OFDM PAPR Reduction Using Selected Mapping Without Side Information

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Abstract—Selected mapping (SLM) is a well-known method for reducing the peak-to-average power ratio (PAPR) in orthogonal frequency-division multiplexing (OFDM) systems. The main drawback of this technique is that, for each data block, it requires the transmission of several side information bits, which results in some data rate loss. These redundant bits are so critical to the error performance of the system that they need in practice to be protected by a powerful channel code. This increases the system complexity and transmission delay, and decreases the data rate even further. In this paper, we propose a novel SLM method for which no side information needs to be sent. By considering the example of an OFDM system using 16-QAM modulation, it is shown that the proposed method performs very well both in terms of PAPR reduction and bit error rate at the receiver output.

I. INTRODUCTION

A major drawback of orthogonal frequency-division multiplexing (OFDM) systems has traditionally been their high peak-to-average power ratio (PAPR). To overcome this problem, a number of techniques have been developed (see, e.g., [1]-[7]). Selected mapping (SLM) is one of the most promising among all these techniques because it is simple to implement (at least from a conceptual viewpoint), introduces no distortion in the transmitted signal, and can achieve significant PAPR reduction [2]. Basically, SLM consists of generating, using any suitable algorithm, a set of signals, all of them representing the same data block, and then transmitting the one with the lowest PAPR. With this technique, the price to pay for a significant decrease in PAPR is a loss in data rate due to the transmission of several side information bits that are required for original data block recovery at the receiver side. The loss of such side information bits during transmission would result in a significant error performance degradation at the receiver output since the whole data block would be lost in this case. This is why it is, in practice, crucial to protect it using a powerful channel code, which makes the system more complex, increases the transmission delay, and further reduces the data rate. To avoid the need for explicit side information transmission in SLM, a few techniques have been proposed such as, for example, the scrambling method described in [8] or, more recently, the maximum likelihood decoding scheme introduced in [9].

In this paper, we introduce a simple SLM technique that does not require the transmission of any additional bit. The key idea is to reliably embed the side information in the transmitted symbols by extending the symbols that are located at some specific positions inside the transmitted frame. The locations of the extended symbols depend on the values of the side information bits. In the receiver, the role of the side information detection block is simply to determine the locations of the extended symbols. Finding the positions of these symbols is equivalent to detecting the values of the side information bits. We will see later that, in practical OFDM environments, this technique allows for the recovery of the original data block with very high probability.

This paper is organized as follows: In Section II, the proposed SLM technique is presented. Various computer simulation results are shown in Section III. Finally, conclusions are drawn in Section IV.

II. THE PROPOSED SLM TECHNIQUE WITHOUT SIDE INFORMATION

In this Section, a notation in the form $V = (v_q)_{Q}$ shall be used to denote a vector $V$ composed of $Q$ scalar quantities $v_q$, $q \in \{0, 1, ..., Q - 1\}$.

A. The proposed SLM transmitter

Consider an OFDM system using $N$ orthogonal subcarriers. A data block is a vector $X = (x_n)_N$ composed of $N$ complex symbols $x_n$, each of them representing a modulation symbol transmitted over a subcarrier. In the classical SLM technique, $X$ is multiplied element by element with $U$ vectors $B_u = (b_{u,n})_N$ composed of $N$ complex numbers $b_{u,n}$, $u \in \{0, 1, ..., U - 1\}$, defined so that $|b_{u,n}| = 1$, where $| \cdot |$ denotes the modulus operator. Each resulting vector $X_u = (x_{u,n})_N$, where $x_{u,n} = b_{u,n} \cdot x_n$, produces, after inverse discrete Fourier transform, a corresponding OFDM signal $s_u(t)$ given by

$$s_u(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x_{u,n} e^{j2\pi n \Delta f t}, \quad 0 \leq t \leq T,$$ (1)

where $T$ is the OFDM signal duration and $\Delta f = 1/T$ is the subcarrier spacing.

Among the $U$ signals $s_u(t)$, the one having the lowest PAPR is selected for transmission. To allow for the recovery of the original vector $X$ at the receiver side, one needs to transmit $\log_2(U)$ side information bits to indicate which particular vector was selected among the $U$ vectors $X_u$ available.
In this paper, we propose a novel SLM technique that allows for the side information to be reliably embedded in $X_u$ so that no additional bits need to be sent to the receiver. In our technique, the vectors $B_u = (b_{u,n})_N$ are defined so that the terms $|b_{u,n}|$ are not all equal to the unit. Actually, these vectors are composed of some elements whose modulus can also be equal to a real-valued constant $C > 1$.

