Abstract

The EKT method is a recent extraction technique of periodic signals from noise. It is a method presenting a great practical interest because of its easy realization and its high signal-to-noise ratio (SNR)—higher than that of the averaging method. However, it is only valid for signals of known period. Since in practice, periods are not always known with good precision, the EKT method remains limited. To enlarge its application field, we propose in this paper an adaptive EKT method, which could be applied to signals whose variation interval of the fundamental frequency is known.

Keywords: Adaptive EKT method; Averaging method; EKT method

1. Introduction

The EKT method is a rather powerful extraction technique of periodic signals from noise. It was shown [1,2] that its SNR remains higher than that of the averaging method (the averaging method is one of the most used method in signal processing [3–5]). Indeed, it is shown [4,5] that the averaging method can be assimilated to a cross-correlation of the periodic signal with a Dirac comb \( p(t) \) of the same period:

\[
p(t) = \sum_{k=-\infty}^{+\infty} \delta(t - kT).
\]

The Dirac comb is then used as reference signal. Only, taking into account its unlimited spectrum and the width of its spectral lines (Fig. 1), the Dirac comb appears unsuitable for the periodic signals extraction. This is because it causes a distortion due to the nonuniformity of its spectral lines amplitude (Fig. 1), and collects noise between two consecutive lines and beyond the signal spectrum (Fig. 2). Due to these limitations, was developed a new type of reference signal noted \( E(t) \), whose cross-correlation with the noisy periodic signal was called “the EKT method.” The reference signal used by the EKT method is a sum of harmonics of equal amplitudes. Contrary to the Dirac comb, the \( E(t) \) harmonics number is selected so as to cover exactly the useful band of the signal to be extracted. Moreover, the \( E(t) \) spectrum is a set of extremely fine lines and does not present ripples (Fig. 3). Its expression is given below:

\[
E(t) = \frac{\sin[(2m + 1)\pi v_0t]}{\sin(\pi v_0t)},
\]

* Corresponding author.
E-mail address: efoudaja@yahoo.fr (J.S.A. Eyebe Fouda).
Fig. 1. Dirac comb: (a) time representation; (b) magnitude spectrum.

Fig. 2. Extraction principle of a periodic signal from noise with \( p(t) \): (a) amplitude spectrum of the signal to be extracted; (b) amplitude spectrum of the noisy signal; (c) amplitude spectrum of the extracted signal with \( p(t) \).

Fig. 3. \( E(t) \) function: (a) time representation; (b) magnitude spectrum.

\( m \) being its harmonics number and \( v_0 \) its fundamental frequency. Of the spectral differences of \( p(t) \) and \( E(t) \) results an improvement of SNR gain of the averaging method. It is shown [2] that the SNR gain of the EKT method can be given by the following relation:

\[
G_E = \frac{F_e}{2mv_0} G_p,
\]

where \( F_e \) indicates the sampling frequency and \( G_p \) the SNR gain of the averaging method, given by [4]

\[
G_p = M,
\]

\( M \) being the number of period considered on the signal to be extracted. However, the averaging method and the EKT method necessitate a prior knowledge on the period of the signal to be extracted, which is not always possible in practice. We present in this paper an adaptive EKT method just as it was done for the averaging method [6–8]. The main problem to solve is avoiding a zero padding to the reference signal \( E(t) \), when varying the integration duration, as it is done traditionally with the other adaptive methods. Avoiding a zero padding to the \( E(t) \) function would allow us to preserve its spectrum properties, as defined in the EKT method. Moreover, when one wishes to work in real time, it can be defined only one observation duration, but we need more integration durations, since the period of the signal to be extracted is unknown. So, an appropriate use of periodic signals properties is required to carry out the adaptive EKT method.
2. Adaptive EKT method

The purpose of the adaptive EKT method that we present in this part is to extend the EKT method to periodic signals of unknown period. It is based on a judicious exploitation of periodic signals properties. The criterion highlighting it is the research of the maximum energy after cross-correlation with a well chosen reference signal. This criterion follows from the fundamental properties of periodic signals which are presented below.

2.1. First property ($P_1$)

2.1.1. Statement

The amplitude of harmonics of a truncated periodic signal spectrum, evaluated over duration higher or equal to one period, depends only on the observation duration of the signal.

