Decentralized Cooperative Control for Multivehicle Formation Without Velocity Measurement

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Abstract—This paper addresses formation control problems using nonlinear cooperative control theory. In particular, incorporating with the inherent passivity of the tracking error dynamics associated with a general vehicle model, the decentralized formation control algorithms are developed for both leaderless and leader-follower formation scenarios. The proposed control algorithm can ensure the desired formation behavior with an intermittently available, time-varying, and uniformly sequentially complete topology. Furthermore, to facilitate the practical implementation, an auxiliary filter is introduced to develop a control input eliminating the velocity measurement, and accounting for the actuator saturation as well. The simulation results demonstrate the effectiveness of the proposed control scheme.

I. INTRODUCTION

Research on control of multivehicle systems performing cooperative task has received extensive studies in the past decades [1]. Numerous solid results have been proposed in application areas ranging from military battle systems to mobile robot/sensor network to civilian commercial highway and air transportation systems, leading to significant theoretical developments [1][2].

Generally, the formation control problems can be further categorized into five categories [2][3]: Multi-input multi-output (MIMO), Leader/follower (L/F), Virtual structure(VS)[4], Cyclic and Behavioral. Among those categories, the L/F and behavioral architectures are the most popular and well studied ones. In [5], a L/F approach (with one leader) has been designed in a differential-flatness model. Recently, this approach has been extended to a more general directed graph case [6], where interactions between followers and leaders are allowed, one limitation of this scheme is the use of the acceleration of the vehicles in the control law. Moreover, another extension to L/F approach is leaderless formation control scheme, where a virtual leader is often assigned to follow the desired trajectory, while all the vehicles in the group are assigned as followers [7][8][9], and information flow between the leader and follower is characterized by a communication matrix, which has similar properties as graph laplacian matrix. The communication requirement using such approach is proved to be less restrictive compared to classical L/F approaches. As for behavior based approach formation control, successful attempts include reactive control [9] and cooperative avoidance control approaches [10].

Another worthy noting advances on this subject is passivity-based formation control of multivehicle in various cooperative control tasks [11][12][13], such as group alignment, attitude coordination, and flocking, where the system is rendered dissipative with specified input and output.

As mentioned earlier, the resulted control input usually includes velocity/acceleration information of the vehicle or its neighbors, this requirement could often be eliminate with full-state measurement. However, in some applications, such measurement is a challenge to the onboard sensor. Therefore, how to compensate for this requirement without sacrificing the performance is another worthy studying topic associated with cooperative control problem. Successful attempts to solve this problem include quaternion filter for the angular velocity [14], passivity lead filter [13], auxiliary filter [15], and Luenberger observer [5], also a rate-free cooperative controller is presented in [5] using stable linear filter.

In this paper, decentralized formation control problem is explored using nonlinear cooperative control theory. In particular, the formation control laws are developed for both leaderless and leader-follower formation cases using local information only to achieve the desired formation behavior. In addition, an auxiliary filter [15] is introduced to compensate for the rate-requirement in the control input, and the resulted control scheme requires position measurement only. The main contribution of this paper is straightforward the decentralized formation control using varying topology with rate estimation using auxiliary filter. Comparing with existing results, the proposed algorithm does not need continuously available communication topology, an intermittently available, uniformly sequentially complete topology [8] is theoretically enough to ensure both Lyapunov and cooperative stabilities.

II. PRELIMINARY RESULTS ON COOPERATIVE CONTROL

In what follows, we assume that vehicles operate by themselves most of the time and exchange of output information among neighbors occurs only intermittently and locally. To capture the nature of information flow, we define the following binary sensing/communication matrix and its corresponding time sequence \( \{t_k : k \in \mathbb{N}\} \) as \( S(t) \in \{0,1\}^{n \times n} = S(k) = S(t_k), \forall t \in [t_k,t_{k+1}), \) where \( \mathbb{N} = \{0,1,...,\infty\} \)

\[
S(t) = \begin{bmatrix}
1 & s_{12}(t) & \ldots & s_{1n}(t) \\
s_{21}(t) & 1 & \ldots & s_{2n}(t) \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & 1 \\
\vdots & \vdots & \ddots & 1 \\
s_n(t) & s_{n1}(t) & \ldots & 1
\end{bmatrix}
\]  

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where elements $s_{ii} \equiv 1$ means the vehicle has access to its local information, and $s_{ij} = 1$ if information of the jth vehicle is available to the ith vehicle, and $s_{ij} = 0$ if otherwise.

