Grey systems for intelligent sensors and information processing

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Abstract: In a measurement system, new representation methods are necessary to maintain the uncertainty and to supply more powerful ability for reasoning and transformation between numerical system and symbolic system. A grey measurement system is discussed from the point of view of intelligent sensors and incomplete information processing compared with a numerical and symbolized measurement system. The methods of grey representation and information processing are proposed for data collection and reasoning. As a case study, multi-ultrasonic sensor systems are demonstrated to verify the effectiveness of the proposed methods.

Keywords: grey system, grey sensors, information processing.

1. Introduction

In measurement science, there are two important components: (1) sensory systems, a physical means of obtaining numbers; and (2) information processing methods, a mathematical means of manipulating them to produce the information required in an appropriate form[1]. In the wide sense, measurement is defined as a process of empirical, objective assignment of symbols to attributes of objects and events of the real world, in such a way as to describe them[2]. However, all measures are always accompanied by some challenges. One is how to deal with uncertainty and the other is which kind of form should be taken to represent the information.

Since no measurement is perfect, it is generally accepted that a measurement result is not a single value but a range of values, which is characterized by the best estimate of the specific quantity subject to measurement, and the quantitative statement of its uncertainty: \( M = \bar{M} \pm \mu \). However, this expanded form with uncertainty is not widely accepted and most researchers propose to express uncertainty with one standard deviation only, instead of with expanded uncertainty[3]. They argue that the expanded uncertainty does not allow direct comparisons of the quality of measurement results and is complicated for calculation. However, we cannot abandon the information of uncertainty in most situations. What we require is a new representation method to maintain the uncertainty and to supply more powerful ability for reasoning and transformation between numerical system and symbolic system.

On the other hand, the grey system theory, which is pioneered by J.L. Deng,[4–5] has been widely applied in several scientific research and engineering fields, such as economy prediction, prospecting mineral and scientific experiments. The information that is either incomplete or undetermined is called grey and the grey system mainly works on system analysis that has poor, incomplete, or uncertain information. Fan et al.[6] proposed several grey-based models for reducing the modeling error of a dynamically tuned gyroscope. Chen and Jou[7] integrated fuzzy logic and grey theory to develop a smart fuzzy-grey-tuning modeler to deal with the problem of probability estimation. A grey control system model has been used for
structural damage identification\cite{8}. Luo and Chen\cite{9} proposed a grey-fuzzy control algorithm for autonomous mobile target tracking. Grey prediction models are used for typical control problems\cite{10} and wireless communication systems\cite{11-12}. Grey relations are used for clustering analysis\cite{13}. Huang\cite{14-15} combined the grey theory and qualitative simulation and put forward the grey qualitative theory to solve simulation and control problems with uncertainty. All these works have proved the grey system theory to be an effective method to solve uncertain problems with incomplete information and show us a proper approach to represent uncertainty and transfer between numerical value and symbolized measurement. In this article, the measurement system is extended to the grey measurement system based on grey representation and incomplete information processing.

2. Grey measurement system and grey sensors

Nowadays, the concept of measurement generally includes numerical measurement with quantitative value and qualitative symbolic measurement. The measurement result can be represented with numerical or symbolized methods, both of which have their own advantages, respectively. The sketch of a numerical and symbolized measurement system is shown as Fig. 1. It is obvious that this kind of measurement system can give two kinds of output: numerical output and symbolized output.

However, the numerical and symbolized measurement system has the following disadvantages: (1) uncertainty and measurement errors are neglected; (2) Numerical and symbolized representations are very different to be integrated effectively; (3) the transformation from numerical values to symbols is irreversible and considerable quantitative information is lost. The key to these problems is to explore a new representation method for the measurement system to supply more powerful ability for reasoning while maintaining uncertainty, which will connect the numerical and symbolic measurement mechanisms. The grey theory is an appropriate candidate.

As seen in Fig. 2, a grey measurement system can give grey outputs after the measuring process. Numerical output

\begin{center}
\begin{tikzpicture}
\node[rectangle, draw] at (0,0) {Grey measurement system};
\node[rectangle, draw] at (0,-1) {Grey value to symbol transform or grey reasoning};
\node[rectangle, draw] at (0,-2) {White noise};
\node[rectangle, draw] at (0,-3) {Numerical output};
\node[rectangle, draw] at (0,-4) {Symbolized output};
\node[rectangle, draw] at (0,-5) {Grey output};
\end{tikzpicture}
\end{center}

Fig. 2 Sketch of the grey measurement system
ical outputs can be acquired through whitening grey results, and symbolized outputs can also be acquired through grey value to symbol transformation or grey reasoning. The relationship of the different representations in the grey measurement system is described as Fig. 3 and the related sets and mapping relations are briefly listed as follows: $X = \{x_1, x_2, \ldots, x_j\}$, set of objects to be measured; $\otimes = \{\otimes_1, \otimes_2, \ldots, \otimes_l\}$, set of grey measurement values; $S = \{s_1, s_2, \ldots, s_m\}$, set of symbolized values; $Q = \{q_1, q_2, \ldots, q_n\}$, set of quantitative numerical values; Mapping relations: $\pi_1 : X \rightarrow \otimes$, $\pi_2 : \otimes \rightarrow S$, $\pi_3 : \otimes \rightarrow Q$.