The procedure to construct the set of vectors $B_u = (b_{u,n})_N$, $u \in \{0, 1, \ldots, U - 1\}$, in the proposed SLM technique is as follows: for a given vector $B_u$, the phases of the complex elements $b_{u,n}$ can first be chosen randomly, as in the classical SLM technique. The next step consists of dividing $B_u$ into $L$ subvectors of length $M = N/L$. Hereafter, we denote by $b_{m,l}$ the $(m+1)^{th}$ element in the $(l+1)^{th}$ subvector, $m \in \{0, 1, \ldots, M - 1\}$, $l \in \{0, 1, \ldots, L - 1\}$. In each subvector, the modulus of $K$ elements is set to a constant $C > 1$, whereas the modulus of the other $(M - K)$ elements remains equal to the unit. The locations in each subvector of the elements $b_{m,l}$ for which $|b_{m,l}| = C$ are identical for each subvector, i.e. the values of the terms $|b_{m,l}|$ do not depend on the index $l \in \{0, 1, \ldots, L - 1\}$.

For a given vector $B_u$, these locations form a set $S_u$ composed of $K$ integers. For instance, if a vector $B_u$ is associated with the set $S_u = \{0, 3\}$, it means that only the complex elements $b_{u,n}$ in positions $m = 0$ and $m = 3$ in each subvector have a modulus equal to $C$. The set $S_u$ actually represents the side information that is not explicitly transmitted since it is embedded in the vector $B_u$. To allow for side information recovery at the receiver side, there must be a one-to-one correspondence between an index $u$ and a set $S_u$. In other words, no two distinct vectors $B_u$ can be associated with identical sets. Since the number of distinct sets $S_u$ that can be generated is given by the binomial coefficient $M! / (M-K)!$, we can implement the SLM technique presented in this paper using up to $U = (M/K)$ vectors $B_u$.

Once all the vectors $B_u$ have been generated, our SLM method works exactly like the classical one, i.e. the data block $X$ is multiplied element by element with each $B_u$ so as to produce $U$ vectors $X_u = (x_{u,n})_N$, with $x_{u,n} = b_{u,n} \cdot x_n$, as well as $U$ corresponding OFDM signals $s_u(t)$. Finally, the signal $s_u(t)$ with the lowest PAPR is transmitted. Throughout this paper, this particular signal is denoted as $s_v(t)$ and is associated with the vectors $X_v = (x_{v,n})_N$ and $B_v = (x_{v,n})_N$. The index $v \in \{0, 1, \ldots, U - 1\}$ represents the side information to be transmitted.

As an illustration, Fig. 1 shows the vectors $B_u$ that can be used to apply our SLM technique to an OFDM system with $N = 12$ subcarriers, when each vector $B_u$ is divided into $L = 3$ subvectors of length $M = 4$ and the parameter $K$ is set to the unit. There are $U = 4^3 = 64$ possible vectors $B_u$ and the one-to-one correspondence between $u$ and $S_u$ is such that $u \rightarrow S_u = \{u\}$. Note that Fig. 1 only shows the modulus of the elements $b_{u,n}$ since their phases can basically take any random value.

It is important to mention that, in the proposed SLM technique, the average energy per transmitted symbol is increased when the data block $X$ is multiplied by the vectors $B_u$ because the fact that $|x_{u,n}| = |b_{u,n}| \cdot |x_n|$, with $|b_{u,n}|$ being either equal to 1 or $C > 1$, implies that $E[|x_{u,n}|^2] > E[|x_n|^2]$, where $E[\cdot]$ designates the expectation operator. This energy increase $G$, expressed in decibels (dB), is given by

$$G = 10 \log_{10} \left[ \frac{M + K \cdot (C^2 - 1)}{M} \right].$$

Such energy increase must be taken into account when assessing the error performance of an OFDM system using the proposed SLM method and comparing it, say, to that of an equivalent OFDM scheme based on the classical SLM technique for which there is no energy increase.

### B. The proposed SLM receiver

In this paper, we assume that, for each transmitted symbol $x_{v,n}$, the corresponding frequency-domain sample, obtained after discrete Fourier transform, is given by

$$y_{v,n} = h_n \cdot x_{v,n} + n_n, \quad n \in \{0, 1, \ldots, N - 1\},$$

where $h_n$ is a real sample representing the fading experienced by the $n$th subcarrier and $n_n$ is a complex Gaussian noise sample with zero-mean and variance $\sigma^2$. All fading and noise samples are independent. We consider that the fading samples are perfectly known at the receiver side, i.e. perfect channel state information (CSI) is assumed.