Time sequence $\{t_k : k \in \mathbb{N}\}$ and the corresponding changes in the row $S_i(t)$ of $S(t)$ are detectable instantaneously by and locally at the ith vehicle, but they are not predictible or prescribed or known apriori or modeled in any way [8][7].

In what follows, systems are said to be cooperatively stable if, for every $\epsilon > 0$, there exist non-empty set $\Omega_0$ and constant $\delta > 0$ and $c \in \mathbb{R}$ such that $z_{\mu}(t_0) \in \Omega_0$ and $\|z_{\mu}(t) - c\| \leq \delta$ imply $\|z_{\mu}(t) - c\| \leq \epsilon$ for all $t \geq t_0$ and for all $\mu$, where 1 is the column vector of 1s. The systems are said to be asymptotically cooperatively stable if they are cooperative stable and if $\lim_{t \to \infty} z_{\mu}(t) = c1$.

Given a nonlinear system given by

$$\dot{x}_i = f(x_i) + g(x_i)u_i \quad y_i = h(x_i) \quad (2)$$

where $x_i \in \mathbb{R}^n$ is the state system, $u_i \in \mathbb{R}^m$ is system input, $y \in \mathbb{R}^l$ is the output, $f(x)$, $g(x)$, and $h(x)$ are the system matrices with proper dimensions.

Without loss of any generality, it is assumed that control input $u_i$ to system (2) consists of an individual self feedback part and a cooperative control part, that is

$$u_i(t) = g^{-1}(x_i)[-\alpha_i(x_i) + R_i(x_i)\sum_{j=1}^n d_{ij}\beta_j(x_j)] \quad (3)$$

where $d_{ij} = \frac{s_{ij}l_{ij}}{\sum_{k=1}^n s_{ik}l_{ik}}$ is the non-negative, piecewise-constant, and row stochastic matrix produced by (1), $l_{ij}$ are the gains designed to improve connectivity with $\sum_{j=1}^n l_{ij} = 1$, $\alpha_i(x_i)$ is the self-feedback term, and $R_i(x_i)$ is defined as

$$R_i(x_i) = \left[\frac{\partial \beta_i(x_i)}{\partial x_i}\right]^{-1}$$

Therefore, under input (3), the closed-loop system is

$$\dot{x}_i = f_i^c(x, D(t)) = f_i^c(d_{i1}x_1, d_{i2}x_2, ..., d_{in}x_n) \quad (4)$$

where $f_i^c(\cdot)$ is the closed-loop dynamics of the ith state variable, $x_i$ is the state of the ith subsystem, and $x = [x_1^T, ..., x_n^T]^T \in \mathbb{R}^n$ is the overall state of the group, and $D_i(t)$ are the ith row entries of $D(t)$. Note that for heterogenous systems, the value of $d_{ij}$ and $d_{ji}$ captures the connectivity of the communication network between the subsystems $i$ and $j$.

Before proceeding to the cooperative analysis, defining the following two conditions for Lyapunov function components $V_i(\cdot)$ with $i \in \{1, ..., n\}$ and $L_{\mu, \kappa}(\cdot)$ with $\mu, \kappa \in \{1, ..., n\}$.

Condition 1: System (4) is said to be amplitude dominant on the diagonal if, for all $i$, differential inequality

$$\frac{d}{dt} V_i(x_i) \leq -\xi_i(|x_i|) + \gamma_i(x_i) \sum_{l=1}^n d_{il}(t)\beta_l(x_l - x_i) \quad (5)$$

holds for some positive definite, radially unbounded and differentiable function $V_i(\cdot)$, piecewise-constant entries $d_{il}(t)$ of non-negative matrix $D(t)$, nonnegative function $\xi_i(\cdot)$, and strictly monotone increasing function $\gamma_i(\cdot)$ and $\beta_l(\cdot)$ with $\gamma(0) = \beta(0) = 0$.

