Distributed Beam Scheduling in Multi-cell Networks via Auction over Competitive Markets

Kai Yang, Doru Calin, Chan-Byoung Chae, and Simon Yiu
Bell Labs, Alcatel Lucent
{kai.yang, doru.calin, chan-byoung.chae, simon.yiu}@alcatel-lucent.com

Abstract—The capacity of a wireless network could be considerably improved by employing directional antennas which are capable of illuminating multiple beams toward different directions. However, more beams from the same BS may lead to stronger inter-cell interference. In this paper, we consider coordinated beam scheduling schemes to mitigate the inter-cell interference. We first formulate this problem as a combinatorial optimization problem. We then reveal that the complexity of this problem hinges upon a single scalar termed as degree of constraint (DoC), which is related to the degree of conflict a beam is subject to. If the DoC is at least 3, the general beam scheduling problem is NP-hard. If DoC is smaller than 3, which corresponds to a relevant subclass of the beam scheduling problem arising in practice, this problem can be solved in polynomial time. We propose an optimal beam switching algorithm based on the auction method to this particular subclass of problem. This algorithm is of low complexity and is well suited for distributed implementation. We then extend the auction algorithm to solve the general multi-cell beam scheduling problem. The performance of the proposed algorithms is finally assessed through extensive simulation studies.

I. INTRODUCTION

A. Motivation

Future wireless networks are envisioned to support high-data-rate transmissions by employing aggressive spatial and frequency reuses. Directional antennas have been shown to be an effective means to increase the throughput of a wireless network. The BS could reuse the whole frequency band to communicate with multiple receivers simultaneously by employing an advanced antenna to radiate different signals toward multiple directions [1]. While multiple beams from the same transmitter has the potential of offering higher throughput, it also brings about stronger interference among neighboring cells. In fact, the inter-cell interference is an major impediment to limit the throughput of a multi-cell system. As a remedy, coordinated beamforming has been promoted as an effective technique to mitigate the inter-cell interference. Closely related technologies are currently being investigated and standardized by the IEEE 802.16m as well as 3GPP partners in long-term-evolution (LTE) release 10 and beyond (LTE-advanced) [1], [2]. It has been shown that the cell throughput could be significantly increased by enforcing beam collaborations among adjacent cells.

The coordinated beamforming schemes can be broadly grouped into two categories, i.e., centralized beam coordination schemes and distributed beam coordination schemes. A centralized beam coordination approach often needs to collect information from all BSs, e.g., the number of users served by each beam, the wireless channel conditions, and traffic conditions, to make a decision on the beam scheduling, i.e., which beam to activate for data transmission. While such centralized schemes could offer good performance in theory, they are in general difficult to implement in practice, as a wireless network is physically distributed and it is practically impossible to collect and process a large amount of real-time information from all BSs across the whole wireless network. Such practical constraints motivate us to seek simple and distributed beam scheduling schemes which only require local message passing between neighboring BSs.

B. Contributions

The purpose this paper is to study the coordinated beam scheduling problem and develop distributed beam scheduling schemes for interference avoidance in a wireless network. To this end, we have made the following contributions.

• NP-hard proof: We formulate the general beam scheduling problem as a combinatorial optimization problem and prove it is NP-hard. In addition, we show that it remains NP-hard for a network with degree of constraint (DoC), i.e., the number of conflicts a beam may suffer from, is at least 3.

• A practically relevant subclass of problem which is easy to solve: We reveal that if the DoC is smaller than 3, then the resultant beam scheduling problem could be efficiently solved.

• Design of distributed optimal Algorithm: We design optimal algorithm for the a subclass of the beam scheduling problem based on auction method. This algorithm is fully distributed and converges both synchronously and asynchronously.

• Design of distributed sub-optimal Algorithm: For the general beam scheduling problem, despite its NP-hardness, we propose a sub-optimal algorithm based on auction over competitive markets. At each step of this algorithm, we obtain a feasible solution and also a dual upper bound to the optimal objective function value of the general beam scheduling problem.

