Jitter in IP Networks: A Cauchy Approach

L. Rizo-Domínguez, D. Torres-Roman, D. Munoz-Rodriguez, and C. Vargas-Rosales, Senior Member, IEEE

Abstract—Jitter is recognized as an important phenomenon that degrades the communication performance. Particularly, in real time services such as voice and video over the Internet, there is evidence that jitter departs from already proposed Laplacian models and that it has a heavy tail behavior. In this paper, we show that an Alpha-Stable jitter model is adequate, and that in some cases the Cauchy distribution provides a satisfactory approximation. Furthermore, this work shows how the jitter dispersion increases with the number of hops in the path, following a power law with scaling exponent dependent on the index of stability $\alpha$. This allows us to predict the expected QoS in terms of the number of nodes and traffic parameters.

Index Terms—Alpha-stable model, jitter, QoS.

I. INTRODUCTION

S mistiming is a major concern in telecommunication systems; it has been addressed, in the literature, from different perspectives. Since jitter impairs severely real-time applications such as videoconferencing, network gaming, VoIP (Voice over IP) and VIP (Video over IP), among others, delay and packet loss have been studied extensively. For instance, Fulton and Li, [1], deal with delay in ATM networks, while Qiong and Mills, [2], consider the jitter-bound estimation problem from the TCP perspective; and Daniel, et. al., [3], consider the Round Trip Time (RTT) in the Internet environment and suggest a Laplacian model.

For this study an ample data set of network delay measurements was obtained and examined. Conducted observations show that jitter has a behavior that departs from the Laplacian distribution, [3], thus a jitter model that matches the heavy tail behavior exhibited by packets traveling in the network is proposed. The model helps to determine the maximum number of allowable hops in an end-to-end path maintaining a specified QoS. This information is relevant for routing purposes, and for resource assignment and reservation. We describe the heavy tail jitter observations by a general alpha-stable representation, and show a description based on the Cauchy distribution that provides an accurate approximation. Applicability to QoS is also presented, and results comparing against network measurements, show strong agreement. The proposed jitter model is described in Section II. The evaluation scenarios are presented in Section III. The jitter accumulation law and its validation are introduced in Section IV. QoS based on the proposed model is discussed in Section V. Concluding remarks are given in Section VI.

II. JITTER MODEL

Jitter is defined as the difference of the trip delays of consecutive packets in an end-to-end connection. Under ideal conditions, all packets should undergo the same delay. However, due to traffic queueing, processing time variations in the nodes and even route changes, packets experience jitter, which can be expressed as

$$J_N(k) = D_N(k) - D_N(k - 1),$$

where $D_N(k)$ is the delay of the $k$-th packet as observed in the $N_i$-th node. A negative jitter is known as a packet clustering phenomenon, and a positive is known as packet spreading. Let $\xi_i(k)$ be the $i$-th stage deterministic delay (i.e., propagation, and processing times), and $W_i(k)$ be the $i$-th stage random delay (i.e., queueing and route change phenomena), then the end-to-end delay through the $N$ hops of the path is given by

$$D_N(k) = \sum_{i=1}^{N} [\xi_i(k) + W_i(k)].$$

Several studies have shown that network traffic exhibits long range dependence, [4]; this implies, according to [5], that the waiting time $W_i(k)$ in the queue is heavy tailed. This has been revealed through delay measurements whose distribution exhibits a Pareitian behavior, [7]; and in 1925 Lévy showed, [8], that Pareto laws belong to the so-called stable-Pareitian or stable non-Gaussian distributions. This implies that $W_i(k)$ can be modeled by an alpha-stable distribution [6], and then $D_N(k)$, in (2), converges to an alpha-stable distribution [6] as well when $W_i(k)$ are independent. This independence assumption is discussed in Section IV.

It is known that if two alpha-stable random variables are independent, then their difference is also alpha-stable, [6]. Thus, jitter becomes alpha-stable with characteristic function given by a symmetrical distribution with $\mu = 1$, $\beta = 0$, as

$$C^{\alpha,\beta}(\zeta)_{DN}(k) = \exp(-\gamma^\alpha |\zeta|^\alpha).$$

$D_N(k)$ has an alpha-stable distribution if its characteristic function is, [6],

$$C^{\alpha,\beta}(\zeta)_{DN}(k) = \exp(j\mu \zeta - \gamma^\alpha |\zeta|^\alpha \{1 - j\beta \text{sign}(\zeta) \alpha \omega(\zeta, \alpha)\}),$$

$$\omega(\zeta, \alpha) = \begin{cases} \tan(\frac{\alpha \pi}{2}), & \alpha \neq 1, \\ \frac{2}{\alpha} \log |\zeta|, & \alpha = 1, \end{cases}$$

where $\alpha$ is the index of stability, $\gamma$ the dispersion parameter, $\beta$ the skewness parameter and $\mu$ the shift parameter. There are three closed forms of alpha-stable distributions: the Gaussian distribution when $\alpha = 2$, the Cauchy distribution when $\alpha = 1$, $\beta = 0$, and Levy distribution when $\alpha = 0.5$, $\beta = 1$.  