Condition 2: System (4) is said to be relative amplitude dominant on the diagonal if, for any index pair $\{\mu, \kappa\}$, the following differential inequality holds:

$$\frac{d}{dt} L_{\mu, \kappa}(x_{\mu} - x_{\kappa}) \leq \gamma_{\mu, \kappa}(x_{\mu} - x_{\kappa}) \sum_{l=1}^n d_{\mu l}(t)\beta_{\mu, \kappa, l} \quad (\xi_{\mu, \kappa}(|x_{\mu} - x_{\kappa}|) \quad (6)$$

where $L_{\mu, \kappa}(\cdot)$ is a positive definite, radially unbounded and differentiable function, piecewise-constant time functions $d_{\mu l}(t)$ are the entries of non-negative matrix $D(t)$, scalar function $\xi_{\mu, \kappa}(\cdot)$ is non-negative, and scalar functions $\gamma_{\mu, \kappa}, \beta_{\mu, \kappa, l}$ are strictly monotone increasing functions and pass through the origin.

Lemma 1: [8] System (4) is both uniformly Lyapunov stable and uniformly asymptotically cooperative stable, if it satisfies both the Condition 1 and Condition 2, and the matrix sequence of $D(t)$ over time is sequentially complete (which, by the structural property of each subsystem and through control design, is true if and only if the sensor/communication sequence of $S(t)$ is uniformly sequentially complete over time). Furthermore, whenever its element $d_{ij}(t) \neq 0$, it is uniformly bounded from below by a positive constant.

Remark 1: While Lyapunov function components $V_i(\cdot)$ and $L_{\mu, \kappa}(\cdot)$ can always be chosen, it is usually too difficult to find or assume a differentiable Lyapunov function because of non-linear dynamics and of time-varying sensing/communication topology whose changes are sequentially complete but otherwise unknown apriori. Also, despite of the unpredictable changes in $S(t)$ and hence in $D(t)$, the two conditions in Lemma 1 can be checked, and they can also be used to guide a cooperative control design.

Using Lemma 1, systematic desigs of cooperative control for nonlinear dynamical system can be done for several classes of nonlinear systems with certain classical PID/PD controllers [17], the details of its application to formation control problem will be provided in the next section.

III. FORMULATION OF THE VEHICLE MODEL

For the sake of brevity, we consider the motion of a group of $n$ vehicles in the 2D scenario in this paper, we assume that each vehicle is represented by a particle, and there is no force acting perpendicular to the vehicle's heading direction.

As illustrated in Fig.1, the motion of vehicle $i$ can be formulated as [5]

$$\dot{x}_i = v_i \cos \phi_i \quad \dot{y}_i = v_i \sin \phi_i \quad \dot{\phi}_i = \omega_i \quad (7)$$

where $x_i$ and $y_i$ are the vehicle’s position components, $\phi_i$ is the heading angle, $v_i$ and $\omega_i$ are respectively the vehicle’s linear velocity and angular velocity, which are referred as control inputs in this paper.

Since (7) representing a underactuated system with three states (i.e., $x_i$, $y_i$, $\phi_i$) and two control inputs. As such, we
have the following transformation between the state and input [5]:
\[ v_i = \sqrt{x_i^2 + y_i^2}, \quad \varphi_i = \tan^{-1}\left(\frac{y_i}{x_i}\right), \quad \omega_i = \frac{y_i \dot{x}_i - x_i \dot{y}_i}{v_i^2} \] (8)

Therefore, we can define the new control input as \( u_i = [\dot{x}_i \quad \dot{y}_i]^T \), and defined the new state for vehicle \( i \) as \( x_i = [x_i \quad y_i \quad \dot{x}_i \quad \dot{y}_i]^T \). Consequently, we have
\[ \dot{x}_i = A \dot{x}_i + B u_i \] (9)
where \( A \) and \( B \) are defined accordingly from (8).

IV. DECENTRALIZED FORMATION CONTROL DESIGN

Intuitively, the purpose of formation control is to achieve and maintain the desired formation behavior along a specified trajectory using as less global information as possible. In this section, the formation control algorithm is developed for both leaderless and leader-follower formation cases using nonlinear cooperative control theory and the dynamics model provided in previous sections.