![Fig. 3 Relationship of the different representations in the grey measurement system](image)

3. Grey system for grey sensors and sensory information

In this section, the grey theory is introduced briefly and a new kind of grey number (frequency grey number) is defined with related operations. Whitening function and grey reasoning with ordinal grey scale are studied through these operations.

3.1 Grey numbers and operations

The essential concept of the grey theory is grey number, which refers to those numbers for which we know only their range instead of precise values. Under most situations, a grey number can be denoted as an interval grey number, $\otimes = [a, b]$, with its whitening function.

**Definition 1**[15] Frequency grey number $X_{[a, b]}$ is an interval grey number with whitening function

$$f_X(x) = N \left[ \frac{1}{2} (a + b), \sigma^2 \right] = \frac{1}{\sqrt{2\pi\sigma}} \exp \left\{ -\frac{(x - 0.5(a + b))^2}{2\sigma^2} \right\}$$

in real interval $[a, b]$ ($a < b$), where $\sigma^2$ satisfies

$$\int_{-\infty}^{0} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-0.5(a+b))^2}{2\sigma^2}} dx = \frac{\alpha}{2} \quad (2)$$

and $1 - \alpha$ implies the measure of frequency grey number $X_{[a, b]}$, in symbol

$$\mu(X_{[a, b]}) = 1 - \alpha, \quad \alpha \in [0, 1] \quad (3)$$

From the above definition, we know that a frequency grey number is decided by three components: interval $[a, b]$, measure $1 - \alpha$, and distribution parameter $\sigma$. These three elements are not independent, and if arbitrarily two of them are known, the third one can be determined.

The frequency grey number is a kind of special interval grey number and its whitening function is a probability density function. For example, when we mention that a person’s age must be between 32 and 35, we can use a frequency grey number with measure 0.9 in interval [32, 35] to represent it.

3.2 Whitening function

Whitening function is the base of grey systems and may have different styles of functions. As introduced in Section 3.1, we generally use frequency grey numbers with whitening function of Eq. (3). This kind of whitening function is defined by the parameters of interval $[a, b]$, measure $1 - \alpha$, and distribution parameter $\sigma$. We list some rules for parameter setting of these
whitening functions in a grey measurement system.

(1) When a grey number is a measuring value based on a certain kind of sensor, the parameters are determined by the characteristics of this sensor and the operation of measurement.

(2) When a grey number is the result of the operation on numbers (grey numbers or real numbers), the parameters are determined by the operator and may be modified when necessary.

(3) Some grey numbers for general concepts can be defined through statistics.

(4) Prior knowledge and experience can be used to give an approximate whitening function.

(5) The parameters will adaptively be adjusted with the running of the whole system through learning.

### 3.3 Grey scale

To determine the relationship between frequency grey numbers, we provide the definition of grey scale: nominal grey scale for equivalent relation and ordinal grey scale for ordinal relation.

**Definition 2** (Similarity between frequency grey numbers) The similarity of two frequency grey numbers ($g_{s1}$ and $g_{s2}$) is defined as (Fig. 4)

$$Sim(g_{s1}, g_{s2}) = \mu(g_{s1}[c, +\infty]) + \mu(g_{s2}[-\infty, c])$$

where $c$ is the point of intersection.

Based on the definition of the similarity of frequency grey numbers, the nominal grey scale and ordinal grey scale are given for measurement scale.

**Definition 3** If the similarity of two frequency grey numbers ($g_{s1[a, b]}$ and $g_{s2[a, b]}$) satisfies

$$Sim(g_{s1}, g_{s2}) = 1$$

$g_{s1}$ is equal to $g_{s2}$ ($g_{s1} = g_{s2}$). If

$$\frac{a_1 + b_1}{2} < \frac{a_2 + b_2}{2}$$

$g_{s1}$ is less than $g_{s2}$ ($g_{s1} < g_{s2}$ or $g_{s2} > g_{s1}$) and

$$d(g_{s1}, g_{s2}) = 1 - Sim(g_{s1}, g_{s2})$$

is the grey distance between $g_{s1}$ and $g_{s2}$.