The receiver must recover the side information index $v$ by processing both vectors $Y_v = (y_{v,n})_N$ and $H = (h_n)_N$. This receiver has the knowledge of the system parameters $M$, $L$, and $K$ as well as the one-to-one correspondence between an index $u$ and a set $S_u$. In other words, it knows that each subvector in $Y_v$ has a length of $M$ symbols and corresponds to a transmitted subvector composed of $K$ symbols that were extended by a factor $C$ and $(M - K)$ symbols that were not extended. For this receiver, determining the $K$ locations associated with the extended symbols in a subvector, i.e. the set $S_v$, is equivalent to detecting the index $v$ of the transmitted vector $X_v$ thanks to the one-to-one correspondence between $v$ and $S_v$.

The detection algorithm consists of first dividing $Y_v$ into $L$ subvectors of length $M$, and then finding the $K$ positions in which the average energy per symbol was increased in each
subvector. Let us denote by $y_{m,l}$ and $h_{m,l}$ the received and fading samples, respectively, in the $(m+1)$th position in the $(l+1)$th subvector, $m \in \{0, 1, ..., M - 1\}$, $l \in \{0, 1, ..., L - 1\}$. We can show that the average energy per symbol, $E_{s,m}$, in position $m$ in a subvector is given by

$$E_{s,m} = E[[y_{m,l}^2] \cdot E[(h_{m,l})^2]^{-1} - \sigma^2 \cdot E((h_{m,l})^2)]^{-1}. \tag{4}$$

Hence, the computation of (4) for each position $m$ allows us to determine the $K$ positions with the highest energy per symbol. The evaluation of the expectation terms in (4) is performed by taking into account the $L$ samples available for each position $m$, i.e., using

$$E[[y_{m,l}^2] \approx \frac{1}{L} \cdot \sum_{l=0}^{L-1} [y_{m,l}]^2 \tag{5}$$

and

$$E[(h_{m,l})^2] \approx \frac{1}{L} \cdot \sum_{l=0}^{L-1} (h_{m,l})^2, \quad m \in \{0, 1, ..., M - 1\}. \tag{6}$$

If $L$ is not large enough, (5) and (6) may yield inaccurate evaluations of the expectation terms. The computation of the $M$ average energies $E_{s,m}$, using (4), based on such evaluations can then lead to erroneous detection of the set $S_0$, i.e., the index $v$. In such case, the whole original vector $X$ is lost since the receiver is not able to recover the side information.

In order to minimize the probability of side information detection failure, $P_{df}$, it is necessary to maximize the value of $L$, for a given number $N$ of subcarriers and a desired number $U$ of vectors $B_u$ available for PAPR reduction, which is equivalent to minimizing the length $M$ of a subvector since $M = N/L$. Therefore, we must choose the value of the parameter $K$ so that the binomial coefficient $U = \binom{M}{K}$ is maximized for a given value of parameter $M$. This can be achieved by taking $K = \frac{M}{2}$ for even values of $M$ and $K = \frac{M+1}{2}$ or $K = \frac{M+1}{2}$ for odd values of $M$. The problem is further complicated by the fact that a higher value of $K$ for a constant $M$, results in a higher energy increase $G$, which degrades the error performance of the system. Ultimately, the issue is thus to find the set of parameters that yields the best trade-off between low probability of side information detection failure, desired PAPR reduction, and overall error performance at the receiver output.

### III. EXAMPLE

To illustrate the proposed SLM technique, we consider in this Section the example of an OFDM system using 16-QAM over $N$ subcarriers. The length of the subvectors is $M = 5$ symbols and $K = 2$ symbols are extended in each subvector. With such parameters, PAPR reduction can thus be achieved using $U = \binom{5}{2} = 10$ possible vectors $B_u$. The phases of the elements $b_{u,n}, u \in \{0, 1, ..., U - 1\}, n \in \{0, 1, ..., N - 1\}$, are randomly set to either 0 or $\pi$ (with equal probabilities). All results given below are obtained by computer simulations assuming that the fading samples follow a Rayleigh distribution defined so that $E[(h_{m,l})^2] = 1$.

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**Fig. 2.** Probability of detection failure ($P_{df}$) of the side information with the proposed algorithm as a function of the constant $C$, for SNR = 8 dB and 25 dB and $N = 125, 255, 510,$ and 1020 subcarriers. For each subcarrier, we consider the uncoded transmission of 16-QAM symbols over a flat Rayleigh fading channel with perfect CSI. The length of the subvectors is $M = 5$ symbols and $K = 2$ symbols are extended in each subvector.

**A. Probability of side information detection failure**

Fig. 2 shows the probability of detection failure, $P_{df}$, as a function of the constant $C$, for two values of the signal-to-noise ratio (SNR = 8 and 25 dB) and four different numbers of subcarriers ($N = 125, 255, 510$, and 1020). Note that the numbers of subcarriers are chosen to be multiples of $M = 5$ close to the usual powers of two used in practice. The parameter $P_{df}$ represents the probability that the receiver cannot recover the side information, i.e., a complete OFDM frame (vector $X$) is lost. The SNR is defined as the ratio between the average energy per transmitted bit, $E_b$, and the one-sided power spectral density, $N_0$, of white Gaussian noise.