C. Related Work

Beam forming technologies have been used to improve the coverage and throughput of a wireless network for outdoor transmissions [1], [2], [3]. The directional antennas have also been employed for indoor transmissions [4], [5]. It has been shown that in spite of the rich-scattering nature of an indoor channel, the beam forming technology has the potential of significantly increasing the throughput, provided a proper beam scheduling scheme is employed to reduce the interference. Different from the previous works, we study the theoretical computational complexity of the general multi-cell beam scheduling problem. Also, the algorithms we proposed here can be realized in a fully distributed manner, making the practical implementations easier.
The rest of this paper is organized as follows. In Section II, the general beam scheduling problem is formulated as a combinatorial optimization problem, and the complexity of this problem is analyzed. Section III develops low-complexity optimal algorithm for a relevant subclass of the beam scheduling problem, and then extend it to the general beam scheduling problem. The simulation results are presented and discussed in Section IV. We conclude this paper in Section V.

II. PROBLEM FORMULATION

A. Interference Model

Consider now a multi-cell wireless network consisting of B BSs. We assume the antenna of the bth BS is capable of forming a number of Eb beams toward different directions using a pre-determined power level. We further assume that due to the limited total power budget at most Mb beams from the bth BS can be activated simultaneously. A pair-wise binary interference model is employed to capture the interference between different wireless links. Such an approach determines whether a pair of simultaneous transmissions from two BSs can be successfully received by measuring the received signal-to-noise-plus-interference-ratio (SINRs) at respective receivers and comparing the measured SINRs with given thresholds. The threshold could differ for different transmissions. Also, the SINR measurement process can be carried out in either synchronous or asynchronous manner [6], [7], and such measurements can be performed periodically to capture the dynamics of the channel, e.g., mobility of mobile users. The binary interference model has been widely used to capture the interference between wireless nodes equipped with omni-directional antenna [8], [9] and has been recently extended to characterize the interference of a wireless network with directional antennas [4]. Let αbi denote a 0-1 indicator variable for the ith beam of the bth BS such that it equals to 1 if this beam is active. rb indicates the weight of activating this beam, which is determined by a collection of factors such as the channel quality and the traffic condition of this beam. Let beam bi denote the ith beam of the bth BS. A group of conflict beams (CB) B(j) = {b1...bj}, 1 ≤ j ≤ J, is defined as a set of beams which are mutually interfered with each other, where J is the total number of CBs. The multi-cell weighted beam scheduling problem can be formulated as follows.

B. Complexity Analysis of the Beam Scheduling Problem

For notational simplicity, above beam scheduling problem is defined as general beam scheduling (GBS) problem. In this section we analyze the complexity the GBS problem and show it is NP-hard. In addition to this, we prove that an important subclass of the GBS problem can be efficiently solved. A common practice to prove an optimization problem is NP-hard is to find an independent set with maximum sum weight.

Definition 1 Single-Beam Scheduling (SBS) problem: An SBS problem is defined as a GBS problem in which each BS is capable of only illuminating one beam, e.g., omni-beam.

Definition 2 Maximum-weight Independent Set (MWIS) problem: Given an undirected graph \( G = (V, E) \). Each node is associated with a positive scalar \( s_e \). An independent set is defined as a set of mutually disjoint nodes. The MWIS problem is to find an independent set with maximum sum weight.

Definition 3 Degree of Constraints (DoC): Notice that in the beam scheduling problem in (1) \( \alpha_{bj} \) is a 0-1 indicator variable for the ith beam of the bth BS. Each indicator variable might be associated with more than one constraint in (1). Let \( \omega(\alpha_{bi}) \) denote the number of constraints associated with \( \alpha_{bi} \). We define \( \omega(\alpha_{bi}) \) as the DoC of the ith beam of the bth BS.

Theorem 1 Both GBS and SBS problem are NP-hard. They remain NP-hard even DoC is 3.

Proof: We show this by converting any MWIS problem into an SBS problem. Give any undirected graph \( G = (V, E) \). We use a BS to represent each vertex. Two BSs are adjacent, i.e., BSs that are subject to mutual interference if the corresponding vertices are connected by an edge. The weight of illuminating a BS is set as the weight of corresponding vertex in \( G \). Likewise, we can convert any SBS problem into a corresponding MWIS problem. Hence, SBS problem is NP-hard. It follows that GBS is also NP-hard, as SBS is a restricted instance of GBS problem. In addition, it has been proved that MWIS problem is NP-hard for a graph with maximum degree 3 [11], [12], which implies that general GBS problem remains difficult to solve even under additional constraints that a BS has one beam and only interferes with at most 3 other BSs.

