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Piecewise function based gravitational search algorithm and its application on parameter identification of AVR system

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\begin{abstract}
Heuristic optimization has shown its superiority in handling identification problem of complicated system, for methods based on heuristic optimization do not have special requirements on model structures of the target system. In this paper, a piecewise function based gravitational search algorithm (PFGSA) is proposed and applied in parameter identification of automatic voltage regulator (AVR) system. In the proposed algorithm, a piecewise function is designed as the gravitational constant function to replace the traditional exponential equation. The piecewise function provides more rational gravitational constant to control the convergence of algorithm, and thus excellent searching ability is likely to be achieved. Moreover a new weighted objective function is proposed in the identification frame. Comparative experimental studies are conducted to test the searching ability of PFGSA and to verify the performance of proposed identification strategy, while genetic algorithm, particle swarm optimization and GSA are employed for comparison. The experimental results show that PFGSA performs the best on term of accuracy and stability in the parameter identification of AVR system, and the proposed identification strategy is effective.
\end{abstract}

1. Introduction

Automatic voltage regulator (AVR) is the main controller of an excitation system, which maintains the voltage of a synchronous generator at a specific level. Parameter identification not only provides accurate models for systems in advanced control system designs, but also could be used in status inspection, fault diagnosis and performance prediction. In China, parameter identifications of power system, including excitation system, generator, prime mover and governing system, are widely applied recently, while parameter identification of AVR system is one of the key tasks.

Many traditional parameter identification algorithms for power system have been proposed in the past [1–3]. However, for many reasons, analytic search methods, e.g. least-squared algorithms based on quadratic error functions and other gradient based, often cannot find optimal solutions when dealing with complicated systems. When analytical approaches either do not apply or do not guarantee a global solution for parameter identification of complicated system, stochastic search algorithms may provide a promising alternative to these traditional approaches. The problem of parameter identification could be treated as a problem of optimization. There are many researches on the applications of intelligent optimization to parameters identification and control system design, such as [4–9].

The genetic algorithm (GA) and particle swarm optimization (PSO) are the most popular intelligent optimization algorithms, which have been widely applied in parameter identification of power system. GA has been applied in parameter identification of excitation system models [10], hysteretic brushless exciter model [11], and Jiles Atherton model [12]. As for PSO, it has been proved to be effective in parameter identification of permanent magnet synchronous motors [13], nonlinear dynamic system [14] and chaotic dynamic systems [15]. As a special optimization problem, the performance of parameter identification depends on two aspects, namely the effectiveness of the objective function, and the searching ability of the optimization algorithm, including accuracy and efficiency.

As a different intelligent optimization algorithm compared with GA and PSO, gravitational search algorithm (GSA) is a newly developed heuristic optimization method based on the law of gravity and mass interactions [16]. GSA has been confirmed higher performance in solving various nonlinear functions, compared with some well-known search methods. In [17,18], GSA was introduced to be applied in parameter identification of hydraulic turbine governing system. In GSA, the balance between exploration and exploitation is dictated by the value of gravitational constant function, and proper control of the inertia weight is very important to find the optimum solution accurately and efficiently.

In this paper, we develop the parameter identification method of AVR system by improving GSA and proposing a new objective function. In the improvement of GSA, we propose a kind of piecewise function based gravitational search algorithm, named...
PFGSA, by designing a piecewise function as the gravitational constant function, for the purpose of enhancing the searching ability. On the other hand, we propose a new kind of objective function for AVR system parameter identification, while more than one outputs of system are taken into consideration and assigned different weights according to the significance of variation of the output responding to the changes of parameters. And then, parameter identification strategy based on PFGSA is designed for the AVR system.

The paper is organized as follows. Section 2 describes the model of AVR system. The GSA is briefly introduced and then the improved PFGSA is proposed in Section 3. In Section 4, we propose the parameter identification strategy of AVR system. The comparative experiments are designed and the results are discussed in Section 5. Finally, the conclusion is drawn in Section 6.

2. Model of AVR system with PID controller

Automatic voltage regulator (AVR) is the central controller within the excitation system that maintains the terminal voltage of a synchronous generator at a specified level. Depending on the method of supplying DC power, different types of excitation systems exist. In Ref. [19], the schematic diagram of an alternator supplied controlled rectifier excitation system was shown, where DC regulator holds constant generator field voltage and is commonly referred to as manual control. It is primarily for testing, start-up and to cater to situations where the AC regulator is faulty. Closed loop voltage control is carried out through the AC regulator. In addition to the AVR, this loop is composed of five main components, namely amplifier, exciter, exciting voltage limiters, generator and measurement and filtering, as shown in Fig. 1. To analyze dynamic performance of AVR, transfer functions of these components are represented as follows [20]:

Amplifier model: The amplifier model is given by

$$\frac{V_R(s)}{V_C(s)} = \frac{K_A}{1 + \tau A s}$$

Typical values of $K_A$ are in the range of 10–400. The amplifier time-constant often ranges from 0.02 to 0.1 s.

Exciter model: Exciter model and parameters greatly depend on its type. A simplified transfer function of a modern exciter is

$$\frac{V_R(s)}{V_(R)(s)} = \frac{K_e}{1 + \tau E s}$$

Typical values of $K_e$ are in the range of 0.8–1 and the time-constant $\tau E$ for an AC exciter is in the range of 0.5–1.0 s.

