Heterogeneous Constellation Based QOSTBC for Improving Detection Property of QRD-MLD

Chang-Jun AHN, Senior Member and Ken-ya HASHIMOTO, Member

SUMMARY Orthogonal space-time block code (OSTBC) can achieve full diversity with a simple MLD, but OSTBC only achieves 3/4 of the maximum rate if more than two transmit antennas are used. To solve this problem, a quasi-orthogonal STBC (QOSTBC) scheme has been proposed. Even though a QOSTBC scheme can achieve the full rate, there are interference terms resulting from neighboring signals during detection. The existing QOSTBC using the pairs of transmitted symbols can be detected with two parallel MLD. Therefore, MLD based QOSTBC has higher complexity than OSTBC. To reduce the detection complexity, in this paper, we propose the heterogeneous constellation based QOSTBC for improving the detection property of QRD-MLD with maintaining a simple decoding structure.

key words: OSTBC, QOSTBC, MIMO, QRD-MLD, heterogeneous constellation

1. Introduction

Space-time processing with multiple antennas is one of the effective schemes and an ideal candidate for improving the system performance. Many schemes based on space-time processing have been proposed. Space-time coding (STC) has been studied as an effective transmit diversity technique to combat fading without wasting the bandwidth by spatio-temporal structure [1], [2]. STC is classified as either space-time trellis code (STTC) or space-time block code (STBC) [3],[4]. STBC has been gaining more attention than STTC since STBC has simplicity of implementation and decoding. In general, STBC is known as orthogonal STBC (OSTBC). However, the symbol rates of OSTBC for more than two antennas are upper bounded by 3/4 [5]. To solve this problem, a quasi-orthogonal STBC (QOSTBC) scheme has been proposed [6], [7]. Even though a QOSTBC scheme can achieve the full rate, there are interference terms resulting from neighboring signals during detection. The existing QOSTBC using the pairs of transmitted symbols can be detected with two parallel maximum-likelihood decoding (MLD). Therefore, MLD based QOSTBC has higher complexity than OSTBC. To reduce the detection complexity, in this paper, we propose the heterogeneous constellation based QOSTBC for improving the detection property of QRD-MLD while maintaining a simple decoding structure.

2. System Model

We assume that a multiple antenna system with M transmit antennas and N receive antennas. A space-time block code encodes the input systems vector of the length $L$, $s = [s_1, s_2, \ldots, s_L]^T$, into a $K \times M$ matrix $X$ where $K$ is the number of time slots. The code rate is $R = L/K$. The received signal can be defined by

$$Y = XH + W, \quad (1)$$

where $H$ is the $M \times 1$ complex channel matrix, $Y$ and $W$ are $K \times 1$ received matrix and noise matrix, respectively. The entries of $H$ and $W$ are assumed to be independent samples of a zero-mean complex Gaussian random variable with variance 1. The fading channel is quasi-static. The average energy of the symbols transmitted from each antenna is normalized to be 1, so that the average power of the received signal at each receive antenna is $M$.

2.1 QOSTBC Scheme

QOSTBC schemes for achieving the full rate in four transmission antenna systems were independently introduced [6], [7]. They both involved a similar concept, and achieved almost identical BER performance. In these schemes, the encoding matrix columns are divided into two orthogonal groups, where the columns within a group are not orthogonal. For example, the encoding matrix for the four transmission antenna system introduced in [6] is expressed as

$$X = \begin{bmatrix} X_{12} & X_{34} \\ -X_{34} & X_{12} \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3 & -x_4 & x_1 & x_2 \\ x_4 & -x_3 & -x_2 & x_1 \end{bmatrix}. \quad (2)$$

Here, the encoding matrix has two copies of the $2 \times 2$ Alamouti block code with symbols $x_1, x_2$ in the diagonal block, and two copies of the Alamouti code with symbols $x_3, x_4$ in the off-diagonal block.