C. Restricted Beam Switching Problem

Due to the NP-hardness of the GBS problem, it seems unlikely we could develop any efficient algorithm which is capable of optimally solving the GBS problem. One question we would like to ask is that: is there any practically relevant subclass of GBS problem which could be efficiently solved? In this section, we give a sufficient condition under which the GBS problem can be efficiently solved.

Definition 4 b-matching problem: Consider an undirected graph \( G = (V, E) \), each edge \( e \in E \) is associated with a weight \( s_e \), and there is a degree constraint \( \xi_i \) for the ith vertex in V. A b-matching M is defined as a sub-graph of G such
that for the \(i^{th}\) vertex in \(G\), the number of incident edges in \(M\) is no larger than \(\zeta_i\). The maximum weighted \(b\)-matching is defined as the \(b\)-matching with maximum sum weight.

**Theorem 2** If the DoC of a GBS problem is no larger than two, then the beam scheduling problem in (1) can be solved in polynomial time. In particular, the DoC of each beam is at most 2 if the interference constraints between beams meet the transitive condition, i.e., if beam \(A\) interferes with beam \(B\), and beam \(B\) interferes with beam \(C\), then beam \(A\) is subject to mutual interference with beam \(C\).

**Proof:** The key idea of the proof is that if the DoC of each beam is at most 2, we can use a \(b\)-matching graphical model to characterize the GBS in (1). To see this, we use an undirected graph \(G = (V, E)\) to model (1), as shown in Figure 1 and Figure 2. If the DoC of each beam is at most 2, we can use an edge to represent this beam, and each vertex corresponds to one constraint in the GBS problem. All beams subject to the same constraint are connected to one vertex. The weight associated with each edge equals to the weight achieved by activating the corresponding beam. Hence the GBS under the DoC-2 constraint can be modeled as a \(b\)-matching problem, which can be solved in polynomial time [13]. In particular, if the interference constraints meet the transitive condition, we can prove that every beam only belongs to a single CB (it is possible that a CB only contains one beam). Assume otherwise, i.e., there exist one beam \(i\) which belongs to both CB1 and CB2. Since CB1 and CB2 are different CBs, at least one beam in CB1 does not interfere with at least one beam in CB2. However, since both CB1 and CB2 contain beam \(i\), it follows from the transitive condition that all beams in CB1 are subject to mutual interference to all beams in CB2, which contradicts the assumption and complete the proof. Hence, the illumination of each beam is subject to only two constraints, i.e., a maximum beam number constraint and an interference constraint.

![Graph illustration of a three cell wireless network, each BS is capable to forming three beams toward different directions](image)

Fig. 1. Graph illustration of a three cell wireless network, each BS is capable to forming three beams toward different directions

To the best of our knowledge, Theorem 2 is the first result on the sufficient condition under which the GBS problem is polynomial time solvable. In an outdoor environment, such a sufficient condition can be satisfied if multiple beams from the same BS are well separated, i.e., no strong inter-beam interference for beams from the same BS and beams from different BSs are properly aligned with each other, as shown in Figure 1.

**III. DISTRIBUTED LOW-COMPLEXITY BEAM SCHEDULING ALGORITHMS**

**A. Auction Algorithms for RBS Problem**

In this section, we show that the RBS problem can be viewed as an assignment (matching) problem. In an assignment problem [14], [15], we aim to assign \(K\) objects to \(N\) users. The assignment problem is called asymmetric assignment problem if \(K \neq N\), otherwise it is defined as a symmetric assignment problem. Also, each user \(b\) is associated with a set of objects \(T(b)\), and the \(j^{th}\) object is associated with a group of users \(U(j)\). There is a reward of assigning the \(j^{th}\) object to the \(b^{th}\) user. Since we assume one BS can illuminate at most \(M_b\) beams at the same time instance, the corresponding RBS problem can be viewed as an assignment problem. In this case, each BS corresponds to a collection of \(M_b\) users and each CB represents an object. For notational simplicity, we generate \(M_b\) virtual BSs (VBS) for each BS. As a consequence each VBS could illuminate at most one beam at a time. The \(b^{th}\) VBS (user) is associated with \(T(b)\) CB (objects). Likewise, all VBSs with one beam in the \(j^{th}\) CB constitutes the set \(U(j)\). The reward of assigning the \(b^{th}\) VBS to the \(j^{th}\) CB is equal to \(w_{bj}\). The total number of users and objects are \(B\) and \(J\) respectively, where \(B = \sum_{b=1}^{B} M_b\). If we use a 0-1 indicator variable \(\beta_{bj}\) to characterize the assignment of the \(j^{th}\) object to the \(b^{th}\) user, we can compactly represent the RBS problem in the sequel.

