Analytical Sub-band Filtering Technique of Two-Dimensional Adaptive Notch Filter

N. Piyachaiyakul and C. Charoenlarpnopparut

Abstract—A Two-Dimensional (2-D) Notch Filter is typically used for removing sinusoidal interferences from the a 2-D broadband image. An adaptive algorithm is subsequently applied to the filters in order to provide more flexibility to cope with multiple unknown interferences. Later on, a sub-band filtering technique is introduced to achieve decent convergence times. This work mainly focuses on an analysis of sub-band filtering methods. On previous work, a conventional 2-D FIR filter is used and the number of sub-band is not clearly recommended. Therefore, this work aims to provide other alternative methods of sub-band filtering and to clarify the issues related to the number of sub-bands. The simulation results is also presented including analysis of each method.

I. INTRODUCTION

A notch filter is basically used to eliminate sinusoidal interferences, often called coherent noise, from broadband signals in the communication fields [1], [2], [3]. Many researchers have purposed various algorithms of notch filter to remove those interferences [4], [5], [6], [2], [7]. This also includes an adaptive algorithm [8], [9], [10], [11], [12]. In 2-D case, a few algorithms have been proposed during recent years. A non-separable form of 2-D notch filter was initially introduced by Pei, et al. [2]. An adaptive approach was added to the system by Hinamoto, et al. [3]. Then a simplified version is subsequently invented by Pei, Wu, and Ding [13]. To achieve fast convergence time, Piyachaiyakul and Charoenlarpnopparut introduce a sub-band filtering technique with 2-D downsampler in each sub-band [14].

II. BACKGROUND

This section provides general ideas on how 2-D notch filter is designed and finally implemented. The conventional adaptive algorithm is also presented together with conventional sub-band filtering technique.

A. Conventional 2-D Notch Filter

The frequency response of an ideal 2-D notch filter is given by:

\[
H_0(e^{j\omega_1}, e^{j\omega_2}) = \begin{cases} 
1, & (\omega_1, \omega_2) = (\omega_1', \omega_2'), \\
0, & (\omega_1, \omega_2) \neq (\omega_1', \omega_2') \end{cases},
\]

where \((\omega_1', \omega_2')\) is notch frequency. By using a Singular Value Decomposition (SVD) [7], a desired frequency response of a 2-D notch filter can be expanded and approximated in terms of 1-D filters as:

\[
H(z_1, z_2) = 1 - \frac{1}{2} H_{b_1}(z_1) H_{b_2}(z_2) [1 - H_{a_1}(z_1) H_{a_2}(z_2)],
\]

where

\[
H_{b_i}(e^{j\omega_i}) = \frac{1}{2} \frac{b_{i2} - b_{i1} z_i^{-1} + z_i^{-2}}{2(1-b_{i1} z_i^{-1} + b_{i2} z_i^{-2})},
\]

\[
H_{a_i}(z_i) = \frac{a_i + z_i^{-1}}{1 + a_i z_i^{-1}}, \quad i = 1, 2,
\]

\[
a_i = \frac{\sin(\frac{\omega_i}{2})}{\sin(\frac{\omega_i}{2} + \frac{\pi}{4})},
\]

\[
b_{i1} = \frac{2 \cos(\omega_i^*)}{1 + \tan(\frac{BW}{2})},
\]

\[
b_{i2} = \frac{1 - \tan(\frac{BW}{2})}{1 + \tan(\frac{BW}{2})}.
\]

With the mentioned transfer function, a 2-D notch filter is successfully achieved by specifying notch frequencies to the system. However, the system still cannot operate in unknown interference situation.

B. Conventional 2-D Adaptive Notch Filter

In order to deal with unknown interference situation, adaptive approach need to be applied. Hinamoto, et al. realized that a bandpass term \(H_{b_i}\) is actually an inverse of 1-D notch filter [3]. As a consequence, the bandpass term is replaced by an adaptive version of 1-D notch filter which is given by:

\[
G_i(e^{j\omega_i}) = \frac{1 + 2 \gamma_i z_i^{-1} + z_i^{-2}}{1 + a_{0i}(1 + a_{1i}) z_i^{-1} + a_{1i} z_i^{-2}},
\]

where

\[
\gamma_i = -\cos(\omega_i^*)
\]

\[
a_{0i} = \frac{2 a_i \gamma_i}{1 + a_i^2}
\]

\[
a_{1i} = a_i^2.
\]

As a result of this approach, the system can deal with a single unknown sinusoidal interference. However, several interferences still causes a trouble to the system.

C. Conventional 2-D Adaptive Notch Filter with Sub-band Filtering Technique

Piyachaiyakul and Charoenlarpnopparut solve this problem of multiple interferences by introducing a sub-band filtering technique [14]. This technique distributes multiple sinusoidal interferences into many sub-band by assuming that one interference per one sub-band. In each sub-band, a 2-D
downsamplers is applied to improve a convergence time. The structure of the system is shown in Fig. 1.

An conventional adaptive notch filter is then applied in each sub-band in order to track a notch frequency. The result of this state is just an intermediate version of notch frequency. This output frequency is subsequently converted back to an actual notch frequency by using:

\[
\omega_{l1}' = \frac{\omega_{h1}}{2}, \quad (7)
\]

\[
\omega_{h1}' = \frac{2\pi - \omega_{h1}}{2}. \quad (8)
\]

If the intermediate notch frequency is below 0.5\(\pi\), Eq. 7 will be selected. Apart from those range, Eq. 8 is chosen instead. These actual notch frequencies are then used to implement final notch filters. Although, fast convergence time and more flexibilities are greatly achieved, there are still some ambiguous situations in conversion from intermediate to actual notch frequencies. The number of sub-bands is not sufficient to identify whether Eq. 7 or Eq. 8 will be used.