Generator model: The generator model used in this paper is the well-known simplified first-order transfer function

$$\frac{V_I(s)}{V_(I)(s)} = \frac{K_G}{1 + \tau G s}$$

$K_G$ depends on load (0.7–1.0); 1.0 s $\leq \tau G \leq 2.0$ s

Although the simplified first-order model described above is a standard model for a AVR system [24,25], three-order model or more complicated model could also be used for AVR system.

A typical three-order model, which represents a synchronous generator connected to an infinite bus through a transmission line, can be expressed as [26]:

$$\begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta E_J \end{bmatrix} = \begin{bmatrix} 0 & \omega_B & 0 \\ -\frac{1}{M} & 0 & -\frac{1}{C_0} \\ \frac{1}{\tau_E} & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta E_J \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{\Delta T_m}{\tau_E} \end{bmatrix}$$

$$\Delta V_I = [K_5 \quad 0 \quad K_6] \begin{bmatrix} \Delta \delta \\ \Delta \omega \\ \Delta E_J \end{bmatrix}$$

(5)

$\delta$, $\omega$ and $E_J$ represent the torque angle, the angular speed and the electro motive force of the generator, respectively. The $T_m$ and $T_{eD}$ are the field voltage and mechanical torque, $\omega_B$, $M$, $D$ and $\tau_E$ denote the frequency, inertia coefficient, damping coefficient and the $d$-axis transient open circuit time constant of the synchronous generator, respectively. $K_1$–$K_6$ represents the modeling constants depending on the system parameters and change according to the operating point.

Measurement model: The voltage measurement block, including PT, rectifier and filter, is often modeled with a single time constant $V(s) = \frac{K_R}{1 + \tau \delta s}$

$\tau \delta$ ranges over 0.001–0.06 s.

PID controller: PID controllers are being extensively used by industries today owing to their simplicity. As modeled in this paper, the transfer function of PID controller is

$$G(s) = K_p + \frac{K_I}{s} + K_D s$$

(7)

The AVR system with first-order generator model is shown by Fig. 1. In this system, some nonlinear factors have to be considered. There are limits to the outputs of amplifier and exciter for the system protection. When the system is subjected to a large disturbance, system limits play an important role to protect the system from damage. In this situation, the AVR system is no longer a linear system.

If three-order generator model is considered, the AVR system with three-order generator model is shown by Fig. 2.

3. The proposed novel gravitational search algorithm

3.1. Brief introduction of gravitational search algorithm

In GSA, agents are considered as objects and their performance are measured by their masses, and all these objects attract each other by the gravity force, while this force causes a global movement of all objects towards the objects with heavier masses [16].

Assumed there are $N$ agents (masses), position of the $i$th agent is $X_i = (x_i^1, x_i^2, \ldots, x_i^n), i = 1, \ldots, N$.

Every agent has three kinds of masses, which are defined as:

Active gravitational mass, $M_a$ is a measure of the strength of the gravitational field due to a particular object. Gravitational field of

![Fig. 1. Block diagram of AVR system with first-order generator model.](image_url)
an object with small active gravitational mass is weaker than the object with more active gravitational mass.

**Passive gravitational mass**, \( M_p \), is a measure of the strength of an object’s interaction with the gravitational field. Within the same gravitational field, an object with a smaller passive gravitational mass experiences a smaller force than an object with a larger passive gravitational mass.

**Inertial mass**, \( M_i \), is a measure of an object resistance to changing its state of motion when a force is applied. An object with large inertial mass changes its motion more slowly, and an object with small inertial mass changes it rapidly.

Inertial mass is calculated according to fitness function value of the agent. It is easy to understand that excellent agent possesses strong gravitational field and moves slowly, because it has big inertial mass. According to fitness function value, the inertial mass of the ith agent is defined

\[
m_i = \frac{f_{i_t} - \text{worst}}{\text{best} - \text{worst}}
\]

where \( f_{i_t} \) is the fitness function value of the ith agent. For optimization problem seeking minimal value, \( \text{best} = \min f_{i_t} \), \( \text{worst} = \max f_{i_t} \).

In order to make it easy, it is assumed that active gravitational mass and passive gravitational mass are equal to inertial mass

\[
M_{ai} = M_{pi} = M_i
\]

where \( M_i = \sum_{j=1}^{N} m_j \).

According to Newton gravitation theory, the force acting on the ith mass from the jth mass is defined

\[
F_{ij}^g(t) = G(t) \frac{M_i(t) \times M_j(t)}{\|X_i(t),X_j(t)\|^2} (x_i^j(t) - x_j^i(t))
\]

where \( M_i(t) \) and \( M_j(t) \) are masses of agents, \( G(t) \) is the gravitational constant at time \( t \).

It must be pointed out that the gravitational constant \( G(t) \) is important in determining the performance of GSA, and \( G(t) \) is defined a function of time \( t \):

\[
G(t) = G_0 \exp \left( -\beta \frac{t}{\text{max}_T} \right)
\]

where \( G_0 \) is the initial value, \( \beta \) is a constant, \( t \) is the current iterations, and \( \text{max}_T \) is the maximum iterations.

For the ith agent, the randomly weighted sum of the forces exerted from other agents

\[
F_i^g(t) = \sum_{j \neq i} \text{rand}_i F_{ij}^g(t)
\]

Based on law of motion, the acceleration of the ith agent is calculated by

\[
a_i^g(t) = \frac{F_i^g(t)}{M_i(t)}
\]

where \( M_i \) is the inertial mass of the ith agent.