**Theorem 1** $g_{s1}, g_{s2},$ and $g_{sk}$ are arbitrary frequency grey numbers; their grey distances satisfy:

(1) Nonnegative: $d(g_{s1}, g_{s2}) \geq 0$ and $d(g_{s1}, g_{s2}) = 0 \iff g_{s1} = g_{s2};$

(2) Symmetric: $d(g_{s1}, g_{s2}) = d(g_{s2}, g_{s1});$

(3) Triangle Inequality

$$d(g_{s1}, g_{s2}) \leq d(g_{s1}, g_{sk}) + d(g_{sk}, g_{s2})$$

**Proof**

(1) $d(g_{s1}, g_{s2}) = 1 - Sim(g_{s1}, g_{s2}) = 0,$ if and only if 

$$Sim(g_{s1}, g_{s2}) = 1$$

$$g_{s1} = g_{s2}$$

(2) $Sim(g_{s1}, g_{s2}) = Sim(g_{s2}, g_{s1}),$ so

$$d(g_{s1}, g_{s2}) = d(g_{s2}, g_{s1})$$

(3) From the definition of the similarity of frequency grey numbers, we know

$$Sim(g_{s1}, g_{sk}) + Sim(g_{sk}, g_{s2}) - Sim(g_{s1}, g_{s2}) \leq$$

$$1 - Sim(g_{sk}, g_{s2}) \iff$$

$$d(g_{s1}, g_{sk}) - Sim(g_{sk}, g_{s2}) \leftrightarrow$$

$$d(g_{s1}, g_{sk}) \leq d(g_{s1}, g_{sk}) + d(g_{sk}, g_{s2})$$

### 3.4 Qualitative reasoning in grey systems

The grey system theory not only can deal with the problems of uncertainty and poor information, but also is a good candidate for qualitative reasoning. Qualitative reasoning is one of the fundamental capabilities of human intelligence and artificial intelligence. Nowadays, qualitative theory such as qualitative simulation, qualitative spatial reasoning, and temporal reasoning has developed rapidly[14–18]. However, one most important open issue and challenge is to develop better engineered and easy-to-use techniques and tools to make qualitative reasoning techniques available to potential users in wider areas and applications[19].

The qualitative state, landmark values, qualitative
reasoning, and behavior based on grey numbers and grey operations will make the qualitative system robust and natural. The grey theory and the grey measurement system will be applied to qualitative methods for knowledge representation, reasoning, learning, and planning. For example, its application in robot exploration, mapping, and navigation in large scale environments is our future study.

4. A case study: multi-ultrasonic sensor systems

To demonstrate the proposed grey systems for intelligent sensors, here we take a multi-ultrasonic sensor system as a case study. Ultrasonic sensors (also called ultrasonic range finders or sonar sensors) measure the time elapsed between the transmission of a signal and the receiving of an echo of the transmitted signal to determine the distance to an obstacle, which is called the time of flight (TOF). As sonar sensors may not provide very accurate information because of low resolution, uncertainty and measurement errors are inevitable. Moreover, sonar sensors are also sensitive to the operation temperature, humidity, the surface of obstacle, etc. Most of these uncertainties should not be neglected.

4.1 Hardware configuration of a multi-ultrasonic sensor system

A multi-ultrasonic sensor system is shown as Fig. 5. The sonar devices being used are EFR-40RSC. Their useful measuring range is 0.2-5.0 m and the center frequency is 40 kHz. Experimental results show that the range accuracy of the sensors is in the order of \(\pm 0.05\) m.

4.2 Sonar data processing and information integration

Although sonar sensors are economic and easy to use, they always suffer some weaknesses, such as poor resolution, specular reflections, and crosstalk, which cannot be well solved through the modification of the hardware configuration. In this grey sensor system, the sonar data processing after the sonar data collection may be described as follows.

(1) Data preprocessing

Suppose the sampling data sequence of a sonar sensor is \(d(i)\) \((1 \leq i \leq n)\), and \(n\) is the length of the sampling data sequence. Using grey numbers, \(d(i)\) is represented as \(d(a_i, b_i)\), \(1 - \alpha_i\). The parameters \(a_i\), \(b_i\), and \(\alpha_i\) are determined and adaptively modified by the performance of the sonar devices, the operation condition, etc.