Manuscript received March 24, 2009. The associate editor coordinating the review of this letter and approving it for publication was N. Nikolaou.

This work was partially sponsored by CONACyT.

L. Rizo and D. Torres are with Research Center and Advanced Studies, CINVESTAV, Guadalajara, Jal., Mexico (e-mail: {lrizo, dtorres}@gdl.cinvestav.mx).

D. Munoz and C. Vargas-Rosales are with ITESM-Campus Monterrey, Monterrey, N.L., 64849, Mexico (e-mail: {dmunoz, cvargas}@itesm.mx).

Digital Object Identifier 10.1109/LCOMM.2010.02.090702

1089-7798/10$25.00 c 2010 IEEE
In order to present a realistic jitter model, extensive delay measurements were conducted for several hops and paths. The observation setup involved international destinations located at six countries in different continents: Argentina, Australia, Japan, Mexico, France, and USA. All packets traveled through the USA. A set of some 7.2 million measurements taken along a 24-day period was examined. A description of the experiment and recorded data are available in [9]. The survival tail was studied, and typical results are presented in Figure 1; it can be seen that jitter does not fit the Laplacian nor Gaussian models, but tail exhibits a slow decay.

From a practical perspective, system performance forecast based on alpha-stable modeling can be cumbersome due to the inverse transform of the characteristic function, which does not have a close expression, but for very limited values of the stability indexes. However, observations show that the parameter can be close to one, thus Cauchy distribution can be considered, [10]. Figure 2 shows an example of a normalized histogram of the parameter alpha in a 21-node path, where the mean stability index is $E(\alpha) = 0.9716$

### III. Evaluation Scenarios

To illustrate the use of the model, we consider as QoS, among other criteria, [13], that the mean jitter be less than 30 ms for VoIP, and be kept under a maximum value $J_{max}$ for at least 99% of the transmitted packets. $J_{max}$ is set at 30 ms for VIP services. However, in heavy tail environments mean and variance may diverge and constraint $J_{max}$ may be more appropriately described in terms of distribution percentiles. This is $Q_{99} \leq P(J_{N} \leq J_{max})$. For alpha-stable jitter distributions, this can be expressed as the infinite series, [14],

$$P(J_{N} \leq J_{max}) = \frac{2}{\pi \alpha} \sum_{k=1}^{\infty} \frac{\Gamma(1+\psi(\alpha,k))}{k!} \frac{(J_{max} \gamma_{p}(n))^{-\alpha k}}{\sin \left(\frac{\alpha \pi k}{2}\right)},$$

(4)
where $\alpha$ and the QoS requirement can be expressed as $E_a$. Jitter-QoS level is given by \( |J_N| \leq J_{max} \)

Fig. 4. \( P(|J_N| \leq J_{max}) \) vs Number of nodes.

\[ \psi(\alpha, k) = \begin{cases} \alpha k, & 0 < \alpha < 1, \\ k/\alpha, & 1 \leq \alpha < 2, \end{cases} \]

where $\Gamma$ is the gamma function. It has been shown that when $\alpha \approx 1$, a Cauchy distribution is an adequate approximation and the QoS requirement can be expressed as

\[ QoS \leq P(|J_N| \leq J_{max}) = \frac{2 \arctan(J_{max}/\gamma_p(N))}{\pi}. \]

Figure 4 shows the percent of observed packets with a jitter below 30 ms for a given hop length in path Mex-USA-France; we see that (6) provides a good QoS approximation.

Also, substituting in (6) the cumulative dispersion $\gamma_p(N) = E(\gamma_i)N^{1/E(\alpha)}$, the maximum number of hops $N$ to guarantee a jitter-QoS level is given by

\[ N < \left[ \frac{J_{max}}{E(\gamma_i)\tan(\pi QoS/2)} \right]^{E(\alpha)}. \]

Since we consider the Cauchy distribution, $E(\alpha) = 1$. In practice, routing protocols must consider the maximum number of hops $N$ permitted in a path as a QoS criterion. This relationship is illustrated in Figure 5 for a QoS = 0.99, 0.96, 0.98 and 0.94 as a function of the mean jitter dispersion. The presented model captures the heavy tail behavior and the dispersion of jitter for different nodes in a path, and describes as well the jitter-QoS for $N$ nodes.

VI. CONCLUSIONS

In this paper, an IP network alpha-stable jitter model that exhibits a better fit than that of the exponential formulation was presented. It was shown through measurements that jitter is better described with our alpha-stable model by comparing it to network measurements.

When the stability index has a mean value close to one, a simplified model based on the Cauchy distribution is adequate. Jitter dispersion follows a power law of the number of nodes in the path with scaling exponent given by the stability index. The proposed models permit the estimation of jitter-QoS as a function of the number of nodes in the path, the stability index and the jitter dispersion.

REFERENCES