Then, the searching strategy on this concept can be described by following equations:

\[
v_i^g(t+1) = v_i^g(t) + a_i^g(t)
\]

\[
x_i^g(t+1) = x_i^g(t) + v_i^g(t+1)
\]

In above equations, \( x_i^g(t) \) represents the position of ith agent in dth dimension, \( v_i^g(t) \) is the velocity, \( a_i^g(t) \) is the acceleration, \( \text{rand}_i \) is a random number among \([0, 1]\).

In GSA, each mass presents a solution, and the algorithm is navigated by properly adjusting the gravitational and inertia masses. With lapse of time, the heaviest mass will present an optimum solution in the search space.

### 3.2. Piecewise function based gravitational search algorithm (PFGSA)

In GSA, proper control of global exploration and local exploitation is crucial in finding the optimum solution efficiently. Obviously, the performance of GSA greatly depends on its parameters. It is clear that the first part of Eq. \((12)\) represents the influence of previous velocity, which provides the necessary momentum for particles to roam across the search space, and the second part represents the influence of current gravitational force put on by other masses, in the form of acceleration. The gravitational constant \( G(t) \) is the modulus that controls the impact of gravitational force. So, the balance between exploration and exploitation in GSA is dictated by the value of \( G(t) \). Thus proper control of the inertia weight is very important to find the optimum solution accurately and efficiently. It is regarded that a larger gravitational constant makes the algorithm move towards global exploration, while a smaller gravitational constant make the algorithm move toward fine-tuning of the current search area.

Due to the effect of decreasing gravity, the actual value of the “gravitational constant” depends on the actual age of the universe. As shown in Eq. \((11)\), exponential function is used to simulate the varying of gravitational constant.

\[
G(t) = G_0 \exp \left( -\beta \frac{t}{\text{max}_T} \right)
\]

where \( G(t) \) is the value of the gravitational constant at time \( t \). \( G_0 \) is the value of the gravitational constant at the first cosmic quantum-interval of time \( t_0 \) and \( \beta \) is a constant controlling decreasing rate of gravitational constant.

In order to improve the searching ability of GSA, a new gravitational constant function is designed in this paper, while a kind of piecewise function is introduced. The piecewise function...
based gravitational constant function is designed as
\[
G(t) = \begin{cases} 
\frac{G_0 - G_1 t + G_0}{t_1} & 0 \leq t < t_1 \\
\frac{G_0 - G_1 t_1 + G_0}{t_1} & t_1 \leq t < t_2 \\
\frac{G_0 - G_1 t_2 + G_0}{t_2} & t_2 \leq t \leq N 
\end{cases}
\] (16)

Fig. 3 shows comparison of exponential function, adopted in GSA, with the proposed piecewise function. In Fig. 3, parameters of exponential function as Eq. (11), are set as \(G_0 = 30, \beta = 9\), while parameters of piecewise function are set as \(G_0 = 30, t_1 = 50, G_1 = 3, t_2 = 200, G_2 = 0.5, G_3 = 1 \times 10^{-4}\).

Compared with exponential function, piecewise function is more flexible to control the decreasing rate of gravitational constant, divided into three stages, namely coarse search stage, moderate search stage and fine search stage. In the first stage, the gravitational constant decreases at a large rate, reducing the searching range rapidly; in the second stage, the decreasing rate of gravitational constant slows down, getting close to the global optima gradually; the last stage is a fine-tuning stage, when gravitational constant is quite small with a low decreasing rate, searing the global optima in a meticulous way.

In this paper, in view of the importance of gravitational constant function, we design a kind of piecewise function as the gravitational constant function, divided into N stages, \(N \geq 3\), to control the decreasing rate of gravitational constant, divided into three stages, namely coarse search stage, moderate search stage and fine search stage. In the first stage, the gravitational constant decreases at a large rate, reducing the searching range rapidly; in the second stage, the decreasing rate of gravitational constant slows down, getting close to the global optima gradually; the last stage is a fine-tuning stage, when gravitational constant is quite small with a low decreasing rate, searing the global optima in a meticulous way.

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The steps of PFGSA are as following:

1. Initialize populations \(X_i = (x_{i1}, x_{i2}, ..., x_{in})\);
2. Calculate objective function; 
3. Update \(G(t)\) as Eq. (16);
4. Update best(\(t\)), wors(\(t\)) and Mi(\(t\)) for \(i = 1, 2, ..., N\);
5. Calculate of the total force \(F_i(t)\) as Eq. (12), acceleration \(a^*_i\) as Eq. (13);
6. Updating of velocity and position as Eqs. (14) and (15);
7. Repeat steps 2–6 until the stop criterion is reached.

In this work, PFGSA will be used to solve parameter identification problem of AVR system. The performance of PFGSA and effectiveness of the improvement made in this section will be demonstrated through experimental results.

For parameter selection of piecewise function, we suggest a range \([20, 100]\) for \(G_0\) and a range \([1e-4, 1e-10]\) for \(G_2\) according to different problems. For other parameters, we suggest: \(G_1 = G_0/10, G_2 = G_1/100, t_1 = N/4, t_2 = N/2.5\). According to our experience, PFGSA will show acceptable performance with the suggested parameters. And we can adjust the parameters around the suggested values by means of trial-and-error method. Thus, setting values for \(G_1\) and \(G_2\) is necessary, but setting values for \(G_1\), \(G_2\), \(t_1\) and \(t_2\) is optional, depending on performance of PFGSA with the default values.