Let us define the received signal vector $Y$, the encoding matrix $X$, and the noise vector $W$. Then the received signals can be written as

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3 & -x_4 & x_1 & x_2 \\ x_4 & -x_3 & -x_2 & x_1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}. \quad (3)$$
where $W = [w_1 \ w_2 \ w_3 \ w_4]^T$. Assuming a flat fading channel over four times slots and a single receive antenna, the received signals are expressed by (3). Analyzing the decoder performance, the Grammian matrix is essential for the error performance

$$G = X^H X = \begin{bmatrix} \alpha & 0 & 0 & \beta \\ 0 & \alpha & -\beta & 0 \\ 0 & -\beta & \alpha & 0 \\ \beta & 0 & 0 & \alpha \end{bmatrix},$$

where $X^H$ is the Hermitian of the matrix $X$, $\alpha = \sum_{n=1}^4 |x_n|^2$, $\beta = (x_1 x_4^* + x_3 x_4^*) - (x_2 x_3^* + x_4 x_1^*)$. $\beta$ represents interference terms by neighboring signals. Interference terms $\beta$ in the detection matrix result in substantial performance degradation, thus, a more complex decoding method is required. Moreover, the minimum rank of difference matrix $(X - \hat{X})$ over all pairs of distinct codeword matrices $X$ and $\hat{X}$ is as large as possible; The minimum of the product of all the nonzero eigenvalues of matrix $(X - \hat{X})^H (X - \hat{X})$ over all pairs of distinct codeword matrices $X$ and $\hat{X}$ is as large as possible [3, 8]. Clearly, when a space-time code has full diversity (or full rank), i.e., any difference matrix of any two distinct codeword matrices has full rank, the product of all the nonzero eigenvalues of matrix $(X - \hat{X})^H (X - \hat{X})$ in the diversity product criterion is the same as the determinant $\det (X - \hat{X})^H (X - \hat{X})$. From (4), the minimum rank of $(X - \hat{X})$ is 2. It means that the encoding matrix (1) does not have full diversity (diversity order of 2) which is connected to the slope of the BER-SNR curve [9].

2.2 MLD Algorithm

Assuming perfect channel state information is available, the received signal can be detected as

$$\hat{X} = \arg \min |Y - XH|^2 = \min f_{14}(x_1, x_4) + \min f_{23}(x_2, x_3),$$

where $f_{ij}$ is a quadratic form of complex variables $x_i$ and $x_j$, $f_{14}$ and $f_{23}$ can be calculated as

$$f_{14}(x_1, x_4) = \arg \min \left( \sum_{m=1}^4 |h_m|^2 |x_1|^2 + |x_4|^2 \right),$$

$$f_{23}(x_2, x_3) = \arg \min \left( \sum_{m=1}^4 |h_m|^2 |x_2|^2 + |x_3|^2 \right),$$

where $\Re(\cdot)$ is the real part. From (6) and (7), since MLD algorithm needs the joint detection of $x_1$, $x_4$, and $x_2$, $x_3$, it is more complex than OSTBC [6, 7]. In general, the channel coefficients of MIMO are correlated with increasing the number of antennas. Therefore, if the channel coefficients $h_1, h_4$ and $h_2, h_3$ are correlated, MLD shows poor detection property. This is because almost the same replicas might be generated, even if the combination of the transmitted signals are different [10]. Figure 1 shows the example of composite signal constellation of two received signals of $y_1, y_4$ with the correlated channel coefficients. When the received signals have different amplitude and phase, the composite constellation with QPSK has 4 x 4 = 16 points. However, the composite constellation has only 9 points when the received signals have the same amplitude and phase. This becomes a cause of detection errors. To mitigate this problem, constellation rotated QOSTBC has been proposed [9]. However, the complexity is still remained.

2.3 QRD-MLD Algorithm

From (3), the channel matrix is derived by applying a complex conjugate operation to the second and fourth elements of the received signal.

$$\vec{Y} = \begin{bmatrix} y_1 & y_2 & y_3 & y_4 \end{bmatrix}^T = \overline{H} \cdot \overline{X} + \overline{W} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3^* & h_4^* & -h_1^* & -h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} w_1^* \\ w_2^* \\ w_3^* \\ w_4^* \end{bmatrix},$$

where $y_k$ is the received signal in the $k$th time slot, $w_k$ is the complex Gaussian noise added in the $k$th time slot, $(\cdot)^T$ is the transpose matrix, and $h_m$ is the channel coefficient between the $m$th transmit antenna and the receiver. Suppose a matrix $Q$ is decomposed as $\overline{H} = QR$. The QR decomposition is a coordinate rotation that left multiplies the vector $\vec{Y}$ by $Q^H$ to produce a sufficient statistic

$$Z = Q^H \cdot \vec{Y} = R \cdot \overline{X} + \hat{W},$$

where $\hat{W} = Q^H \overline{W}$ and $Z = [z_1 \ z_2 \ z_3 \ z_4]^T$. Since $R$ is an upper triangular matrix, the detected signal can be obtained with the following M-algorithm,
is higher than \( P_1 \). From (8), the signal expression at the receiver can be transformed as