2.1.2. Verification of $P_1$

Let

$$x(t) = \sum_{l=0}^{m} a_l \cos(2\pi lv_0 t)$$

be a periodic signal of period $T = 1/v_0$, its spectrum evaluated on a multiple positive integer of its period is given by the expression:

$$X_T(v) = \frac{a_0}{2} \delta(v) + \frac{1}{2} \sum_{l=-m}^{m} a_l \delta(v-lv_0).$$

It is a line spectrum. This signal is considered to be truncated if it is prolonged with zero samples or if its observation duration $D$ is different from a multiple integer of its period. If the spectrum of $x(t)$ is computed over a duration $D$ such as $D > T$, it becomes

$$X(v) = D \frac{\sin(\pi vD)}{\pi vD} * X_T(v).$$

According to this expression, it appears that $X(v)$ depends only on $D$. Figure 4 shows the amplitude variations of the $l = 3$ harmonic of $u(t)$ defined in (8), versus the observation duration $D$ and the interpolation duration $D_i$ (obtained by prolonging $D$ with zeros).

$$u(t) = \cos(4\pi t) + \sum_{l=0}^{7} \cos(2\pi kt).$$

Fig. 4. Amplitude variations of the $l = 3$ harmonic of $u(t)$ versus $D$ and $D_i$. 
As it can be observed on this figure, for a given observation duration, the amplitude of the considered harmonic remains unchanged for all interpolation duration, i.e., for any number of zeros added to the observed signal. Also the amplitude of \( l = 3 \) harmonic tends towards a finite value (its effective amplitude) when \( D \gg T \).

2.2. Second property (\( P_2 \))

2.2.1. Statement

The amplitude of the harmonic frequencies, in spite of truncation, remains unchanged if the observation duration \( D \) of the signal is taking as \( D = nT, \ n \in \mathbb{N}^* \).

2.2.2. Verification of \( P_2 \)

Expression (7) generally allows estimating the spectrum of a truncated periodic signal. However, when the duration \( D \) becomes a multiple positive integer of the signal period \( T \), the zero padding is reduced to a simple interpolation. So the spectrum of \( x(t) \) in (7) becomes

\[
X(v) = \frac{\sin(\pi vD)}{\pi vD} * X_T(v).
\]  
(9)

Equation (9) can still be written as

\[
X(v) = \frac{a_0}{2} \sin(\frac{\pi Dv}{v Dv}) + \frac{1}{2} \sum_{l=-m}^{m} a_l \left[ \frac{\sin(\pi D(v - l v_0))}{\pi D(v - l v_0)} \right].
\]  
(10)

While taking \( v = nv_0, \ n \in \mathbb{N} \) in (10), it becomes

\[
X(n) = \frac{a_0}{2} \sin(\frac{\pi D v_0 n}{D v_0}) + \frac{1}{2} \sum_{l=-m}^{m} a_l \left[ \frac{\sin(\pi D v_0(n - l))}{\pi D v_0(n - l)} \right].
\]  
(11)

\( D \) being a multiple integer of \( T \), (11) can also be written as

\[
X(n) = \frac{a_0}{2} \delta(n) + \frac{1}{2} \sum_{l=-m}^{m} a_l \delta(n - l).
\]  
(12)

\( X(n) \) are the Fourier coefficients of the \( x(t) \) harmonic lines. We verify through this expression that amplitudes of the harmonic lines remain unchanged. Figure 5 shows the amplitude variations of \( l = 2 \) and 3 harmonics of \( u(t) \) versus the observation duration of \( D = MT \) (\( M \) being a whole number) and the interpolation duration \( D_i \).

![Fig. 5. Amplitude variations of the \( l = 2 \) and 3 harmonics of \( u(t) \) versus \( D \) and \( D_i \), \( D \) being a whole number of \( T \).](image-url)
2.3. Third property \((P_3)\)

2.3.1. Statement

Let \(x(t)\) be a periodic signal of period \(T = 1/v_0\) made up of \(m\) harmonics, observed on a sufficiently large duration, \(D = NT\), and a reference signal \(E(t)\) of fundamental frequency \(v_1 = 1/T_1\) made up of \(m\) harmonics. The energy of the cross-correlation of \(x(t)\) and \(E(t)\) evaluated on \(D_i = MT\) such that, \(M \geq N\) \((M, N \in \mathbb{N}^+)\) is maximum if \(v_1 = v_0\).