4. Parameter identification of AVR system using PFGSA

In parameter identification, the objective function is usually a function of unknown parameters, which is defined to measure the difference between original system and identified system. By minimizing of objective function, the identified system trends to be equal with the original system, while the unknown parameters tend to be equal with real values. In this section, objective function for parameter identification of AVR system is defined. And then, the identification strategy based on PFGSA and the proposed objective function is illustrated.

4.1. Objective function

The objective function of solving the problem of parameter identification is usually a function measuring the discrepancy between the system and the model outputs. As minimizing the objective function, model outputs fit system outputs with smaller and smaller errors. In literatures, system output is usually the final output of the system in definition of objective function [17,20]. The traditional objective function (TOF) for parameter identification of HTGS is defined as Eq. (17), while only speed of generator \(x\) is selected as system output for objective function construction.

\[
C_{TOF}(\theta) = \sum_{k=1}^{N} (y(k) - \hat{y}(k))^2
\] (17)

However, in applications, more than one signal could be sampled in engineering applications, and multiple signals may contain different information of parameters. So, we take the output of amplifier \(V_a\), output of exciter \(V_e\), voltage of generator \(V_g\) and output of sensor \(V_s\) as compared outputs to construct the objective function. Although four outputs are selected, their contribution and importance in solving the problem of parameter identification may be different. At the same variation of parameters, the more significant the output of identified model deviates from that of original system, the more important the output in the objective function will be. And the weight for this output pairs is heavy. In this way, weights of errors of outputs are designed, according to the significance of deviation.

The objective function is adopted in defined as:

\[
C_{OF}(\theta) = \sum_{k=1}^{N} \sum_{j=1}^{s} w_j (y_j(k) - \hat{y}_j(k))^2
\] (18)

where, system output vector \(y = [V_e, V_f, V_r, V_s]\), the weight vector \(w = [w_1, w_2, w_3, w_4]\).

The weights are calculated according to following steps:

1. set a basis of system parameter \(\theta_i, i = 1, ..., m\), calculate system outputs \(\hat{y}_j(k), k = 1, ..., N, j = 1, ..., s\);
2. loop A: \(j = 1 : s\)
   loop B: \(i = 1 : m\)
   vary the ith parameter, and get \(\theta_{new} = [\theta_1, \theta_2, 0, 0, ..., \theta_m]\);
   calculate system outputs \(\hat{y}_j(k)\);
   \(w_j = |y_j - \hat{y}_j|/\text{max}(\hat{y}_j)\);
   the jth weight \(w_j = \text{average}(w_j)\);
End loop B;
End loop A;

4.2. Identification strategy

The structure of the PFGSA-based parameter identification approach can be illustrated by Fig. 3. The identified model is the
AVR system model described in Section 2, of which parameters are undetermined. At first, the inputs and output of amplifier \( V_p \), output of exciter \( V_s \), voltage of generator \( V_t \) and output of sensor \( V_r \) of the original system are sampled. Then, with an initial set of parameters, the identified model is excited by the same inputs. The corresponding outputs of identified model are sampled. And then, outputs of original system and identified system are introduced to the performance evaluator, where the objective function \( c(\theta) \), measuring the difference of the two system, will be calculated. PFGSA-based optimizer is used to adjust the unknown parameter vector \( \theta \). The estimated \( \hat{\theta} \) is adopted to update the identified system. The above process will be repeated until \( \hat{\theta} \) has been successfully identified.

5. Simulation experiments

In this section, the AVR system is simulated by MATLAB, and the proposed PFGSA based approach is applied to identify the parameters of simulated system. The simulation model structure of AVR system is illustrated in Fig. 1. Eight key parameters are chosen to be identified in simulation experiments, which are \( K_p, \tau_p, K_e, \tau_e, K_g, \tau_g, K_s, \tau_s \). In experiments, parameters of simulated model are set with values, and the eight parameters are expected to be identified. In experiments, simulation model of AVR system is excited by step signal with amplitude of 1 p.u. and 2 p.u. The simulation time is set to be 3 s. The sampling time is set to be 0.01 s. The sampling time is fast enough to capture the system dynamics. The output of amplifier \( V_p \), output of exciter \( V_e \), voltage of generator \( V_t \) and output of sensor \( V_r \) are sampled Fig. 4.

Parameter identification accuracy is measured by parameter error (PE)

\[
PE = \frac{|\theta_i - \hat{\theta}_i|}{\theta_i}, \quad i = 1, ..., m
\]

and the average parameter error (APE)

\[
APE = \frac{1}{m} \sum_{i=1}^{m} \frac{|\theta_i - \hat{\theta}_i|}{\theta_i}
\]

In this part of experiments, PFGSA, GSA, PSO and GA have been employed to identify the parameters in the dynamic model of AVR system. IOP has been adopted in identification structure since the effectiveness has been verified.