\[
\bar{\mathbf{y}} = \mathbf{H}'\mathbf{y} = \mathbf{H}'\mathbf{r}\mathbf{x} + \mathbf{H}'\mathbf{W} = \bar{\mathbf{G}}\mathbf{x} + \bar{\mathbf{W}}
\]

where \( \bar{\mathbf{W}} = \mathbf{W}'\mathbf{W} \). Suppose a matrix is decomposed as \( \bar{\mathbf{G}} = \bar{\mathbf{Q}}\bar{\mathbf{R}} \). The QR decomposition is a coordinate rotation that left multiplies the vector \( \bar{\mathbf{y}} \) by \( \bar{\mathbf{Q}}' \) to produce a sufficient statistic

\[
\bar{\mathbf{Z}} = \bar{\mathbf{Q}}' \cdot \bar{\mathbf{y}} = \bar{\mathbf{R}} \cdot \mathbf{x} + \bar{\mathbf{W}},
\]

where \( \bar{\mathbf{W}} = \mathbf{W}'\mathbf{W} \). \( x_3 \) and \( x_4 \) can be easily and accurately detected due to the heterogeneous transmit powers as \( \Theta(\bar{r}_{1,3}), \Theta(\bar{r}_{4,4}) \), since \( \bar{\mathbf{R}} \) is an upper triangular matrix where

\[
\bar{\mathbf{R}} = \begin{bmatrix}
\bar{r}_{1,1} & 0 & 0 & \bar{r}_{1,4} \\
0 & \bar{r}_{2,2} & \bar{r}_{2,3} & 0 \\
0 & 0 & \bar{r}_{3,3} & 0 \\
0 & 0 & 0 & \bar{r}_{4,4}
\end{bmatrix}.
\]

Moreover, the detection of \( x_1 \) and \( x_2 \) is operated with the joint detection of \( (x_1, x_4) \) and \( (x_2, x_3) \) in order to achieve best performance of MLD algorithm. \( x_1, x_2 \) can be detected as

\[
x_1 = \Theta(\bar{z}_1 - \bar{r}_{1,4}x_4), \quad x_2 = \Theta(\bar{z}_2 - \bar{r}_{2,3}x_3) \bar{r}_{2,2}.
\]

Therefore, if \( x_3, x_4 \) are correctly detected, the detection of \( x_1, x_2 \) are also successful detected.

4. Simulation Results

In this section, we provide the simulation results for the proposed algorithm and compare them with the results for ZF, QRD-MLD and full MLD. In all simulations, we consider four transmit antennas and one receive antenna with a quasi-static flat fading channel [6].

Figure 3 shows the BER performance of uncoded, ZF,
QRD-MLD, full MLD and the proposed algorithm with power imbalance of 3.3 dB for QOSTBC. The system structure of uncoded means single input single output (SISO) systems. For the sake of achieving full rate for four transmit antennas, QOSTBC was proposed with loss in achieving full diversity and symbol-wise decoding. QOSTBC only provides a practical diversity of 2 and allow pairwise decoding which causes the increasing of receiver complexity. From the simulation results, ZF and QRD-MLD exhibit 2 dB and 1.5 dB penalty compared with full MLD at the BER of $10^{-5}$. These performance penalties of QOSTBC are smaller that those of OSTBC due to the reduced diversity order. On the other hand, the proposed algorithm shows the same BER of QRD-MLD at low SNR values. However, it offers high detection accuracy at high SNR values; the proposed algorithm shows the same BER performance as full MLD at the BER of $10^{-5}$ with maintaining a simple decoding structure.

Figure 4 shows the BER performance of the proposed algorithm with various transmit power ratios for QOSTBC. If $x_3$, $x_4$ are correctly detected, $x_1$, $x_2$ are also successfully detected. With small power imbalance, the detection of $x_3$, $x_4$ is highly dependent on the channel condition. On the other hand, $x_3$, $x_4$ are successfully detected with high power imbalance, but $x_1$, $x_2$ are poorly detected due to low SNR. For this reason, our proposed algorithm achieves the best performance when the power ratio is fixed at 3.3 dB. In this paper, we use the different transmit power ratio of 3.3 dB

5. Conclusion

In this paper, we have proposed the heterogeneous constellation based QOSTBC for improving the detection property of QRD-MLD. From the simulation results, the proposed algorithm with different transmit power ratio as 3.3 dB shows the same BER performance as full MLD at the BER of $10^{-5}$ while maintaining a simple decoding structure.

References