A. Leaderless formation control algorithm

In the leaderless formation control problem, it is often assumed one virtual leader is along with the desired trajectory (provided by path planning algorithms, such as [18]), and the information of virtual leader is available to all the vehicle at every time \( t_k \). As such, the desired position for vehicle \( i \), \( z_{di}(t) \in \mathbb{R}^2 \), is expressed in the moving frame \( \mathcal{F}_d(t) \) locating at the virtual leader. Then, the tracking error for the \( i \)th vehicle is
\[ e_i(t) = q_i(t) - z_{di}(t) \] (10)
where \( q_i(t) = [x_i \quad y_i]^T \).

Consequently, we have
\[ \dot{e}_i(t) = \dot{q}_i(t) - \dot{z}_{di}(t) \quad \ddot{e}_i(t) = u_i - \ddot{z}_{di}(t) \] (11)

Therefore, the error dynamics can be described in the form
\[ \dot{E} = A_e E + B_e U \] (12)
with
\[ E = [e_1 \quad \dot{e}_1 \quad e_2 \quad \dot{e}_2 \quad \ldots \quad e_n \quad \dot{e}_n]^T \]
\[ U = [\dot{z}_{d1} \quad u_1 \quad \dot{z}_{d2} \quad u_2 \quad \ldots \quad \dot{z}_{dn} \quad u_n]^T \] (13)

where \( A_e \in \mathbb{R}^{2n \times 2n} \) and \( B_e \in \mathbb{R}^{2n \times 2n} \) can be derived accordingly.

It should be pointed out that, if we choose \( E \) as the output and \( U \) as input, the error dynamics (12) is passivity with the storage function \( V_c = E^T E/2 \) in the sense [11]
\[ \int_0^T U^T E dt = V_c(E(T)) - V_c(E(0)) \geq -V_c(E(0)) \] (14)
which indicates that the error dynamics (12) is dissipative with the specified control input \( U \) and output \( E \), the tracking error will converge to its equilibrium state (i.e., \( E = 0 \) asymptotically. In other words, the formation control could be achieved in a finite time. That is
\[ \lim_{t \to \infty} e_i(t) \to 0 \quad \text{and} \quad \lim_{t \to \infty} [e_i(t) - e_j(t)] \to 0 \] (15)

Motivated by the passivity property, a PD type control law for the \( i \)th vehicle could be designed intuitively as
\[ u_i = \dot{z}_{di} - (e_i + \gamma_i \dot{e}_i) - \sum_{j=1}^n d_{ij}(e_i - e_j) \] (16)
where \( \gamma_i \) are positive control gains to be specified later.

Theorem 1: Given system (12) under the control input (16), with the restriction that \( S(t) \) is sequentially complete over time. Then, all the vehicles in the group will achieve desired formation behavior asymptotically.

Proof: In order to prove Theorem 1, we need to prove the closed-loop system of under (16) satisfies both Condition 1 and Condition 2. Defining the Candidate Lyapunov function
\[ V_i = \frac{1}{2} E_i^T E_i \]
Taking its time derivative along (12), and substituting (16) into the resulted equation, we have
\[ \dot{V}_i = -\gamma_i \dot{e}_i^T \dot{e}_i + \dot{e}_i^T \sum_{j=1}^n d_{ij}(e_i - e_j) \] (17)

Therefore, Condition 1 is satisfied with the following functions
\[ \xi_i(x) = 0, \quad \gamma_i(x) = [0 \ 1]^T, \quad \beta_i(x) = [1 \ 0]^T \]

To verify Condition 2, defining the following cooperative control Lyapunov function
\[ L_i = \frac{1}{2}(e_i - e_j)^T (e_i - e_j) \] (18)

Again taking time derivative of (18) along (12) and substituting (16) into the resulted equation, we have
\[ \dot{L}_i = -\gamma_i (\dot{e}_i - \dot{e}_j)^T (\dot{e}_i - \dot{e}_j) + (\dot{e}_i - \dot{e}_j)^T \sum_{k=1}^n [d_{ik}(e_k - e_i) - d_{jk}(e_k - e_j)] \] (19)

It follows that Condition 2 is verified with the functions
\[ \gamma'(x) = [0 \ 1]^T, \quad \beta'(x) = \beta''(x) = [1 \ 0]^T, \quad \xi'(x) = [0 \ 1]^T \]

With both conditions satisfied, invoking Lemma 1, it follows that (15) is ensured in an asymptotically manner.
The main advantage using nonlinear cooperative control theory is that we need not to feedback linearize the nonlinear dynamics. In particular, the communication requirement is less restrictive, an intermittently available, piecewise constant, and time-varying topology is theoretically enough to ensure the cooperative stability provided (1) is uniformly sequentially complete over time, which, from graph point of view, means from any time $t_k$ on, the union of the communication matrix has a globally reachable node.