5.1. Linear model identification

In this part of experiments, the AVR system is excited by step signal with amplitude of 1 p.u., when \( V_{ref} \) is set as 1. In this situation, the system limits have not been excited, and the system is a linear system. The parameters of GA are set as: population size is 50, iteration number is 500, crossover rate \( P_c = 0.7 \), mutational rate \( P_m = 0.06 \). Parameters of PSO are set as: population size is 50, iteration number is 500, \( c_1 = c_2 = 2.0 \). For GSA, the parameters are set as: population size is 30, iteration number is 500, \( G_o = 30 \), \( \beta = 9 \). Parameters of PFGSA are set as: population size is 30, iteration number is 500, \( G = 30 \), \( G_3 = 1e-4 \), \( G_2 = 0.5 \), other values are default setting. All the optimization method applied in experiments are stochastic search algorithm, in order to test the stability and effectiveness of algorithms, identification experiments are repeated 20 times, and average results are obtained.

In group 1, parameter vector of simulated system is set as \( \theta = [8, 0.12, 1, 0.4, 1.1, 0.8, 1.0, 0.1] \). In order to compare identification accuracy achieved by different methods, results on term of PE are listed in Table 1, and results considering mean best cost and mean APE are listed in Table 2. In Tables 1 and 2, it is shown that, compared with GA and PSO, GSA and PFGSA achieve better parameter identification accuracy. Although, PE achieved by GSA of several parameters are slightly better than those of PFGSA, the PFGSA shows the best performance of identification accuracy in an overall view in Table 1. In Table 2, it is clear that PFGSA performs best in searching the optimal solution of objective function. The parameter identification accuracies are in accord with the optimal value of objective function, which means the objective function designed in this paper is effective. We could get better identification accuracy by searching better solution of objective function. The convergence of algorithms is compared in Fig. 5, which exhibits the average convergence of 20 times. The figure shows that PFGSA could converge on the best optimal, although its convergence rate is not the best.

In Figs. 6 and 7, identified AVR system are simulated by using average identified parameters, and then compared with the original system outputs, where output of amplifier \( V_p \), output of generator \( V_t \) and output of sensor \( V_r \) are compared. It is seen that the identified system obtained by PFGSA is more accurate than those identified by GA, while comparative curves of original system and identified system obtained by PFGSA are very close. It is difficult to distinguish the compared curves, which means that the parameter identification based on PFGSA is effective and with high accuracy.

Table 3 compares the efficiency of different methods for parameter identification. The computations are finished by a common PC, with

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>System value</th>
<th>GA</th>
<th>PSO</th>
<th>GSA</th>
<th>PFGSA</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_p )</td>
<td>8</td>
<td>7.719</td>
<td>0.035</td>
<td>8.323</td>
<td>0.040</td>
</tr>
<tr>
<td>( c_v )</td>
<td>0.12</td>
<td>0.116</td>
<td>0.029</td>
<td>0.134</td>
<td>0.121</td>
</tr>
<tr>
<td>( K_e )</td>
<td>1</td>
<td>1.100</td>
<td>0.100</td>
<td>1.018</td>
<td>0.018</td>
</tr>
<tr>
<td>( c_e )</td>
<td>0.4</td>
<td>0.446</td>
<td>0.115</td>
<td>0.886</td>
<td>1.217</td>
</tr>
<tr>
<td>( K_g )</td>
<td>1.1</td>
<td>1.296</td>
<td>0.178</td>
<td>1.465</td>
<td>0.331</td>
</tr>
<tr>
<td>( c_g )</td>
<td>0.8</td>
<td>1.008</td>
<td>0.260</td>
<td>0.381</td>
<td>0.523</td>
</tr>
<tr>
<td>( K_s )</td>
<td>1</td>
<td>1.166</td>
<td>0.160</td>
<td>1.331</td>
<td>0.331</td>
</tr>
<tr>
<td>( c_s )</td>
<td>0.1</td>
<td>0.108</td>
<td>0.080</td>
<td>0.122</td>
<td>0.279</td>
</tr>
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</table>

Table 2 Comparison of optimal objective function value of different methods in group 1.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Objective function value</th>
<th>APE</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA</td>
<td>0.31445</td>
<td>19.811</td>
</tr>
<tr>
<td>PSO</td>
<td>1.15567e–016</td>
<td>0.82541</td>
</tr>
<tr>
<td>GSA</td>
<td>0.026109</td>
<td>2.2894</td>
</tr>
<tr>
<td>PFGSA</td>
<td>0.00087725</td>
<td>0.68332</td>
</tr>
</tbody>
</table>

Fig. 4. Block diagram of PFGSA based parameter identification strategy.
a 2.4 G dual-core CPU and 2 G RAM. It is showed that GA and PSO need much more time to search the optimal solution, for the population sizes of GA and PSO are bigger. The executing time of PFGSA is similar with GSA, and both of them are acceptable. In group 2, parameter vector of simulated system is set as \( \theta = [8.5, 0.1, 1.2, 0.5, 1, 1.5, 1, 0.1] \). The excitation signal is also step voltage disturbance with amplitude of 1 p.u. In experiment, corresponding signals are sampled for parameter identification.

Tables 4 and 5 show the identification accuracy achieved by different methods, where average PE, APE and best cost are taken into consideration. The results show clear that PFGSA perform best on all indices. The average APE obtained by PFGSA is as small as 0.00044, much better than those achieved by other methods, which shows the proposed parameter identification method is effective. The average convergence process of algorithms are compared in Fig. 8, which shows PFGSA possesses excellent ability in optimizing the objective function compared with other algorithms, while PFGSA could converge on the best optimal value. In order to check the accuracy of identified system further, average identified parameters are used to simulate the identified system. Identified system and original system are compared in Figs. 9 and 10, while GSA and PFGSA are applied respectively. Figs. 9 and 10 show that PFGSA exhibits higher accuracy in system approximation. It seems that the identified system almost coincides with the original system in three compared outputs in Fig. 10.