\[
\text{maximize } \sum_{b} \sum_{i} w_{bi} \beta_{ij} \\
\text{subject to } \sum_{j \in T(b)} \beta_{bj} \leq 1, \forall b, \\
\sum_{b \in U(j)} \beta_{bj} \leq 1, \forall j, \\
\text{variables } \beta_{bj} \in \{0, 1\}.
\]

The first and second group of constraints in the above formulation correspond to the maximum beam number constraints and interference constraints respectively. The auction algorithm is an efficient means to solve the assignment problem. It is inspired by the process of auction in the real world and has shown impressive performance in solving various assignment problems [14], [15]. The algorithm proceeds in a multi-stage manner. Each stage consists of two steps, i.e., bidding step and assignment step. In the first stage, users bid for object by comparing the profit of obtaining an object, i.e., the quantity obtained by subtracting the reward of assigning an object to this user from the price of the object [c.f. step a and step b in Algorithm 1]. After this comparison, the assignment with highest profit is carried out, and the price for this assigned object is adjusted [c.f. step c and step d in Algorithm 1].
Without loss of generality, we consider the case in which there are more objects than users. In addition, the terms VBS and user, as well as CB and object are used interchangeably throughout this section. The assignment is considered partial if any VBS remains unassigned. Also, a price variable \( p_j \) is introduced for the \( j^{th} \) CB. A VBS is \textit{happy} if it is either assigned to an object \( j_0 \) which satisfies the \( \epsilon \)-complementary slackness (CS), i.e.,

\[
    w_{b,j_0} - p_{j_0} \geq \max_{j \in U(b)}\{ w_{b,j} - p_j \} - \epsilon,
\]

or it is not assigned to any object but still satisfies a generalized (CS) condition, i.e.,

\[
    \max_{j \in U(b)}\{ w_{b,j} - p_j \} \leq \epsilon.
\]

Otherwise, a VBS is \textit{unhappy}.

\textbf{Algorithm 1 : Distributed Auction for RBS problem}

1) Initialization: Set the object price vector \( p = [p_1, ..., p_J]^T \) as an all-zero vector. Set a positive scalar \( \epsilon \). Assume all VBSs are unhappy VBSs.

2) \textit{While not all VBSs are happy}

a) Each unhappy VBS \( b \) searches over all CBS in the set \( T(b) \) and chooses the CB \( j_b \) with maximum profit \( \eta_{b,j_b} = \max_j \{ w_{b,j} - p_j \} \). If VBS \( b \) satisfies the generalized CS condition, i.e., \( \eta_{b,j_b} \leq \epsilon \), set it as a happy user. Otherwise, the same VBS also calculates the second maximum profit \( \eta_{b,j} = \max_{j \neq j_b} \{ w_{b,j} - p_j \} \).

b) Each unhappy VBS \( b \) bids for CB \( j_b \) by sending the price \( p_{j_b} = p_{j_b} + \theta_{b,j_b} - \epsilon \).

c) After receiving all bids, each CB \( j \) calculates its maximum bids and assign itself to the maximum bidder \( b_j \). If CB \( j \) has been assigned to another VBS \( b \) before, remove that assignment.

d) Set each VBS \( b_j \) as an happy VBS.

3) \textit{End while}

The convergence property of the parallel auction algorithm is established in the following theorem.

\textbf{Theorem 3} Assume the RBS problem is feasible, the parallel auction algorithm is guaranteed to converge to a solution within \( O(\max(B, J)\epsilon) \) from the optimal objective function value in \( O(\max(B, J)\max(w_{b,j})) \) iterations.

