application in geographic information systems (GIS)[16], while mobile robot navigation mostly use’s quantitative information for years. The recent cognitive maps (e.g. SSH) start using qualitative reasoning for way finding or path planning. Kuipers[7] uses first-order logic to deduce the topology relationship of place or region in his SSH model.

Traditional qualitative knowledge in cognitive map is reasoned in a more logical way without any metric information. Because the proposal of qualitative representation is begin with the idea that precision is not always desirable, and precise, quantitative data is not always available [25]. However, Stolzenburg[22] and other researchers find it difficult to deal with exploration problem when only qualitative information is available. Hence in the area of robot navigation, the combination with quantitative information is necessary, which results in the method of qualitative reasoning with the consideration of quantitative information.

#### 5.2 Semi-Quantitative Approaches on Fuzzy Set

Early researches using fuzzy notions combine topological and metric information[26], which are similarly the basic idea of retaining the connection between qualitative concepts and metric representations. The author believes that a fuzzy map captures facts about objects by recording their relative positions, orientations, and scales in convenient frames of reference, and coordinates are specified to lie in a range rather than having fixed values in terms of fuzzy sets. While fuzzy map allows easy retrieval of information, constructing a fuzzy map is much more difficult.

Fuzzy approaches for qualitative modeling have been discussed by Sugeno and Yasukawa. They propose a qualitative model based on fuzzy representations from a set of sample input-output datas describing the system behavior, as well as a set of fuzzy rules to describe the mapping from the given input values to the output values [27].

Schiffer and Ferrein present models for qualitative notions of space, i.e. qualitative distance and orientation based on fuzzy sets, and integrate them into the situation calculus[28]. By introducing the so-called fuzzy fluents, they can take qualitative categories as function values and formalize membership to a qualitative category, then a defuzzifier function are proposed that allows to requantify a qualitative category. Madhava and Kalra modeled the environment of the robot by classifying temporal sequences of spatial sensory patterns. This classification is determined by fuzzy ART network[29].

The fuzzy approach combined with qualitative reason-
ing is employed in many other approaches of cognitive map building and navigation. And these fuzzy approaches differ from each other due to different purposes of their application. However, these approaches of fuzzy bridge the gap between the qualitative representation which is familiar and easy to understand to humans and quantitative robot control. What’s more, fuzzy approach is also combined the non-metric spatial representation and metric representation in perfect way.

5.3 Semi-Quantitative Approaches on Rough Mereology

Polkowski [30] thinks that humans have to reasoning even when their knowledge is uncertain, incomplete, and proposes a method of representation with the idea that humans outline a decomposition scheme by approximating [31]. He then first brought the notion of rough inclusion into the mereology and thus applied them in qualitative reasoning. The rough inclusion is a parameterized relation \( \mu \), such that for any pair of objects \( u, v \) the formula \( \mu \) is \( \mu_r \). \( v \) means \( \mu \) is a part of \( v \) to a degree \( r \), where \( r \in [0, 1] \). The distance in rough mereology is defined as:

\[
\kappa(u, v) = \min\{\max u, \max w : u \in \mu_v, v \in \mu_u, w \}.
\]

And the closer objects, the greater is the value of \( \kappa \). The relationship between \( \kappa(u, v) \) is defined as:

\[
z \in T(u, v) \Leftrightarrow \text{for all } w \kappa(z, v) \in [\kappa(u, w), \kappa(v, w)].
\]

The qualitative spatial knowledge is represented by rough mereology, and spatial relationship of robots can be reasoned.

6 Conclusion and Open Questions

The goal of this paper was to survey qualitative representation and reasoning in cognitive maps. We first discussed the necessity of qualitative representation for robot navigation and map building, and then put emphasis on different forms of qualitative information representation in different cognitive maps. The main spatial representation surveyed here included topological representation, directional representation, distance representation, and ordering form of representation. We conclude that the non-metric qualitative representation would sometimes not be competent in robot navigation and exploration and the combination with metric information is much more helpful.

Two main classes of methods in qualitative reasoning had been discussed. The traditional non-metric qualitative reasoning with logic is considered as old fashioned for the incapability in robot navigation. Fuzzy approaches are considered a bridge between qualitative information and quantitative information. Works of researchers have proved the convenience brought by fuzzy approaches dealing with qualitative representation and quantitative robot control. Rough mereology is another approach dealing the gap of qualitative representation and quantitative information by approximating.

Further researches could draw attention on the following aspects:

- Most spatial object mentioned above are represented by lines and dot without consideration of size and shape, a region can be used as a representation of object and size or shape will be an important information for reasoning.
- The form of qualitative representation differs from each other due to the purpose of them varies. A uniform representation help the robot finish the tasks of modeling, navigation and path planning with a much simpler system.
- Learning is a promising approach in spatial knowledge representation as well as in robot navigation, and there has been a mounts of researches on learning but still far from maturity.

References

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