B. Leader-follower formation control

In the leaderless formation case, the information of the virtual leader is assumed to be available to all vehicles throughout the engagement, this assumption is not reasonable due to the complex environment in the real world leader-follower applications. In what follows, we consider a general case where the leader is only intermittently accessible to some of the vehicles at every $t_k$, and all the vehicles can only cooperate according to its local information and the available information receiving from its neighbors. Thereby, the leader is introduced as an additional node in (1) (termed as vehicle 0), that is

\[
\bar{S}(t) = \begin{bmatrix}
1 & s_{01}(t) & \cdots & s_{0n}(t) \\
s_{10}(t) & \ddots & \cdots & \vdots \\
\vdots & \ddots & \ddots & \ddots \\
s_{n0}(t) & \cdots & \cdots & 1
\end{bmatrix}
\]

where $s_{ij} = 1$ if vehicle $i$ has access to the leader, and $s_{ij} = 0$ if otherwise. $s_{0i} = 1$ if the leader receives information from vehicle $i$, and $s_{0i} = 0$ if otherwise.

In this way, the formation control problem is rendered to consensus problem, and is decomposed into geometrical relation with respect to any neighboring vehicles. In particular, the moving frame is defined at each vehicle instead of at the leader. As such, $z_{di}$ is developed explicitly from the individual behavior incorporating with the communication topology for the $i$th vehicle. We assume the desired offset of vehicle $i$ with respect to the moving frame $\mathcal{F}_i(t)$ at vehicle $j$ is known apriori and can be treated as constant in a rigid formation. Thus, the desired position for the $i$th vehicle is

\[
z_{di} = \sum_{j=0}^{n} d_{ij}(q_j(t) + T_{ji}(t)c_{ij})
\]

where $T_{ji}(t)$ is the transformation matrix from the moving framing locating at $j$th vehicle to the inertial frame, $c_{ij} \in \mathbb{R}^2$ is the offset vector of vehicle $i$ with respect to vehicle $j$, and

\[
T_{ji}(t) = \begin{bmatrix}
\cos \vartheta_j(t) & \sin \vartheta_j(t) \\
-\sin \vartheta_j(t) & \cos \vartheta_j(t)
\end{bmatrix}
\]

where $\vartheta_j(t) = \tan^{-1}(\dot{y}_j(t)/\dot{x}_j(t))$.

It is clear that, if $s_{ik} = 1$, vehicle $i$ can receive information from vehicle $k$ about its state $q_k(t)$ (or simply $q_k(t) - q_i(t)$) and its current heading basis vectors $c_{ik}$. Therefore, the cooperative control input (16) can be revised to

\[
u_i = -(q_i + \gamma_i \dot{q}_i) + \sum_{j=0}^{n} d_{ij}(q_j - q_i + T_{ji}(t)c_{ij})
\]

As such, the motion of vehicle $i$ is solely determined by the local information and the feedback from its accessible neighbors. Moreover, the Lyapunov stability and cooperative stability of the closed-loop system under input (23) could be proved similarly by verifying Condition 1 and Condition 2 with the same candidate Lyapunov functions (except in this case using (21) in $e_i$ and its derivatives) and provided (20) is uniformly sequentially complete as well.

Remark 2: With the augmented communication matrix (20), the interact between the leader and follower can be straightforward formulated and analyzed using the same approach provided above, and the input (23) also applies in this case.

If only one member of group has access to the leader, this problem is rendered to classical L/F structure. In which, the input for the leader can be designed exactly as in (16), while for the followers, the input in (23) applies, except in this case the communication matrix should be revised. In particular, the index of the leader should be termed as node 0 in (1).