2.3.2. Verification of \(P_3\)

The spectrum of \(E(t)\) is given by

\[
E(v) = \sum_{l=-m}^{m} \delta(v - lv_1). \tag{13}
\]

Let \(c(t)\) be the cross-correlation of \(E(t)\) and \(x(t)\), the integration duration being a multiple positive integer of \(T_1\) and the observation duration of \(x(t)\) a multiple positive integer of \(T\), its spectrum will be given by the expression:

\[
C(v) = \left(\frac{a_0 \sin(\pi Dv)}{\pi Dv} + \frac{1}{2} \sum_{l=-m}^{m} a_l \left[\frac{\sin(\pi D(v - lv_0))}{\pi D(v - lv_0)}\right]\right) \sum_{p=-m}^{m} \delta(v - pv_1). \tag{14}
\]

Equation (14) can also be written as

\[
C(v) = \sum_{p=-m}^{m} \left(\frac{a_0 \sin(\pi Dv)}{\pi Dv} + \frac{1}{2} \sum_{l=-m}^{m} a_l \left[\frac{\sin(\pi D(v - lv_0))}{\pi D(v - lv_0)}\right]\right) \delta(v - pv_1). \tag{15}
\]

Expression (15) shows that \(C(v)\) is a line spectrum, so \(c(t)\) is a periodic signal of period \(T_1\) whose power is given by

\[
\sigma^2(v_1) = a_0^2 + 2 \sum_{n=1}^{m} \left(\frac{a_0 \sin(\pi Dnv_1)}{2 \pi Dn} + \frac{1}{2} \sum_{l=-m}^{m} a_l \left[\frac{\sin(\pi D(nv_1 - lv_0))}{\pi D(nv_1 - lv_0)}\right]\right)^2. \tag{16}
\]

If \(v_1 \neq v_0\), apart of the harmonic of rank zero, the other \(c(t)\) spectrum lines appear on the sides of the \(x(t)\) spectrum main lobes. This justifies, according to \(P_2\) that, if \(D \gg T\), their amplitudes remain lower than that of the corresponding lines of the \(x(t)\) spectrum (Fig. 6). Hence, the power of \(c(t)\) remains lower than that of \(x(t)\). In fact, the power can only be maximal if the \(E(t)\) spectrum lines coincide with all the highest peaks of the \(x(t)\) spectrum, therefore the \(x(t)\) harmonics. In other words, according to Eq. (16), the power of \(c(t)\), when \(v_1 = v_0\) is reduced to the expression:

\[
\sigma^2 = a_0^2 + \sum_{n=1}^{m} \frac{a_n^2}{2}. \tag{17}
\]
Figure 7 shows the variations of the $c(t)$ power versus $v_1$ for some examples of signals whose characteristics are given below:

$$x_1(t) = \frac{1}{\sqrt{7}} \sum_{l=0}^{7} \cos \left( 2\pi lt + \frac{2\pi}{l+1} \right),$$

$$x_2(t) = \sum_{l=0}^{7} a_l \cos (2\pi l v_0 t), \quad v_0 = 1.5 \text{ Hz}, \quad \{a_l\} = \{0.1, 0.5, 0.3, 1, 2, 0.4, 0.8, 0.2\}. $$

$$x_3(t) = \sum_{l=0}^{7} a_l \cos (2\pi l v_0 t), \quad v_0 = 2 \text{ Hz}, \quad \{a_l\} = \{0.2, 0.4, 0.3, 0.8, 4, 0.4, 5, 0.2\}. $$

According to Fig. 7, it is noted that the power of each cross-correlation presents a maximum which is equal to the power obtained with expression (17).