5.2 Nonlinear model identification

In this part of experiments, the AVR system is excited by step signal with amplitude of 2 p.u., when \( V_{ref} \) is set as 2. In this situation, the system limits have been excited, and the system is a nonlinear system. Parameter vector of simulated system is set as \( \theta = [10, 0.1, 1, 0.4, 1, 1, 1, 0.1] \). In this part of experiments, PSO, GSA and PFGSA have been adopted in experiment of parameter identification. The Parameters of PSO are set as: population size is 30, iteration number is 50, and \( \beta = 9 \). The parameters of PFGSA are set as: population size is 30, iteration number is 50, other values are default setting. In order to test the stability and effectiveness of algorithms, identification experiments are repeated 20 times.

Tables 6 and 7 show the identification accuracy achieved by different methods, where average PE, APE and best cost are taken into consideration. By comparing identification results of linear system and nonlinear system, it is found that the nonlinear system is more difficult to identify and the identification accuracies obtained by different methods are lower than those of linear AVR system. From Table 7, we find that PFGSA perform the best on terms of optimal objective function value and mean parameter error. The APE obtained by PFGSA is 0.0154, which still indicates good identification accuracy.

In order to test the robustness of algorithms for parameter identification of the nonlinear AVR system, we add white noise with

![Fig. 5. Comparison of average iteration process of different algorithms in group 1.](image)

![Fig. 6. Comparison of original system and system with average identified parameters by GA in group 1.](image)

![Fig. 7. Comparison of original system and system with average identified parameters by PFGSA in group 1.](image)

![Fig. 8. Comparison of average iteration process of different algorithms in group 1.](image)
that the noise concomitant of sampling makes it more difficult for parameter identification of AVR system. The identification accuracies of all methods decrease, while PFGSA still show its advantage in comparison with other methods. All experiments are repeated 20 times, and the average results are calculated. The identification results are listed in Tables 8 and 9. By comparing Table 9 with Table 7, it is obviously that the noise concomitant of sampling makes it more difficult for parameter identification of AVR system under this condition. Figs. 11–13 exhibit the simulated system with average parameters over 20 trials obtained by different methods with 25 dB white noise. From these figures, we find the output limits have worked to keep the output of exciter under a level. System identified by PFGSA coincide the best with the original system.

5.3. AVR system identification with three-order generator model

The experiments discussed above are based on the AVR system with first-order generator model. Although the first-order generator model is used most for a AVR system, a three-order generator model can also be used in AVR system identification. In this situation, we can identify more parameters for the generator. In this part of experiments, we try to identify parameters of a simulated AVR system, while a three-order generator model is adopted in the AVR system, as illustrated in Fig. 2. Nine key parameters are chosen to be identified in simulation experiments, which are $K_a$, $r_g$, $K_s$, $r_f$, $K_e$, $r_v$, $r_a$, $M$, $D$ and $K_c$. In experiments, simulation model of AVR system is excited by step signal with amplitude of 1 p.u. The simulation time is set to be 3 s.
The parameters of PFGSA are set as: population size is 30, iteration number is 500, $\tau_a = 0.14055$, $\tau_e = 0.40545$, $\theta_i = 0.09876$, $\beta = 0.0205$, $\gamma = 0.07651$. In this part of experiments, PSO, GSA and PFGSA have been adopted in experiment of parameter identification. The parameters of PSO are set as: population size is 30, iteration number is 500, $\tau_a = 0.14055$, $\tau_e = 0.40545$, $\theta_i = 0.09876$, $\beta = 0.0205$, $\gamma = 0.07651$.

### Table 6
Comparison of average identification results of nonlinear AVR system without white noise.

<table>
<thead>
<tr>
<th>$i$</th>
<th>System value</th>
<th>PSO</th>
<th>GSA</th>
<th>PFGSA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta_i$</td>
<td>$\sigma_i$</td>
<td>$\delta_i$</td>
<td>$\epsilon_i$</td>
</tr>
<tr>
<td>$K_a$</td>
<td>10</td>
<td>9.9377</td>
<td>0.002625</td>
<td>9.9544</td>
</tr>
<tr>
<td>$r_a$</td>
<td>0.1</td>
<td>0.0914</td>
<td>0.008582</td>
<td>0.0994</td>
</tr>
<tr>
<td>$K_a$</td>
<td>0.4</td>
<td>0.4001</td>
<td>0.025026</td>
<td>0.4039</td>
</tr>
<tr>
<td>$K_a$</td>
<td>1</td>
<td>0.7889</td>
<td>0.21107</td>
<td>1.0474</td>
</tr>
<tr>
<td>$r_a$</td>
<td>0.1</td>
<td>1.0259</td>
<td>0.025822</td>
<td>1.0366</td>
</tr>
<tr>
<td>$r_e$</td>
<td>0.1</td>
<td>1.5040</td>
<td>0.560404</td>
<td>0.9740</td>
</tr>
</tbody>
</table>