### III. NEW RESULTS

This section aims to provide the analysis of three alternative methods and introduce the solution to identify the ambiguous situations by changing the sub-band filtering method.

#### A. Diamond Sub-band Filtering

This method is applied based on the first norm criterion which is given by:

\[
\omega_c = |\omega_1| + |\omega_2|. \quad (9)
\]

\(\omega_c\) denotes a cut-off frequency. In this work, \(\omega_c\) is set to be 0.5\(\pi\). The frequency response can be plotted as a "Diamond" shape as shown in Fig. 2.

If the intermediate notch frequency is below 0.5\(\pi\), Eq. 7 will be selected. Apart from those range, Eq. 8 is chosen instead. These actual notch frequencies are then used to implement final notch filters. Although, fast convergence time and more flexibilities are greatly achieved, there are still some ambiguous situations in conversion from intermediate to actual notch frequencies. The number of sub-bands is not sufficient to identify whether Eq. 7 or Eq. 8 will be used.

#### B. Circular Sub-band Filtering

Another alternative method is implemented based on the second norm criterion which is given by:

\[
\omega_c^2 = \omega_1^2 + \omega_2^2. \quad (10)
\]

\(\omega_c\) denotes a cut-off frequency which is set to be 0.5\(\pi\). The frequency response can be plotted as a "Circular" shape as shown in Fig. 3.

The area outside and inside a circle express the high and low sub-band respectively. Fig. 4 shows the ambiguous area which causes the problem to the system when conversion equation is used.
4 regions (cosets) as: Low-Low, Low-High, High-Low, and High-High. Fig. 6 illustrates a structure of 2-D ANF with quad sub-band filtering technique. The input signal is divided into 4 cosets according to 4 regions in frequency plane as shown in Fig. 5.

Then, the signal in each coset will be downsampled as usual to improve a adaptive convergence speed. A 2-D adaptive notch is subsequently conducted to reach the intermediate notch frequencies.

All intermediate notch frequencies in each coset will be converted to the actual notch frequencies by using conversion equation based on Table I.

<table>
<thead>
<tr>
<th>Dimensional Frequency</th>
<th>Low-Low</th>
<th>Low-High</th>
<th>High-Low</th>
<th>High-High</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1$</td>
<td>Eq. 7</td>
<td>Eq. 7</td>
<td>Eq. 8</td>
<td>Eq. 8</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>Eq. 7</td>
<td>Eq. 8</td>
<td>Eq. 7</td>
<td>Eq. 8</td>
</tr>
</tbody>
</table>

These four region of frequency plane are all possible cases of the input signal frequency. Therefore, the conversion equation can be assigned to each coset easily and correctly. Consequently, all ambiguous situation are solved.

IV. SIMULATIONS

A. Example 1: Ambiguous Situation in Diamond and Circular Sub-band Filtering

Assume $x(m, n)$ is a noisy image.

$$x(m, n) = 30 \sin(0.1\pi m + 0.2\pi n) + 30 \sin(0.4\pi m + 0.8\pi n) + v'(m, n).$$

By using the "Diamond" or "Circular" sub-band filtering, $(0.1\pi, 0.2\pi)$ and $(0.4\pi, 0.8\pi)$ will be classified as Low and High sub-band respectively. Eq. 7 is set to be used on low sub-band while Eq. 8 is conducted in another. As a result of downsamplers, the intermediate notch frequency are $(0.2\pi, 0.4\pi)$ and $(0.8\pi, 0.4\pi)$. On the low sub-band, the actual notch frequency is obtained as $(0.1\pi, 0.2\pi)$ by using Eq. 7 while the high sub-band results $(0.6\pi, 0.8\pi)$. It is obviously that wrong result is obtained in the high sub-band because of wrong conversion equation. Consequently, the output image is obtained as shown in Fig. 7.

B. Example 2: Solution to Ambiguous Situation by using Quad-band Filtering

This example the noisy image $x(m, n)$ is given by:

$$x(m, n) = 30 \sin(0.1\pi m + 0.2\pi n) + 30 \sin(0.4\pi m + 0.8\pi n) + 30 \sin(0.7\pi m + 0.2\pi n) + v'(m, n).$$

Each sub-band will convert the detected notch frequency to the actual notch frequency by using Table I. Fig. 8 shows the final output image.
V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

In conclusion, the quad sub-band filtering technique provides the solution to the ambiguous cases. By dividing input 2-D signal into four cosets (group of sub-bands), all possible ambiguous cases are clarified. The conversion equations are correctly specified for each coset. It is noticed that the number of sufficient sub-bands can be computed by $2^n$ where $n$ denotes a number of dimensions of an input signal.

B. Future Works

In the case of several cosets, the number of cosets indicates how complicate a hardware need to be built. Therefore, the smaller number of cosets seem to be preferred. A future work may lay on the reduction of the sub-band while the ambiguous cases can be classified simultaneously.

VI. ACKNOWLEDGMENTS

The authors gratefully acknowledge Telecommunications Research and Industrial Development Institute (TRIDI), National Broadcasting and Telecommunications Commissions for the financial support of this work.

REFERENCES


Fig. 6. A Block diagram of 2-D ANF with quad sub-band filtering technique.


Fig. 7. A implementation Result of Diamond Sub-band Filtering ANF with \((0.1\pi,0.2\pi)\) and \((0.4\pi,0.8\pi)\).

Fig. 8. A implementation Result of Quad-band Filtering Technique.