V. RATE ESTIMATION

It is clear the resulted control inputs (16) and (23) require local velocity information. However, such measurement may not be available due to the limited sensor packages, such as in the applications of small robotic vehicles or inexpensive deployable vehicles. In this section, we proposed formation control inputs integrating with rate-estimation as well as actuator saturation using an auxiliary filter suggested in [15], and the resulted input only needs position measurement and the estimation error is guaranteed to approach zero asymptotically.

Design a filtered tracking error vector $\eta_i \in \mathbb{R}^n$ for vehicle $i$, as

\[
\eta_i = -\dot{e}_i - \tanh(e_i) + \tanh(\chi_i)
\]

where $\tanh(x)$ represents the hyperbolic function of $x$, and $\chi_i$ is an auxiliary variable defined as

\[
\chi_i = -\cosh^2(\chi_i)[\gamma_i \eta_i + \tanh(e_i) + \gamma_i \tanh(\chi_i)]
\]

where $\gamma_i$ is chosen as the same gain as in (16) and (23), $\cosh(x)$ is also the hyperbolic function for $x$.

Note that $\tanh(\chi_i)$ is the filtered $\dot{e}_i + \tanh(e_i)$, and if $\lim_{t \to \infty} \eta_i = 0$, it in turn indicates $\tanh(\chi_i) \Rightarrow \dot{e}_i + \tanh(e_i)$. Hence, we can revised the input in (16) accounting for the rate estimation as follows

\[
u_i = \ddot{z}_{di} - \tanh(e_i) - \gamma_i \tanh(\chi_i) + \sum_{j=1}^{n} d_{ij} \tanh(e_j - e_i)
\]

Moreover, we have the time derivative of $\eta_i$ as

\[
\dot{\eta}_i = -\dot{e}_i - \operatorname{sech}^2(e_i)\dot{e}_i + \operatorname{sech}^2(\chi_i)\dot{\chi}_i
\]

Then, substituting (26) and (25) into (27), results

\[
\dot{\eta}_i = -\gamma_i \dot{\eta}_i - \sum_{j=1}^{n} d_{ij} \tanh(e_j - e_i) - \operatorname{sech}^2(e_i)\dot{e}_i
\]
Defining the Lyapunov function as
\[ V_f = \sum_{i=1}^{n} \frac{1}{2} \eta_i^T \eta_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \ln(\cosh(e_j - e_i)) \]  
(29)

Note that \( \ln(\cosh(0)) = 0 \) and that \( \ln(\cosh(x)) \) is a radially unbounded, globally positive definite function for all \( x \). Hence, \( V_f \) is a radially unbounded and globally positive definite function.

Taking time derivative of \( V_f \) along (27) results
\[ \dot{V}_f = \sum_{i=1}^{n} \eta_i^T [-\gamma_i \eta_i - \sum_{j=1}^{n} d_{ij} \tanh(e_j - e_i)] \]
\[ -\text{sech}^2(e_i) \dot{\eta}_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \tanh(e_i - e_j)^T (\dot{e}_i - \dot{e}_j) \]  
(30)

Since the reference trajectory is bounded by the given waypoints, and the inherent property of the hyperbolic function, we have
\[ \sum_{j=1}^{n} d_{ij} \tanh(e_j - e_i) + \text{sech}^2(e_i) \dot{\eta}_i \leq \beta_1 \|z_i\| \]  
(31)
\[ \sum_{j=1}^{n} \tanh(e_j - e_i)^T (\dot{e}_j - \dot{e}_i) \leq \beta_2 \|z_i\| \]  
(32)

where \( \beta_i \) are positive constants, and \( z_i = [e_i \dot{e}_i]^T \).

Consequently,
\[ \dot{V}_f \leq -\sum_{i=1}^{n} [\gamma_i \|\eta_i\|^2 + \beta_1 \|\eta_i\||z_i| - \beta_2 \|\eta_i\||z_i|] \]  
(33)

Then, with the proper tuning of \( \beta_i \) and \( \gamma_i \), we have \( \dot{V}_f < 0 \), which means \( \tanh(\chi_i) \to \dot{e}_i + \tanh(e_i) \) asymptotically, as well as \( e_i \to e_j \), the formation control objective (15) is thus achieved.