\textbf{Proof:} To see this, we use the theorem on the optimality properties of the auction algorithm for the symmetric assignment problem from [14] (proposition 1), i.e., for a symmetric assignment problem with \( N \) users and \( N \) objects, any complete assignment satisfying the \( \epsilon \)-CS condition leads to an objective function value at most \( N\epsilon \) from the optimal objective function value. The basic technique we use for proof is to show that we can convert the solution obtained by the parallel auction algorithm into a complete assignment of a corresponding symmetric assignment problem, and such a complete assignment satisfies the \( \epsilon \)-CS condition. Note that Algorithm 1 may terminate with a partial assignment \( S_1 \), i.e., there exists at least one VBS which does not activate any beam. However, we can introduce one dummy beam with zero reward and price, i.e., \( w_u = 0 \) and \( p_u = 0 \), for each VBS. Any unassigned VBS can then be allocated to this dummy beam, leading to a new assignment \( S_2 \). Since the unassigned VBS is happy, it must satisfy the generalized \( \epsilon \)-CS condition. If we assign it to a dummy beam, such an assignment must satisfy the \( \epsilon \)-CS condition, as

\[
    \max_{j \in U(b)}\{ w_{b,j} - p_j \} \leq \epsilon - \epsilon = 0 = w_{b,\emptyset} - p_{\emptyset}.
\]

Hence, all users are assigned in \( S_2 \) and \( S_2 \) satisfies the \( \epsilon \)-CS condition. Likewise, for any unallocated beam, we can introduce dummy users with zero reward and price and assign the unallocated beam to a dummy user, leading to the assignment \( S_3 \). Following a similar approach, we can show that \( S_3 \) also satisfies the \( \epsilon \)-CS condition. By introducing the concepts of dummy users and dummy beams, we can obtain a complete assignment \( S_4 \) and it satisfies the \( \epsilon \)-CS condition. In addition, the assignment \( S_1 \) and \( S_4 \) have identical objective function value since the reward and price of both dummy users and objects are zero, which shows that the assignment \( S_1 \) can obtain an objective function value at least \( \max\{B, J\}\epsilon \) from being optimal. We next analyze the computational complexity of the parallel auction method. We employ similar techniques as used in Proposition 1 of [14] to understand the number of iterations required for the parallel auction algorithm to converge. Notice that the price variable is non-decreasing throughout the iterations of Algorithm 1. Also, at the beginning of this algorithm each price value is set to zero, and upon convergence the value of each price variable is upper bounded by \( \max(w_{b,j}) \). Combining the fact that at each iteration, at least one price value changes at an amount no smaller than \( \epsilon \) [cf. step b of Algorithm 1], we finally conclude that Algorithm 1 converges in \( O(\max(B, J)\max(w_{b,j})) \) iterations.

Note that similar to other auction algorithms [14], we can tradeoff the optimality and converge speed of Algorithm 1 by tuning the parameter \( \epsilon \), leading to flexible implementations.

\textbf{B. Distributed Auction Algorithm over Competitive Markets}

In practice, the transitive condition given in Theorem 2 could be violated due to, for example, the rich-scattering nature of the wireless transmission environment or different beam shapes of neighboring BSs. In this section, we propose a modified auction algorithm to solve the GBS problem. At each stage of this algorithm, we obtain a feasible solution as well as an upper bound to the optimal objective function value of the GBS problem. This upper bound could be used to quantify the suboptimality of the obtained solution. Let \( D(b, j) \) denote the number of interference constraints the beam \( (b, j) \) is associated with in the GBS problem (1). For any beam with \( D(b, j) > 1 \), we introduce a group of virtual beams \( (b, j, \ell) \), \( 1 \leq \ell \leq |D(b, j)| - 1 \). Each virtual beam is associated with an indicator variable \( \beta_{b,j,\ell} \). Also, for notational simplicity, the original beam \( (b, j) \) is redefined as beam \( (b, j_0) \). The weight of virtual beams \( (b, j_\ell) \) is set as zero. The role of virtual beams is to convert the original GBS problem into a group of assignment problems excluding the constraints between one beam and all its virtual beams. As an example, assuming the constraints in (1) is given by \( \alpha_{11} + \alpha_{21} + 1 \), \( \alpha_{12} + \alpha_{22} \leq 1 \), \( \alpha_{21} + \alpha_{22} \leq 1 \). By introducing a virtual beam \((2, 1, 0)\), the above constraints could be rewritten as \( \alpha_{11} + \alpha_{21} + \alpha_{22} \leq 1 \), \( \alpha_{22} + \alpha_{21} \leq 1 \), \( \alpha_{12} + \alpha_{21} \leq 1 \), \( \alpha_{12} + \alpha_{21} \leq 1 \). Notice that if we exclude the equality constraint, beam \((2, 1, 0)\) and...
beam \((2,1_3)\) can be viewed as objects over two independent markets. Recall that \(T(b)\) denote all beams associated with the \(b^{th}\) VBS, and \(U(j)\) is the set of all VBSs associated with beam \(j\). The GBS problem can then be reformulated as follows,