### Table 7
Comparison of optimal objective function value of nonlinear AVR system without white noise.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Objective function value</th>
<th>$\Delta$PE</th>
<th>APE</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSO</td>
<td>2.1986</td>
<td>18.061</td>
<td>9.7221</td>
<td>0.158</td>
</tr>
<tr>
<td>GSA</td>
<td>0.022595</td>
<td>0.63022</td>
<td>0.60341</td>
<td>0.0205</td>
</tr>
<tr>
<td>PFGSA</td>
<td>0.0024935</td>
<td>0.19371</td>
<td>0.25534</td>
<td>0.0154</td>
</tr>
</tbody>
</table>

### Table 8
Comparison of average identification results of nonlinear AVR system with white noise.

<table>
<thead>
<tr>
<th>$i$</th>
<th>System value</th>
<th>PSO</th>
<th>GSA</th>
<th>PFGSA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\theta_i$</td>
<td>$\sigma_i$</td>
<td>$\delta_i$</td>
<td>$\epsilon_i$</td>
</tr>
<tr>
<td>$K_a$</td>
<td>10</td>
<td>10.228</td>
<td>0.22813</td>
<td>9.781</td>
</tr>
<tr>
<td>$r_a$</td>
<td>0.1</td>
<td>0.08928</td>
<td>0.07141</td>
<td>0.09675</td>
</tr>
<tr>
<td>$K_a$</td>
<td>1</td>
<td>1.0002</td>
<td>0.000167</td>
<td>0.94308</td>
</tr>
<tr>
<td>$r_a$</td>
<td>0.4</td>
<td>0.38069</td>
<td>0.48283</td>
<td>0.40333</td>
</tr>
<tr>
<td>$K_a$</td>
<td>1</td>
<td>0.85966</td>
<td>0.14034</td>
<td>1.0694</td>
</tr>
<tr>
<td>$r_a$</td>
<td>1</td>
<td>1.0842</td>
<td>0.08419</td>
<td>1.1095</td>
</tr>
<tr>
<td>$K_a$</td>
<td>1</td>
<td>1.4154</td>
<td>0.41539</td>
<td>0.99243</td>
</tr>
<tr>
<td>$r_a$</td>
<td>0.1</td>
<td>1.2782</td>
<td>0.27796</td>
<td>0.1027</td>
</tr>
</tbody>
</table>

### Table 9
Comparison of optimal objective function value of nonlinear AVR system with white noise.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Objective function value</th>
<th>$\Delta$PE</th>
<th>APE</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSO</td>
<td>0.49466</td>
<td>19.206</td>
<td>9.1097</td>
<td>0.13704</td>
</tr>
<tr>
<td>GSA</td>
<td>0.44311</td>
<td>2.2416</td>
<td>0.99308</td>
<td>0.044446</td>
</tr>
<tr>
<td>PFGSA</td>
<td>0.11914</td>
<td>0.2612</td>
<td>0.10468</td>
<td>0.024837</td>
</tr>
</tbody>
</table>

The output of amplifier $V_a$, output of exciter $V_F$, voltage of generator $V_t$ and output of sensor $V_s$ are sampled. Parameter vector of simulated system is set as $\theta = [10, 0.15, 1.2, 0.6, 5.9, 4.74, 2, 1, 0.1]$. In this part of experiments, PSO, GSA and PFGSA have been adopted in experiment of parameter identification. The parameters of PSO are set as: population size is 30, iteration number is 500, $C_1 = C_2 = 2.0$. For GSA, the parameters are set as: population size is 30, iteration number is 500, $C_1 = C_2 = 30$, $\beta = 9$. The parameters of PFGSA are set as: population size is 30, iteration number is 500, $C_1 = 40$, $C_2 = 10$, other values are default setting. In order to test the stability and effectiveness of algorithms, identification experiments are repeated 20 times.

Tables 10 and 11 show the identification accuracy achieved by different methods, where average PE, APE and best cost are taken into consideration. By comparing identification results of AVR system with first-order generator model and three-order model, it is found that the latter system is more difficult to identify and the identification accuracies obtained by different methods are lower. From Table 11, we find that PFGSA perform the best on terms of optimal objective function value and mean parameter error, compared with PSO and GSA.
5.4. Discussion on experimental results

From the experimental results above, we find PFGSA show obvious advantage over other typical heuristic search algorithms in parameter identification for AVR system. No matter nonlinear factor or sampling noise considered, the proposed algorithm performs the best on term of optimal objective function value and identification accuracy. GSA and some variants of GSA have already showed excellent searching ability over traditional heuristic optimization method, like GA and PSO [21–23], due to its unique searching mechanism inspired by law of gravity. Here, we need to compare and analyze the improvement of PFGSA over traditional GSA. Compared with the traditional GSA, the gravitational constant function of PFGSA is different, while a piecewise function is adopted instead of the original exponential function. The parameter identification of synchronous generators with three-order model is a complicated work, and several papers have studied this problem specially [27,28]. In these studies, the system is excited by pseudo-random binary sequence and the system is an open-loop system, that is the reason that the high-order model could be identified successfully. In this paper, we focus on the parameter identification of the closed-loop control system of AVR, and we excite this system by a step input value, which often occur in the control process. In this situation, the high-order AVR system could not be identified effectively.

However, from Table 10, it is found that the identifications for the three parameters, \( \tau_d \), \( M \) and \( D \), have not achieved the desired results, although the overall identification performance of PFGSA is acceptable. The results will be discussed in the following.