From Fig. 2, it is observed that the SNR does not have a great influence on the value of $P_{df}$. The latter is, on the other hand, much more dependent on the values of parameters $C$ and $N$. As $C$ is increased, the performance of our algorithm improves, simply because a higher value of $C$ allows for a better distinction between extended and non-extended symbols after transmission through the channel. Increasing $N$ also results in a better performance because the number of subvectors $L = N/M$ is then also increased, which provides more reliable estimates of the average energies per symbol $E_{s,m}$ in positions $m \in \{0, 1, ..., M - 1\}$. In any case, it is clear that small values of $C$ suffice to obtain very low probabilities of side information detection failure.

**B. Bit error rate performance**

It is also important to study the error performance degradation caused by the application of the technique proposed in this paper. Such degradation is due to both the energy increase $G$ and the occasional side information detection failure events. In fact, for the example considered in this Section, the bit error probability $P_{eb}$ at the receiver output is given by

$$P_{eb} = P \cdot (1 - P_{df}) + \frac{P_{df}}{4}, \tag{7}$$

where $P$ designates the bit error probability obtained when the side information is properly detected. As $E_b/N_0 \rightarrow +\infty, P \rightarrow$
BER performance of an OFDM system using 16-QAM over $N = 255$ subcarriers, for different values of the constant $C$. For each subcarrier, we assume transmission over a flat Rayleigh fading channel with perfect CSI. The length of the subvectors is $M = 5$ symbols and $K = 2$ symbols are extended in each subvector. For comparison purposes, the BER plot obtained with the classical SLM method with perfect side information (SI) is also displayed.

![BER performance plot](image1)

Fig. 3

CCDF of the PAPR of the proposed SLM technique for $N = 255$ subcarriers and an extension factor $C$ equal to 1.5. The length of the subvectors is $M = 5$ symbols and $K = 2$ symbols are extended in each subvector. For comparison purposes, we also display the plot obtained with the classical SLM method as well as that associated with an OFDM system without any PAPR reduction.

![CCDF of PAPR plot](image2)

Fig. 4

BER curve obtained with an equivalent OFDM system using the classical SLM technique with perfect, i.e. error-free, side information.

Fig. 3 confirms the presence of an error floor in the BER plots. As expected, the error floor can be lowered by increasing the value of $C$, i.e. reducing the probability of detection failure. Fig. 3 also shows that, if $C \geq 1.4$, the error floor appears only at very high SNRs ($> 30$ dB), which indicates that it may not be a major issue for many practical OFDM systems that tend to operate at lower SNRs. For instance, if we target a BER of $10^{-2}$ at the 16-QAM demodulator output, as may well be the case in coded OFDM systems, then the application of our technique with $C = 1.4$ or 1.5 degrades the error performance by only 0.5 dB when compared to the classical SLM method with perfect side information.

**C. PAPR reduction performance**

Finally, Fig. 4 shows the complementary cumulative distribution function (CCDF) of the PAPR obtained with the proposed SLM technique, for $N = 255$ subcarriers and $C = 1.5$. The plots corresponding to the ordinary OFDM without PAPR reduction and the classical SLM method are also depicted for comparison purposes. These results are obtained by using an oversampling factor equal to 4 [10]. We observe that the performance of our SLM method in terms of PAPR reduction is identical to that of the classical SLM technique. In both cases, we achieve 3.6 dB of PAPR reduction at the CCDF level of $10^{-4}$.

**IV. CONCLUSION**

We have proposed a simple SLM technique that does not require the explicit transmission of side information. A study of our method has been carried out by considering the use of 16-QAM over each subcarrier. When compared to the classical SLM method using error-free side information, we found that the only price to pay for the implementation of our method is a slight degradation in error performance as well as a small complexity increase at the receiver side (presence of a side information detection block). We showed that the probability of erroneous detection of the side information can actually be made very small in a realistic OFDM transmission environment. For a given set of system parameters (channel type, number of subcarriers, length of subvectors, etc.), we believe that this probability will be even smaller if 16-QAM is replaced by QPSK but will worsen in case 64-QAM or 256-QAM is used in place of 16-QAM. The application of our method to higher-order signal constellations is thus an issue that needs to be investigated.

We recently discovered that another independent study, performed on a new method for combining channel estimation and PAPR reduction via SLM, also considered the use of extended symbols (in that case, the pilot tones) as a way to embed side information inside an OFDM frame [11]. We actually found that, from a conceptual viewpoint, our work may somewhat be considered as a generalization of that reported in [11].
REFERENCES