\[
\begin{align*}
\text{maximize} & \quad \sum_{b} \sum_{j} w_{bj} \beta_{bj}, \\
\text{subject to} & \quad \sum_{j \in T(b)} \beta_{bj} \leq 1, \forall b, \\
& \quad \sum_{j \in U(j)} \beta_{bj} \leq 1, \forall j, \\
& \quad \beta_{bj} = \beta_{b0}, \forall 1 \leq \ell \leq D(b,j), \\
\text{variables} & \quad \beta_{bj} \in \{0,1\}.
\end{align*}
\]

We then introduce a group of dual variables and move the equality constraints into the objective function to obtain a partial Lagrange problem in the sequel,

\[
\begin{align*}
\text{maximize} & \quad \sum_{b} \sum_{j} w_{bj} \beta_{bj} + \sum_{j} \lambda_{j} (\beta_{bj} - \beta_{b0}) \\
\text{subject to} & \quad \sum_{j \in T(b)} \beta_{bj} \leq 1, \forall b, \\
& \quad \sum_{j \in U(j)} \beta_{bj} \leq 1, \forall j, \\
\text{variables} & \quad \beta_{bj} \in \{0,1\}.
\end{align*}
\]

Notice that the above problem can be decomposed into a collection of subproblems, and each subproblem can be represented as an assignment problem. Consequently, we can employ the auction algorithm to solve each subproblem. For given \(\{\lambda_{j}\}\), let \(L(\lambda)\) denote the optimal objective function value achieved by (7). All these subproblems are coordinated by a high-level master problem given by,

\[
\begin{align*}
\text{maximize} & \quad \lambda_{0} L(\lambda), \\
\text{subject to} & \quad \lambda_{0} = 0, \\
\text{variables} & \quad \lambda_{j} \geq 0.
\end{align*}
\]

where \(\lambda \geq 0\) denotes component-wise non-negativity. Here the vector \(\lambda\) can be interpreted as a group of pricing variables balancing the auctions over a group of competitive markets. A fully distributed subgradient method can be invoked to solve (8). Let \(g(\lambda)\) denote the subgradient of \(L(\lambda)\). For \(\lambda_{0j}\), the corresponding component in the subgradient function \(g(\lambda)\) is given by \(\beta_{b0}^* - \beta_{b0}^j\), where \(\beta_{b0}^*\) is the optimal solution to (7). After obtaining the subgradient, the pricing vector at next iteration is given by \(\lambda^{k+1}\) is calculated as \(\lambda^{k} - \gamma_k g(\lambda^{k})\), where \(\gamma_k\) is a stepsize. Common choices of the stepsize include the diminish stepsize and constant stepsize [16].

**Algorithm 2 : Auction Algorithm over Competitive Markets**

1) **Initialization**: Generate \(D(b,j) = 1\) virtual beams for each beam \(D(b,j) \geq 2\). Set the number of iterations \(k = 0\). Set the maximum number of iterations \(k_{max}\).

2) **Repeat**

a) \(k \rightarrow k + 1\).

b) **Carry out the auction algorithm (Algorithm 1)** to obtain the optimal solution to the assignment problem in each individual market, i.e., problem (7). Assume the optimal solution is \(\beta^* = \{\beta_{b0}^j\}\). Calculate the dual objective function \(U_D(k)\) by substituting \(\beta^*\) into the objective function of (7).

c) Let \(C(b,j_o)\) denote the set of beams interfering with beam \((b,j)\). Beam \((b,j_o)\) communicates with all its virtual beams. If \(\beta_{b0}^* = \beta_{b0}^j\), \(\forall \ell\), stop. Otherwise, using the following greedy approach to recover a feasible solution to the GBS problem.