### 5.4. Discussion on experimental results

From the experimental results above, we find PFGSA show obvious advantage over other typical heuristic search algorithms in parameter identification for AVR system. No matter nonlinear factor or sampling noise considered, the proposed algorithm performs the best on term of optimal objective function value and identification accuracy. GSA and some variants of GSA have already showed excellent searching ability over traditional heuristic optimization method, like GA and PSO [21–23], due to its unique searching mechanism inspired by law of gravity. Here, we need to compare and analyze the improvement of PFGSA over traditional GSA. Compared with the traditional GSA, the gravitational constant function of PFGSA is different, while a piecewise function is adopted instead of the original exponential function. The piecewise function is more flexible to control the decreasing rate of gravitational constant. By controlling the gravitational constant, the searching mode of GSA could be adjusted. Compared with traditional GSA, PFGSA decreases the gravitational constant at a large rate at the first stage, which makes the algorithm reduce the searching range rapidly. At the second stage, decreasing rate of gravitational constant of PFGSA is smaller, which makes the algorithm more easily get close to the global optima gradually. By comparing the iteration process of GSA and PFGSA in Figs. 5 and 8, it is found that the convergence rate and convergence precision of PFGSA have been improved compared with GSA.

In the above experiments, we have compared the performances of different methods on identifying AVR system with first-order generator model and that with three-order generator model. It is found that AVR system with three-order generator model is more difficult to identify. We could deduce the reason and give some solutions on the parameter identification of AVR system. In our experiments, the closed-loop system is excited by a step signal, the system parameters could be identified by the proposed method for AVR system with first-order generator model. However, the parameters of generator of AVR system with three-order generator model could not be identified effectively, the reason may be that the state of the system, especially the generator section, has not be excited fully by the step signal. The parameter identification of synchronous generators with three-order model is a complicated work, and several papers have studied this problem specially [27,28]. In these studies, the system is excited by pseudo-random binary sequence and the system is an open-loop system, that is the reason that the high-order model could be identified successfully. In this paper, we focus on the parameter identification of the closed-loop control system of AVR, and we excite this system by a step input value, which often occur in the control process. In this situation, the high-order AVR system could not be identified effectively.

### 6. Conclusion

In this paper, a piecewise function based gravitational search algorithm is proposed and applied in the parameter identification of AVR system. In order to improve the performance of identification, a new weighted objective function is proposed in the identification strategy. Comparative simulation experiments are conducted to validate optimization performance of PFGSA and the effectiveness of the proposed identification method. The results show that the proposed objective function is more suitable in solving this problem of parameter identification of AVR system. Furthermore, the results verify that GSA possesses superiority over GA and PSO in handling the optimization problem in parameter identification. Results also show the searching ability of PFGSA has been improved obviously over GSA by designing new gravitational constant function, while the PFGSA possesses the best results with high accuracy and stability in parameter identification experiments. Generally, the proposed objective function and the improvement on GSA are effective, and the proposed identification approach achieves desired results in parameter identification of AVR system.

In this paper, the convergence of PFGSA have been proved by plenty experiments. As a swarm based optimization algorithm, the theoretic convergence proof could be deduced, as for the convergence proof of the standard GSA being developed by Ghorbani [29]. On line

**Table 10** Comparison of average identification results of AVR system with three-order generator model.

<table>
<thead>
<tr>
<th>( \theta_i )</th>
<th>PSO</th>
<th>GSA</th>
<th>PFGSA</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_a )</td>
<td>8.5</td>
<td>8.5332</td>
<td>0.0039041</td>
</tr>
<tr>
<td>( r_a )</td>
<td>0.15</td>
<td>0.15484</td>
<td>0.032239</td>
</tr>
<tr>
<td>( K_r )</td>
<td>1.2</td>
<td>1.1075</td>
<td>0.077084</td>
</tr>
<tr>
<td>( r_r )</td>
<td>0.6</td>
<td>0.5297</td>
<td>0.17171</td>
</tr>
<tr>
<td>( K_M )</td>
<td>5.9</td>
<td>9.734</td>
<td>0.68488</td>
</tr>
<tr>
<td>( r_M )</td>
<td>4.74</td>
<td>7.3733</td>
<td>0.55555</td>
</tr>
<tr>
<td>( D )</td>
<td>2</td>
<td>2.6988</td>
<td>0.34946</td>
</tr>
<tr>
<td>( K_1 )</td>
<td>1</td>
<td>1.5669</td>
<td>0.5669</td>
</tr>
<tr>
<td>( r_1 )</td>
<td>0.1</td>
<td>0.00091</td>
<td>0.000102</td>
</tr>
</tbody>
</table>

**Table 11** Comparison of optimal objective function value for identification of AVR system with three-order generator model.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Objective function value</th>
<th>APE</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSO</td>
<td>2.1028</td>
<td>4.0206</td>
</tr>
<tr>
<td>GSA</td>
<td>0.13573</td>
<td>0.47106</td>
</tr>
<tr>
<td>PFGSA</td>
<td>0.0082069</td>
<td>0.45379</td>
</tr>
</tbody>
</table>

Fig. 13. Comparison of original system and system identified by PFGSA for nonlinear AVR system with 25 dB white noise.
parameter identification deserves our attention, because there will be some uncertainties (e.g. parameter change) in the system. In future, we will keep on the study, and the theoretic convergence proof and online parameter of AVR system are the interesting issues.

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References


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