2.4. Adaptive EKT method principle

The adaptive EKT method is based on the exploitation of the above presented properties. It consists of considering a duration $D$ of the noisy signal and varying the fundamental frequency of $E(t)$ in an interval which contains the fundamental frequency of the periodic signal to be extracted. The integration duration must however be equal to a multiple positive integer of the $E(t)$ period, in order to avoid a zero padding to the $E(t)$ function, which could modify its spectrum. So, adapting the EKT method to the extraction of an unknown periodic signal requires a choice of many integration durations. But, since user wishes to work in real time, integration durations must be different (except for the observation duration covering a multiple integer of the observed signal period) from the observation duration because, in this case, it can be defined only one observation duration. For this purpose, integration durations are obtained by zero padding to the observed signal such as it corresponds to a whole number of the reference signal period. After processing, the result to be retained is that presenting the highest energy. Figure 8 shows on the first line, from left to right, signals $x_1(t), x_2(t),$ and $x_3(t);$ on the second line, from left to right, the noisy versions of $x_1(t), x_2(t),$ and $x_3(t);$ on the third line, from left to right, the extracted versions of $x_1(t), x_2(t),$ and $x_3(t)$ by the adaptive EKT method. Figure 9 also indicates from left to right, power variations of the extracted versions of $x_1(t), x_2(t),$ and $x_3(t),$ versus the reference signal frequencies.

3. Discussion

The principle of the adaptive EKT method imposes (even if the fundamental frequency of the signal to be extracted is not known) having at least a prior knowledge of the interval in which the fundamental frequency could be com-
prised. Else, the maximum power research becomes painful and long, taking into account the computation duration of different iterations. For this reason, the frequencies intervals must preferably be as small as possible, which is not obvious in practice. Moreover, it is difficult, for a given observation, to choose $D$ equal to a multiple integer of the signal period $T$. When this condition is not respected, for $D$ not very large compared to $T$, it could appear an appreciable error on the fundamental frequency, taking into account the modification of the respective positions and amplitudes of the signal spectrum peaks. It is important to note that the adaptive technique above presented can be assimilated to
the EKT method only if there is no zero padding to the reference signal $E(t)$. When this condition is not respected, the result does not procure an important SNR gain as the EKT method.

The application of the adaptive EKT method to nonstationary signals however, remains difficult. In fact, the cross-correlation of $x(t)$ with $E(t)$ is a periodic signal. For a large observation duration $D$, the result producing the maximum energy will be a periodic signal of period equal to the average period of the various occurrences of the nonstationary signal. To avoid this periodization of the nonstationary signal, it would be necessary to choose short observation durations, preferably $T < D < 2T$. Unfortunately, this condition limits SNR gain of the method. Thus, it can only be applied to weakly deteriorated signals. There is thus a compromise between the observation duration and the SNR gain. Taking into account the signal contamination rate, in practice we will consider relatively large durations of nonstationarity. Figure 10 shows an example of a processed ECG. One sees on this figure that the original signal was filtered while maintaining positions of its amplitude peaks.

The practical realization of the adaptive EKT method needs choosing a variation step $\Delta v$ of the reference signal fundamental frequency. The choice of this variation step introduces an error $\sigma_v = \Delta v$ on the estimation of the signal to be extracted period. The extraction will be better for small values of $\Delta v$.

4. Conclusion

The EKT method is certainly powerful but, when the period of the signal to be extracted is not known or when the signal is nonstationary, it remains very limited. To give the possibility to the users of extending its use to the case of the unknown period signal still not controlled, we have developed in this paper an adaptive EKT method. It is simply the EKT method where one varies the period of the reference signal without zero padding, in order to obtain the maximum energy. The adaptive EKT method allows extracting a periodic signal of unknown period from noise and estimating its period. This technique supplements the effectiveness of the EKT method for periodic signals extraction. In prospect, we will apply the adaptive EKT method to the processing of the real signals.

References


Jean Sire Armand Eyebe Fouda obtained his Master degree with thesis in 2003 and his Ph.D. degree in 2007 in the University of Yaoundé I, Cameroon. His research interest is digital signal processing.

B.Z. Essimbi received the “Doctorat d’Etat” degree in electronics from the University of Yaoundé I, Cameroon. He is currently an Associate Professor in the Faculty of Science, University of Yaoundé I, Cameroon. His research interest includes optoelectronics and signal processing. Professor Essimbi is a Fellow of the Alexander von Humboldt-Shiftung.

Mbane Biouele was born in Abong-Mbang, Cameroon. He obtained his First Thesis in atmosphere physics in 1989 and his “Doctorat d’Etat” in 2005, both in atmosphere physics. He is currently senior lecturer at the University of Yaoundé I, Cameroon.