i) Let \(\beta_{b0}^* = \max \beta_{b0}^j\). Also, the beam \((b,j)\) is defined as active beam if \(\beta_{b0}^* = 1\).

ii) For any active beam \((b,j_o)\), if the the reward of illuminating this beam, i.e., \(w_{b0j_o}\), is less than that of any active beam in \(C(b,j)\), set \(\beta_{b0j_o} = 0\); otherwise, set \(\beta_{b0j_o} = 1\) and set \(\beta_{b0j_o} = 1 - \beta_{b0j_o}\), \(\forall \ell\), and \(\beta_{b0j_o} = 1 - \beta_{b0j_o}\), \(\forall \ell\). If \((b,j) \in C(b,j)\).

iii) Assume the solution obtained by the greedy approach is \(\{\beta_{b0j}\}\). Calculate \(U_D(k) = \sum_{j} w_{bj} \beta_{bj}\).

3) **Until** (the duality gap is smaller than a pre-determined threshold i.e., \(U_D(k) - U_P(k) \leq D_p\) or \(k \geq k_{max}\)).

The above algorithm can be performed in a fully distributed manner. In addition, let \(U^*\) denote the optimal objective function value to the GBS problem. We have \(U_D(k) \leq U^* \leq U_P(k)\). The first part of this inequality holds because \(\{\beta_{b0j}\}\) is a feasible solution to the GBS problem. The second inequality comes directly from the theorem of weak duality [16].

**IV. SIMULATION STUDIES**

Simulation examples are presented in this section to illustrate the performance of proposed algorithms. Unless we state otherwise, the weight of a beam is calculated as the product of its capacity and the queue length, which equals to the sum of data packets of associated users. The number of users served by each beam is randomly generated between 1 to 3, and the data packets of each user is randomly generated between 1 to 10. The capacity of each beam is randomly generated and ranges from 12.5 to 100 Mbps, which corresponds to the peak downlink transmission rate of an SISO LTE system with 20 MHz bandwidth [17].

We first consider a seven-cell cellular network, each BS is capable of illuminating six equally spaced beams toward different directions, as shown in Figure 3. Also, the weight of each beam is normalized by the maximum beam weight. Each BS could activate up to four beams for data transmission simultaneously. Maximum number of iterations in the auction algorithm is set as 10 and \(c\) [c.f. Step 1 in Algorithm 1] is set as 0.1. The DoC of this wireless network is 2. Hence the beam scheduling problem can be optimally solved.

The performance comparison between the auction algorithm and greedy method is presented in Figure 4. It is compared with the greedy algorithm employed in [4]. A total of fifteen realizations of randomly generated examples are evaluated. The optimal objective function value is obtained by converting the asymmetric assignment problem into a symmetric assignment problem and solve its linear programming relaxation [15]. The sum weight achieved by both algorithms are normalized by the optimal objective function value. It is seen the proposed parallel auction algorithm can achieve a close-to-optimal solution after a few iterations and perform much better than the greedy algorithm. This auction algorithm is of low complexity and requires only local message passing between
adjacent BSs, which makes it much easier to implement in practice.

We next consider a cellular network shown in Figure 5. There are two types of BSs in this topology. A group of six beams are associated with the first type of BSs (BS a, b, c, e, g). We also assume this type of BSs could illuminate at most four beams simultaneously. The second type of BSs (BS d and f) are equipped with an omni-directional antenna. The sufficient condition in Theorem 2 is not satisfied in this topology. Figure 6 shows the performance of the proposed auction algorithm over competitive methods. It is seen that in spite of the irregular structure of this topology, the proposed algorithm could achieve near-optimal performance.

V. CONCLUSIONS

Coordinated beam scheduling plays an important role in interference mitigation of multi-cell wireless networks. In this paper, we have shown that under the binary interference model, the general beam scheduling problem is NP-hard. We have revealed a sufficient condition under which the general beam scheduling problem can be efficiently solved. A low-complexity optimal beam scheduling algorithm has been developed to solve this subclass of beam scheduling problem, which demands only local message passing among adjacent BSs. We have also proposed an auction algorithm over competitive markets to efficiently achieve a near-optimal solution to the general beam scheduling problem.

REFERENCES