We now illustrate how the input (26) can be implemented without rate calculation, only requires the calculation of \( \tanh(\chi_i) \). Firstly, introducing the new vector as
\[ y_i = \tanh(\chi_i) \]  
(34)
Then taking derivative of \( y_i \) and utilizing the results in this section, we have
\[ \dot{y}_i = \gamma_i \dot{e}_i + (\gamma_i - 1) \tanh(e_i) - 2\gamma_i y_i, \quad y_i(0) = 0 \]  
(35)
Thus, it is straightforward to use (35) to design the following filter which computes \( y_i \)
\[ y_i = p_i + \gamma_i e_i \]
\[ p_i = (\gamma_i - 1) \tanh(e_i) - 2\gamma_i y_i, \quad p_i(0) = 0 \]  
(36)
where \( p_i \) is an auxiliary variable which allows (26) to be implemented with \( \tanh(\chi_i) \) measurement only.

Remark 3: For the cooperative input (23), similar procedure applies with definition of similar auxiliary filters using \( q_i \) instead of \( e_i \), and the input is revised to
\[ u_i = -\tanh(q_i) - \gamma_i \tanh(\chi_i) + \sum_{j=0}^{n} \delta_{ij} \tanh(q_j - q_i + T_{ij} e_{ij}) \]
where \( \tanh(\chi_i) \) can be derived analogously using similar filter as in (36).

VI. SIMULATION RESULTS AND DISCUSSIONS

In this section, the effectiveness of the proposed formation control algorithms is examined in numerical simulations. In the simulations hereafter, we assume a group of 3 vehicles performing formations on a circular trajectory. The initial positions for 3 vehicles are \( x_1 = [0 \ 0]^T \), \( x_2 = [-2 \ 4]^T \), \( x_3 = [0 \ 2]^T \), with zero initial linear velocities and zero initial heading angles. The desired trajectory for the (virtual) leader is \( q_d = [5 \sin(t) \ 5 \cos(t)]^T \).

In leaderless formation control case, we assume the offset for three vehicles are \( d_1 = [1 \ -1]^T \), \( d_2 = [0 \ 0]^T \), \( d_3 = [1 \ 1]^T \), and the communication matrix is switching randomly between the following two topologies:

\[ S_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad S_2 = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \]

In leader-follower formation case, the relative coordinates between any pair of vehicles can be derived from \( d_i \), in previous case, the communication matrix is switching randomly between the following two topologies

\[ S_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \quad S_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \]

The formation performance is illustrated in Figures 2 and 3, it follows that without the filter the desired formation can be ensured less than 7 seconds, while with the filter the desired formation achieves in about 10 seconds, this delay is mainly caused by the convergence of the filter, which is consistent to the results in Figure 4. Also, it is clear that, despite the initial oscillation of the formation, the proposed formation control laws can ensure a stable formation with or without the filter, and every vehicle could track the desired trajectory smoothly.

The time history of \( \eta_i \) is provided in Figure 4. Since \( \eta_i \) generally means estimation error of the proposed filter in this paper, it is clear that, except the initial transient phase, the estimation error is approaching to zero asymptotically and will stay within the neighborhood of zero in about 10 seconds, this clearly demonstrates the effectiveness of the proposed filter algorithm.

The time history of \( u_i \) as in (26) is presented in Figure 5, which shows the in order to counteract the estimation error as well as the initial position discrepancy, \( u_i \) exhibits its maximum value at the beginning, and with the decreasing of \( \eta_i \) and the stable of formation, \( u_i \) tends to stay constant at the end.

VII. CONCLUSION

This paper focuses on formation control design for both leaderless and leader-follower cases. The information requirement is less restrictive, a sequentially complete and
time-varying is theoretically enough to ensure both Lyapunov stable and cooperative stable. Furthermore, a rate estimation control scheme is introduced using auxiliary filter, and the resulted control input do not need the rate measurement.

Simulation results indicated clearly the performances of the algorithms are satisfactory. However, for general formation control case, where the communication delay is presented, the algorithm needs revised to account for such challenge, as well as to test its robustness under this assumption. Moreover, how to speed up the convergence of the filter is also expected in the further version of this paper.

REFERENCES