To make it simple, here we suppose that \(d(i)\) has been filtered and has no dirt data. The sonar reading is

\[
s : [a, b], 1 - \alpha
\]

where

\[
[a, b] = \left[ \frac{\sum_{i=1}^{n} a_i}{n}, \frac{\sum_{i=1}^{n} b_i}{n} \right],
\]

\[
1 - \alpha = 1 - \max_i \alpha_i, i = 1, 2, ..., n
\]

(2) Distance information

After data preprocessing, the distance information provided by the multi-ultrasonic system is (Fig. 5).

\[
D = \{RF, Right, RB, LB, Left, LF\} = \{s_1, s_2, ..., s_{18}\}
\]
where Right front (RF): \{s_1, s_2, s_3\}, Right: \{s_4, s_5, s_6\}, Right back (RB): \{s_7, s_8, s_9\}, Left back (LB): \{s_{10}, s_{11}, s_{12}\}, Left: \{s_{13}, s_{14}, s_{15}\}, Left front (LF): \{s_{16}, s_{17}, s_{18}\}, and thus, these 18 sonar sensors are grouped as 6 segments. In fact, the 3 sonar sensors in each segment are designed to avoid crosstalk using grouped-cycling sending techniques. Moreover, in several occasions, we do not need such explicit information as D. Hence, a simplified and abstract form of distance information \(D'\) is given as follows.

\[
D' = \{\text{RF}', \text{Right}', \text{RB}', \text{LB}', \text{Left}', \text{LF}'\} = \\
\{S_1, S_2, S_3, S_4, S_5, S_6\}
\]

Since the center sonar of each segment has the least disturbance, we set

\[
S_{i/3} = 0.6 \times s_{i-1} + 0.2 \times (s_{i-2} + s_i), \quad i = 3, 6, 9, 12, 15, 18
\]

Then, we have

\[
S_{i/3} : [a_{i/3}, b_{i/3}], 1 - \alpha_{i/3}, \quad i = 3, 6, 9, 12, 15, 18
\]

where

\[
[a_{i/3}, b_{i/3}] = [0.6 \times a_{i-1} + 0.2 \times (a_{i-2} + a_i), \\
0.6 \times b_{i-1} + 0.2 \times (b_{i-2} + b_i)]
\]

\[
1 - \alpha_{i/3} = 1 - \max(a_{i-2}, a_{i-1}, a_i)
\]

4.3 Grey operation and qualitative reasoning

In a grey system, qualitative reasoning is based on the input/output space and reasoning rules. Grey operation between grey numbers is critical for all these processes. Suppose the multi-ultrasonic sensor system is used for a mobile robot and the behavior-based controller is such rules as “if distance information is \(X_1\), then take action \(A_1\); if distance information is \(X_2\), then take action \(A_2\); ...”. The grey operations will indicate how to parse the distance information \(D\) to the eigenvalue of the distance space \(\{X_1, X_2, ..., X_n\}\).

\[
D* = \arg \max(Sim(D, X_i)), \quad i = 1, 2, ..., n
\]

that is to say, the distance information \(D\) is parsed to the eigenvalue \(X_i\), which has the maximum similarity with \(D\). Generally speaking, the eigenvalue of the distance space \(\{X_1, X_2, ..., X_n\}\) is adaptive and can be adjusted to acquire better performance through learning.

5. Conclusions

In this article, a systematic grey measurement system is presented based on the concept of grey sensors. It bridges the numerical measurement system to the symbolized measurement system and provides a new representation method to maintain the uncertainty of a measurement. In this grey system, the proposed methods supply more powerful ability for incomplete information processing and qualitative reasoning, which is an important and human-like thinking mode for intelligent systems. Besides the basic concepts and knowledge representation methods, we also introduce related issues of the grey measurement system, such as the definition of whitening function, frequency grey numbers, operations of grey numbers, grey scale, and qualitative reasoning. As a case study, the multi-ultrasonic sensor system is introduced to demonstrate how a grey measurement system works, which indicates the feasibility and effectiveness of the proposed methods.

There are two main advantages of the grey measurement system: (1) it can deal with uncertainty and incomplete information processing. The uncertainty may lie in the raw sensory data or may arise when integrated with inexplicit commands such as “around the coordinate \((100, 100)\)”, “between point A and B”, “go straight for about 10 meters”, etc. (2) it can improve the learning and reasoning performance of intelligent systems. Appropriate knowledge representation methods are the key to new machine learning and control algorithms and the grey theory is a new and effective approach. The grey measurement systems inspire and promote the related research areas.

In this article, we have seen that the grey measurement system has great potential in related research areas and in application; however, this method is far from satisfaction and more issues should be explored and addressed deeply. Besides all the theoretical research, we need more practical techniques for real sensors to apply to intelligent systems, such as robots and mobile systems\([20-21]\). Therefore, based on the present study, the next aim is to develop an autonomous mobile robot, which can move in an unknown environment with its grey sensors.

References


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